16. STATISTICAL TABLES

The following tables are presented in a format that is compatible with the needs of analytical chemists: the significance level $P = 0.05$ has been used in most cases, and it has been assumed that the number of measurements available is fairly small. Except where stated otherwise, these abbreviated tables have been taken, with permission, from *Elementary Statistics Tables* by Henry R. Neave, published by George Allen & Unwin Ltd. (Tables 1–3, 5–6, and 7–11). The reader requiring statistical data corresponding to significance levels and/or numbers of measurements not covered in the tables is referred to these sources.

Table 1 The *t*-distribution

Value of |t| for a confidence interval of:	*90%*	*95%*	*98%*	*99%*
Critical value of t for P values of:	*0.10*	*0.05*	*0.02*	*0.01*
Number of degrees of freedom				
1	6.31	12.71	31.82	63.66
2	2.92	4.30	6.96	9.92
3	2.35	3.18	4.54	5.84
4	2.13	2.78	3.75	4.60
5	2.02	2.57	3.36	4.03
6	1.94	2.45	3.14	3.71
7	1.89	2.36	3.00	3.50
8	1.86	2.31	2.90	3.36
9	1.83	2.26	2.82	3.25
10	1.81	2.23	2.76	3.17
12	1.78	2.18	2.68	3.05
14	1.76	2.14	2.62	2.98
16	1.75	2.12	2.58	2.92
18	1.73	2.10	2.55	2.88
20	1.72	2.09	2.53	2.85
30	1.70	2.04	2.46	2.75
50	1.68	2.01	2.40	2.68
∞	1.64	1.96	2.33	2.58

The critical values of $|t|$ are appropriate for a *two*-tailed test. For a *one*-tailed test the value is taken from the column for *twice* the desired *P*-value, e.g. for a one-tailed test, $P = 0.05$, 5 degrees of freedom, the critical value is read from the $P = 0.10$ column and is equal to 2.02.

Table 2 Critical values of F for a one-tailed test ($P = 0.05$)

v_1: v_2	1	2	3	4	5	6	7	8	9	10	12	15	20
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.619	4.558
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	3.284	3.218	3.150
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.936
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.845	2.774
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719	2.646
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.403	2.328
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276
17	4.451	3.592	3.197	2.965	2.810	2.699	3.614	2.548	2.494	2.450	2.381	2.308	2.230
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.155
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124

v_1 = number of degrees of freedom of the numerator and v_2 = number of degrees of freedom of the denominator.

Table 3 Critical values of F for a two-tailed test ($P = 0.05$)

v_1: v_2	1	2	3	4	5	6	7	8	9	10	12	15	20
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1
2	38.51	39.00	39.17	39.25	39.30	39.33.	39.36	39.37	39.39	39.40	39.41	39.43	39.45
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17
4	12.22	10.65	9.979	9.605	9.364	9.197	9.074	8.980	8.905	8.844	8.751	8.657	8.560
5	10.01	8.434	7.764	7.388	7.146	6.978	6.853	6.757	6.681	6.619	6.525	6.428	6.329
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461	5.366	5.269	5.168
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761	4.666	4.568	4.467
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295	4.200	4.101	3.999
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964	3.868	3.769	3.667
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717	3.621	3.522	3.419
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526	3.430	3.330	3.226
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374	3.277	3.177	3.073
13	6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250	3.153	3.053	2.948
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	2.209	3.147	3.050	2.949	2.844
15	6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060	2.963	2.862	2.756
16	6.115	4.687	4.077	3.729	3.502	3.341	3.219	3.125	3.049	2.986	2.889	2.788	2.681
17	6.042	4.619	4.011	3.665	3.438	3.277	3.156	3.061	2.985	2.922	2.825	2.723	2.616
18	5.978	4.560	3.954	3.608	3.382	3.221	3.100	3.005	2.929	2.866	2.769	2.667	2.559
19	5.922	4.508	3.903	3.559	3.333	3.172	3.051	2.956	2.880	2.817	2.720	2.617	2.509
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774	2.676	2.573	2.464

v_1 = number of degrees of freedom of the numerator and v_2 = number of degrees of freedom of the denominator.

Table 4 Critical values of Q ($P = 0.05$)

Sample size	*Critical value*
4	0.831
5	0.717
6	0.621
7	0.570
8	0.524
9	0.492
10	0.464

Taken from E.P. King, *J. Am. Statist. Assoc.*, 1958, **48**, 531, by permission of the American Statistical Association.

Table 5 Critical values of χ^2 ($P = 0.05$)

Number of degrees of freedom	*Critical value*
1	3.84
2	5.99
3	7.81
4	9.49
5	11.07
6	12.59
7	14.07
8	15.51
9	16.92
10	18.31

Table 6 The sign test

n	*r = 0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
4	0.063	0.313	0.688					
5	0.031	0.188	0.500					
6	0.016	0.109	0.344	0.656				
7	0.008	0.063	0.227	0.500				
8	0.004	0.035	0.144	0.363	0.637			
9	0.002	0.020	0.090	0.254	0.500			
10	0.001	0.011	0.055	0.172	0.377	0.623		
11	0.001	0.006	0.033	0.113	0.274	0.500		
12	0.000	0.003	0.019	0.073	0.194	0.387	0.613	
13	0.000	0.002	0.011	0.046	0.133	0.290	0.500	
14	0.000	0.001	0.006	0.029	0.090	0.212	0.395	0.605
15	0.000	0.000	0.004	0.018	0.059	0.151	0.304	0.500

The table uses the binomial distribution with $P = 0.5$ to give the probabilities of r or less successes for $n = 4$–15. These values correspond to a one-tailed sign test and should be doubled for a two-tailed test.

Table 7 Wilcoxon signed rank test. Critical values for the test statistic at $P = 0.05$

n	*One-tailed test*	*Two-tailed test*
5	0	NA
6	2	0
7	3	2
8	5	3
9	8	5
10	10	8
11	13	10
12	17	13
13	21	17
14	25	21
15	30	25

The null hypothesis can be rejected when the test statistic is $\leq$ the tabulated value. NA indicates that the test cannot be applied.

Table 8 Wilcoxon rank sum test; Mann-Whitney U-test. Critical values for U or the lower of T_1 and T_2 at $P = 0.05$

n_1	n_2	*One-tailed test*	*Two-tailed test*
3	3	0	NA
3	4	0	NA
3	5	1	0
3	6	2	1
4	4	1	0
4	5	2	1
4	6	3	2
4	7	4	3
5	5	4	2
5	6	5	3
5	7	6	5
6	6	7	5
6	7	8	6
7	7	11	8

The null hypothesis can be rejected when U or the lower T value is $\leq$ the tabulated value. NA indicates that the test cannot be applied.

Table 9 The Spearman rank correlation coefficient. Critical values for ρ at $P = 0.05$

n	*One-tailed test*	*Two-tailed test*
5	0.900	1.000
6	0.829	0.886
7	0.714	0.786
8	0.643	0.738
9	0.600	0.700
10	0.564	0.649
11	0.536	0.618
12	0.504	0.587
13	0.483	0.560
14	0.464	0.538
15	0.446	0.521
16	0.429	0.503
17	0.414	0.488
18	0.401	0.472
19	0.391	0.460
20	0.380	0.447

Table 10 The Kolmogorov goodness of fit test

n	*One-tailed test*	*Two-tailed test*
1	0.950	0.975
2	0.776	0.842
3	0.636	0.708
4	0.565	0.624
5	0.509	0.563
6	0.468	0.519
7	0.436	0.483
8	0.410	0.454
9	0.388	0.430
10	0.369	0.409
11	0.352	0.392
12	0.338	0.375
13	0.326	0.361
14	0.314	0.349
15	0.304	0.338
16	0.295	0.327
17	0.286	0.318
18	0.278	0.309
19	0.271	0.301
20	0.265	0.294

Critical values for one-tailed and two-tailed tests at $P = 0.05$. The appropriate value is compared with the maximum difference between the experimental and theoretical cumulative frequency curves.

Table 11 The Kolomogorov test for normality

n	*One-tailed test*	*Two-tailed test*
3	0.367	0.376
4	0.345	0.375
5	0.319	0.343
6	0.297	0.323
7	0.280	0.304
8	0.265	0.288
9	0.252	0.274
10	0.241	0.262
11	0.231	0.251
12	0.222	0.242
13	0.215	0.234
14	0.208	0.226
15	0.201	0.219
16	0.195	0.213
17	0.190	0.207
18	0.185	0.202
19	0.181	0.197
20	0.176	0.192

Critical values for one-tailed and two-tailed tests at $P = 0.05$. The approapriate value is compared with the maximum difference between the experimental and theoretical cumulative frequency curves.