CRYPTOGRAPHY ENGINEERING

Design Principles and Practical Applications

Niels Ferguson
Bruce Schneier
Tadayoshi Kohno

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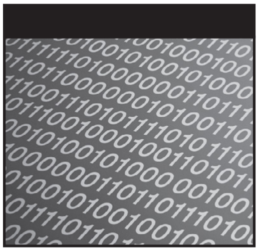

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To Denise, who has made me truly happy.

—Niels Ferguson

To Karen; still, after all these years.

—Bruce Schneier

To Taryn, for making everything possible.

—Tadayoshi Kohno

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About the Authors

Niels Ferguson has spent his entire career working as a cryptographic engineer. After studying mathematics in Eindhoven, he worked for DigiCash analyzing, designing, and implementing advanced electronic payment systems that protect the privacy of the user. Later he worked as a cryptographic consultant for Counterpane and MacFergus, analyzing hundreds of systems and designing dozens. He was part of the team that designed the Twofish block cipher, performed some of the best initial analysis of AES, and co-designed the encryption system currently used by WiFi. Since 2004 he works at Microsoft where he helped design and implement the BitLocker disk encryption system. He currently works in the Windows cryptography team that is responsible for the cryptographic implementations in Windows and other Microsoft products.

Bruce Schneier is an internationally renowned security technologist, referred to by *The Economist* as a ''security guru.'' He is the author of eight books—including the best sellers *Beyond Fear: Thinking Sensibly about Security in an Uncertain World*, *Secrets and Lies*, and *Applied Cryptography*—as well as hundreds of articles and essays in national and international publications, and many more academic papers. His influential newsletter *Crypto-Gram*, and his blog *Schneier on Security*, are read by over 250,000 people. He is a frequent guest on television and radio, and is regularly quoted in the press on issues surrounding security and privacy. He has testified before Congress on multiple occasions, and has served on several government technical committees. Schneier is the Chief Security Technology Officer of BT.

Tadayoshi Kohno (Yoshi) is an assistant professor of computer science and engineering at the University of Washington. His research focuses on improving the security and privacy properties of current and future technologies. He conducted the initial security analysis of the Diebold AccuVote-TS electronic voting machine source code in 2003, and has since turned his attention to securing emerging technologies ranging from wireless implantable pacemakers and defibrillators to cloud computing. He is the recipient of a National Science Foundation CAREER Award and an Alfred P. Sloan Research Fellowship. In 2007 he was awarded the MIT Technology Review TR-35 Award for his work in applied cryptography, recognizing him as one of the world's top innovators under the age of 35. He received his PhD in computer science from the University of California at San Diego.

Niels, Bruce, and Yoshi are part of the team that designed the Skein hash function, one of the competitors in NIST's SHA-3 competition.

Acknowledgments for *Cryptography Engineering*

We are deeply indebted to the cryptography and security community at large. This book would not have been possible without all of their efforts in advancing the field. This book also reflects our knowledge and experience as cryptographers, and we are deeply grateful to our peers and mentors for helping shape our understanding of cryptography.

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We are also indebted to all the other wonderful people in our lives who worked silently behind the scenes to make this book possible.

Acknowledgments for *Practical Cryptography* (the 1st Edition)

This book is based on our collective experience over the many years we have worked in cryptography. We are heavily indebted to all the people we worked with. They made our work fun and helped us reach the insights that fill this book. We would also like to thank our customers, both for providing the funding that enabled us to continue our cryptography research and for providing the real-world experiences necessary to write this book.

Certain individuals deserve special mention. Beth Friedman conducted an invaluable copyediting job, and Denise Dick greatly improved our manuscript by proofreading it. John Kelsey provided valuable feedback on the cryptographic contents. And the Internet made our collaboration possible. We would also like to thank Carol Long and the rest of the team at Wiley for bringing our ideas to reality.

And finally, we would like to thank all of the programmers in the world who continue to write cryptographic code and make it available, free of charge, to the world.

Contents at a Glance

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Preface to *Cryptography Engineering*

Most books cover what cryptography is—what current cryptographic designs are and how existing cryptographic protocols, like SSL/TLS, work. Bruce Schneier's earlier book, *Applied Cryptography*, is like this. Such books serve as invaluable references for anyone working with cryptography. But such books are also one step removed from the needs of cryptography and security engineers in practice. Cryptography and security engineers need to know more than how current cryptographic protocols work; they need to know how to use cryptography.

To know how to use cryptography, one must learn to think like a cryptographer. This book is designed to help you achieve that goal. We do this through immersion. Rather than broadly discuss all the protocols one might encounter in cryptography, we dive deeply into the design and analysis of specific, concrete protocols. We walk you—hand-in-hand—through how we go about designing cryptographic protocols. We share with you the reasons we make certain design decisions over others, and point out potential pitfalls along the way.

By learning how to think like a cryptographer, you will also learn how to be a more intelligent user of cryptography. You will be able to look at existing cryptography toolkits, understand their core functionality, and know how to use them. You will also better understand the challenges involved with cryptography, and how to think about and overcome those challenges.

This book also serves as a gateway to learning about computer security. Computer security is, in many ways, a superset of cryptography. Both computer security and cryptography are about designing and evaluating objects (systems or algorithms) intended to behave in certain ways even in the presence of an adversary. In this book, you will learn how to think about the adversary in the context of cryptography. Once you know how to think like adversaries, you can apply that mindset to the security of computer systems in general.

History

This book began with *Practical Cryptography* by Niels Ferguson and Bruce Schneier, and evolved with the addition of Tadayoshi Kohno—Yoshi—as an author. Yoshi is a professor of computer science and engineering at the University of Washington, and also a past colleague of Niels and Bruce. Yoshi took *Practical Cryptography* and revised it to be suitable for classroom use and self-study, while staying true to the goals and themes of Niels's and Bruce's original book.

Example Syllabi

There are numerous ways to read this book. You can use it as a self-study guide for applied cryptographic engineering, or you can use it in a course. A quarter- or semester-long course on computer security might use this book as the foundation for a 6-week intensive unit on cryptography. This book could also serve as the foundation for a full quarter- or semester-long course on cryptography, augmented with additional advanced material if time allows. To facilitate classroom use, we present several possible syllabi below.

The following syllabus is appropriate for a 6-week intensive unit on cryptography. For this 6-week unit, we assume that the contents of Chapter 1 are discussed separately, in the broader context of computer security in general.

- **Week 1:** Chapters 2, 3, and 4;
- **Week 2:** Chapters 5, 6, and 7;
- **Week 3:** Chapters 8, 9, and 10;
- **Week 4:** Chapters 11, 12, and 13;
- **Week 5:** Chapters 14, 15, 16, and 17;
- **Week 6:** Chapters 18, 19, 20, and 21.

The following syllabus is for a 10-week quarter on cryptography engineering.

- **Week 1:** Chapters 1 and 2;
- **Week 2:** Chapters 3 and 4;

- **Week 3:** Chapters 5 and 6;
- **Week 4:** Chapters 7 and 8;
- **Week 5:** Chapters 9 and 10;
- **Week 6:** Chapters 11 and 12;
- **Week 7:** Chapters 13 and 14;
- **Week 8:** Chapters 15, 16, and 17;
- **Week 9:** Chapters 18, 19, 20;
- **Week 10:** Chapter 21.

The following syllabus is appropriate for schools with 12-week semesters. It can also be augmented with advanced materials in cryptography or computer security for longer semesters.

- **Week 1:** Chapters 1 and 2;
- **Week 2:** Chapters 3 and 4;
- **Week 3:** Chapters 5 and 6;
- **Week 4:** Chapter 7;
- **Week 5:** Chapters 8 and 9;
- **Week 6:** Chapters 9 (continued) and 10;
- **Week 7:** Chapters 11 and 12;
- **Week 8:** Chapters 13 and 14;
- **Week 9:** Chapters 15 and 16;
- **Week 10:** Chapters 17 and 18;
- **Week 11:** Chapters 19 and 20;
- **Week 12:** Chapter 21.

This book has several types of exercises, and we encourage readers to complete as many of these exercises as possible. There are traditional exercises designed to test your understanding of the technical properties of cryptography. However, since our goal is to help you learn how to think about cryptography in real systems, we have also introduced a set of non-traditional exercises (see Section 1.12). Cryptography doesn't exist in isolation; rather, cryptography is only part of a larger ecosystem consisting of other hardware and software systems, people, economics, ethics, cultural differences, politics, law, and so on. Our non-traditional exercises are explicitly designed to force you to think about cryptography in the context of real systems and the surrounding ecosystem. These exercises will provide you with an opportunity to directly apply the contents of this book as thought exercises to real systems. Moreover, by weaving these exercises together throughout this book, you will be able to see your knowledge grow as you progress from chapter to chapter.

Additional Information

While we strove to make this book as error-free as possible, errors have undoubtedly crept in. We maintain an online errata list for this book. The procedure for using this errata list is below.

- Before reading this book, go to `http://www.schneier.com/ce.html` and download the current list of corrections.
- If you find an error in the book, please check to see if it is already on the list.
- If it is not on the list, please alert us at `cryptographyengineering@schneier.com`. We will add the error to the list.

We wish you a wonderful journey through cryptography engineering. Cryptography is a wonderful and fascinating topic. We hope you learn a great deal from this book, and come to enjoy cryptography engineering as much as we do.

October 2009

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Preface to *Practical Cryptography* (the 1st Edition)

In the past decade, cryptography has done more to damage the security of digital systems than it has to enhance it. Cryptography burst onto the world stage in the early 1990s as the securer of the Internet. Some saw cryptography as a great technological equalizer, a mathematical tool that would put the lowliest privacy-seeking individual on the same footing as the greatest national intelligence agencies. Some saw it as the weapon that would bring about the downfall of nations when governments lost the ability to police people in cyberspace. Others saw it as the perfect and terrifying tool of drug dealers, terrorists, and child pornographers, who would be able to communicate in perfect secrecy. Even those with more realistic attitudes imagined cryptography as a technology that would enable global commerce in this new online world.

Ten years later, none of this has come to pass. Despite the prevalence of cryptography, the Internet's national borders are more apparent than ever. The ability to detect and eavesdrop on criminal communications has more to do with politics and human resources than mathematics. Individuals still don't stand a chance against powerful and well-funded government agencies. And the rise of global commerce had nothing to do with the prevalence of cryptography.

For the most part, cryptography has done little more than give Internet users a false sense of security by promising security but not delivering it. And that's not good for anyone except the attackers.

The reasons for this have less to do with cryptography as a mathematical science, and much more to do with cryptography as an engineering discipline. We have developed, implemented, and fielded cryptographic systems over the past decade. What we've been less effective at is converting the mathematical promise of cryptographic security into a reality of security. As it turns out, this is the hard part.

Too many engineers consider cryptography to be a sort of magic security dust that they can sprinkle over their hardware or software, and which will imbue those products with the mythical property of ''security.'' Too many consumers read product claims like ''encrypted'' and believe in that same magic security dust. Reviewers are no better, comparing things like key lengths and on that basis, pronouncing one product to be more secure than another.

Security is only as strong as the weakest link, and the mathematics of cryptography is almost never the weakest link. The fundamentals of cryptography are important, but far more important is how those fundamentals are implemented and used. Arguing about whether a key should be 112 bits or 128 bits long is rather like pounding a huge stake into the ground and hoping the attacker runs right into it. You can argue whether the stake should be a mile or a mile-and-a-half high, but the attacker is simply going to walk around the stake. Security is a broad stockade: it's the things around the cryptography that make the cryptography effective.

The cryptographic books of the last decade have contributed to that aura of magic. Book after book extolled the virtues of, say, 112-bit triple-DES without saying much about how its keys should be generated or used. Book after book presented complicated protocols for this or that without any mention of the business and social constraints within which those protocols would have to work. Book after book explained cryptography as a pure mathematical ideal, unsullied by real-world constraints and realities. But it's exactly those real-world constraints and realities that mean the difference between the promise of cryptographic magic and the reality of digital security.

Practical Cryptography is also a book about cryptography, but it's a book about sullied cryptography. Our goal is to explicitly describe the real-world constraints and realities of cryptography, and to talk about how to engineer secure cryptographic systems. In some ways, this book is a sequel to Bruce Schneier's first book, *Applied Cryptography*, which was first published ten years ago. But while *Applied Cryptography* gives a broad overview of cryptography and the myriad possibilities cryptography can offer, this book is narrow and focused. We don't give you dozens of choices; we give you one option and tell you how to implement it correctly. *Applied Cryptography* displays the wondrous possibilities of cryptography as a mathematical science—what is possible and what is attainable; *Practical Cryptography* gives concrete advice to people who design and implement cryptographic systems.

Practical Cryptography is our attempt to bridge the gap between the promise of cryptography and the reality of cryptography. It's our attempt to teach engineers how to use cryptography to increase security.

We're qualified to write this book because we're both seasoned cryptographers. Bruce is well known from his books *Applied Cryptography* and *Secrets and Lies*, and from his newsletter ''Crypto-Gram.'' Niels Ferguson cut his cryptographic teeth building cryptographic payment systems at the CWI (Dutch National Research Institute for Mathematics and Computer Science) in Amsterdam, and later at a Dutch company called DigiCash. Bruce designed the Blowfish encryption algorithm, and both of us were on the team that designed Twofish. Niels's research led to the first example of the current generation of efficient anonymous payment protocols. Our combined list of academic papers runs into three digits.

More importantly, we both have extensive experience in designing and building cryptographic systems. From 1991 to 1999, Bruce's consulting company Counterpane Systems provided design and analysis services to some of the largest computer and financial companies in the world. More recently, Counterpane Internet Security, Inc., has provided Managed Security Monitoring services to large corporations and government agencies worldwide. Niels also worked at Counterpane before founding his own consulting company, MacFergus. We've seen cryptography as it lives and breathes in the real world, as it flounders against the realities of engineering or even worse, against the realities of business. We're qualified to write this book because we've had to write it again and again for our consulting clients.

How to Read this Book

Practical Cryptography is more a narrative than a reference. It follows the design of a cryptographic system from the specific algorithm choices, outwards through concentric rings to the infrastructure required to make it work. We discuss a single cryptographic problem—one of establishing a means for two people to communicate securely—that's at the heart of almost every cryptographic application. By focusing on one problem and one design philosophy for solving that problem, it is our belief that we can teach more about the realities of cryptographic engineering.

We think cryptography is just about the most fun you can have with mathematics. We've tried to imbue this book with that feeling of fun, and we hope you enjoy the results. Thanks for coming along on our ride.

Niels Ferguson
Bruce Schneier
January 2003

Part

I

Introduction

In This Part

CHAPTER

1

The Context of Cryptography

Cryptography is the art and science of encryption. At least, that is how it started out. Nowadays it is much broader, covering authentication, digital signatures, and many more elementary security functions. It is still both an art and a science: to build good cryptographic systems requires a scientific background and a healthy dose of the black magic that is a combination of experience and the right mentality for thinking about security problems. This book is designed to help you cultivate these critical ingredients.

Cryptography is an extremely varied field. At a cryptography research conference, you can encounter a wide range of topics, including computer security, higher algebra, economics, quantum physics, civil and criminal law, statistics, chip designs, extreme software optimization, politics, user interface design, and everything in between. In some ways, this book concentrates on only a very small part of cryptography: the practical side. We aim to teach you how to implement cryptography in real-world systems. In other ways, this book is much broader, helping you gain experience in security engineering and nurturing your ability to think about cryptography and security issues like a security professional. These broader lessons will help you successfully tackle security challenges, whether directly related to cryptography or not.

The variety in this field is what makes cryptography such a fascinating area to work in. It is really a mixture of widely different fields. There is always something new to learn, and new ideas come from all directions. It is also one of the reasons why cryptography is so difficult. It is impossible to understand it all. There is nobody in the world who knows everything about cryptography. There isn't even anybody who knows most of it. We certainly don't know everything there is to know about the subject of this book. So here is your first lesson in cryptography: keep a critical mind. Don't blindly trust anything, even if it is in print. You'll soon see that having this critical mind is an essential ingredient of what we call ''professional paranoia.''

1.1 The Role of Cryptography

Cryptography by itself is fairly useless. It has to be part of a much larger system. We like to compare cryptography to locks in the physical world. A lock by itself is a singularly useless thing. It needs to be part of a much larger system. This larger system can be a door on a building, a chain, a safe, or something else. This larger system even extends to the people who are supposed to use the lock: they need to remember to actually lock it and to not leave the key around for anyone to find. The same goes for cryptography: it is just a small part of a much larger security system.

Even though cryptography is only a small part of the security system, it is a very critical part. Cryptography is the part that has to provide access to some people but not to others. This is very tricky. Most parts of the security system are like walls and fences in that they are designed to keep everybody out. Cryptography takes on the role of the lock: it has to distinguish between ''good'' access and ''bad'' access. This is much more difficult than just keeping everybody out. Therefore, the cryptography and its surrounding elements form a natural point of attack for any security system.

This does not imply that cryptography is always the weak point of a system. In some cases, even bad cryptography can be much better than the rest of the security system. You have probably seen the door to a bank vault, at least in the movies. You know, 10-inch-thick, hardened steel, with huge bolts to lock it in place. It certainly looks impressive. We often find the digital equivalent of such a vault door installed in a tent. The people standing around it are arguing over how thick the door should be, rather than spending their time looking at the tent. It is all too easy to spend hours arguing over the exact key length of cryptographic systems, but fail to notice or fix buffer overflow vulnerabilities in a Web application. The result is predictable: the attackers find a buffer overflow and never bother attacking the cryptography. Cryptography is only truly useful if the rest of the system is also sufficiently secure against the attackers.

There are, however, reasons why cryptography is important to get right, even in systems that have other weaknesses. Different weaknesses are useful to different attackers in different ways. For example, an attacker who breaks the cryptography has a low chance of being detected. There will be no traces of the attack, since the attacker's access will look just like a ''good'' access. This is comparable to a real-life break-in. If the burglar uses a crowbar to break in, you will at least see that a break-in has occurred. If the burglar picks the lock, you might never find out that a burglary occurred. Many modes of attack leave traces, or disturb the system in some way. An attack on the cryptography can be fleeting and invisible, allowing the attacker to come back again and again.

1.2 The Weakest Link Property

Print the following sentence in a very large font and paste it along the top of your monitor.

A security system is only as strong as its weakest link.

Look at it every day, and try to understand the implications. The weakest link property is one of the main reasons why security systems are so fiendishly hard to get right.

Every security system consists of a large number of parts. We must assume that our opponent is smart and that he is going to attack the system at the weakest part. It doesn't matter how strong the other parts are. Just as in a chain, the weakest link will break first. It doesn't matter how strong the other links in the chain are.

Niels used to work in an office building where all the office doors were locked every night. Sounds very safe, right? The only problem was that the building had a false ceiling. You could lift up the ceiling panels and climb over any door or wall. If you took out the ceiling panels, the whole floor looked like a set of tall cubicles with doors on them. And these doors had locks. Sure, locking the doors made it slightly harder for the burglar, but it also made it harder for the security guard to check the offices during his nightly rounds. It isn't clear at all whether the overall security was improved or made worse by locking the doors. In this example, the weakest link property prevented the locking of the doors from being very effective. It might have improved the strength of a particular link (the door), but there was another link (the ceiling) that was still weak. The overall effect of locking the doors was at best very small, and its negative side effects could well have exceeded its positive contribution.

To improve the security of a system, we must improve the weakest link. But to do that, we need to know what the links are and which ones are weak. This is best done using a hierarchical tree structure. Each part of a system has multiple links, and each link in turn has sublinks. We can organize the links into what we call an *attack tree* [113]. We give an example in Figure 1.1. Let's say that we want to break into a bank vault. The first-level links are the walls, the floor, the door, and the ceiling. Breaking through any one of them gets us into the vault. Let’s look at the door in more detail. The door system has its own links: the connection between the door frame and the walls, the lock, the door itself, the bolts that keep the door in the door frame, and the hinges. We could continue by discussing individual lines of attack on the lock, one of which is to acquire a key, which in turn leads to a whole tree about stealing the key in some way.

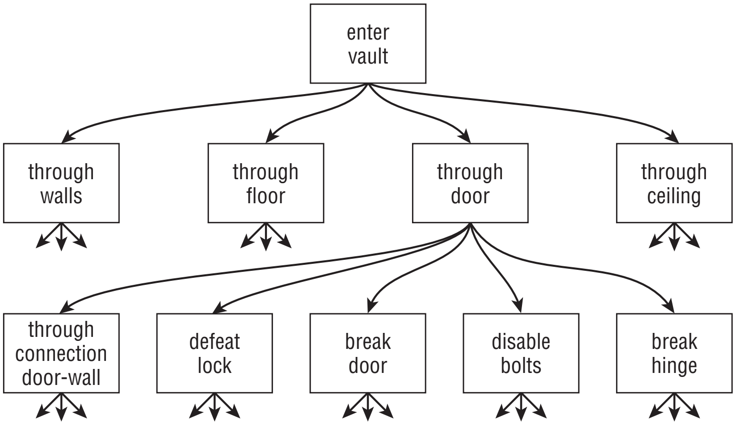

Figure 1.1: Example attack tree for a vault

We can analyze each link and split it up into other links until we are left with single components. Doing this for a real system can be an enormous amount of work. If we were concerned about an attacker stealing the diamonds stored in the vault, then Figure 1.1 is also just one piece of a larger attack tree; an attacker could trick an employee into removing the diamonds from the vault and steal them once removed. Attack trees provide valuable insight as to possible lines of attack. Trying to secure a system without first doing such an analysis very often leads to useless work. In this book, we work only on limited components—the ones that can be solved with cryptography—and we will not explicitly talk about their attack trees. But you should be certain to understand how to use an attack tree to study a larger system and to assess the role of cryptography in that system.

The weakest link property affects our work in many ways. For example, it is tempting to assume that users have proper passwords, but in practice they don’t. They often choose simple short passwords. Users may go to almost any length not to be bothered by security systems. Writing a password on a sticky note and attaching it to their monitor is just one of many things they might do. You can never ignore issues like this because they always affect the end result. If you design a system that gives users a new 12-digit random password every week, you can be sure they will stick it on their monitors. This weakens an already weak link, and is bad for the overall security of the system.

Strictly speaking, strengthening anything but the weakest link is useless. In practice, things are not so clear-cut. The attacker may not know what the weakest link is and attack a slightly stronger one. The weakest link may be different for different types of attackers. The strength of any link depends on the attacker's skill and tools and access to the system. The link an attacker might exploit may also depend on the attacker's goals. So which link is the weakest depends on the situation. It is therefore worthwhile to strengthen any link that could in a particular situation be the weakest. Moreover, it's worth strengthening multiple links so that if one link does fail, the remaining links can still provide security—a property known as *defense in depth*.

1.3 The Adversarial Setting

One of the biggest differences between security systems and almost any other type of engineering is the adversarial setting. Most engineers have to contend with problems like storms, heat, and wear and tear. All of these factors affect designs, but their effect is fairly predictable to an experienced engineer. Not so in security systems. Our opponents are intelligent, clever, malicious, and devious; they'll do things nobody had ever thought of before. They don't play by the rules, and they are completely unpredictable. That is a much harder environment to work in.

Many of us remember the film in which the Tacoma Narrows suspension bridge wobbles and twists in a steady wind until it breaks and falls into the water. It is a famous piece of film, and the collapse taught bridge engineers a valuable lesson. Slender suspension bridges can have a resonance mode in which a steady wind can cause the whole structure to oscillate, and finally break. How do they prevent the same thing from happening with newer bridges? Making the bridge significantly stronger to resist the oscillations would be too expensive. The most common technique used is to change the aerodynamics of the bridge. The deck is made thicker, which makes it much harder for the wind to push up and down on the deck. Sometimes railings are used as spoilers to make the bridge deck behave less like a wing that lifts up in the wind. This works because wind is fairly predictable, and does not change its behavior in an active attempt to destroy the bridge.

A security engineer has to take a malicious wind into account. What if the wind blows up and down instead of just from the side, and what if it changes directions at the right frequency for the bridge to resonate? Bridge engineers will dismiss this kind of talk out of hand: ''Don't be silly, the wind doesn't blow that way.'' That certainly makes the bridge engineers' jobs much easier. Cryptographers don't have that luxury. Security systems are attacked by clever and malicious attackers. We have to consider all types of attack.

The adversarial setting is a very harsh environment to work in. There are no rules in this game, and the deck is stacked against us. We talk about an ''attacker'' in an abstract sense, but we don't know who she is, what she knows, what her goal is, when she will attack, or what her resources are. Since the attack may occur long after we design the system, she has the advantage of five or ten years' more research, and can use technology of the future that is not available to us. And with all those advantages, she only has to find a single weak spot in our system, whereas we have to protect all areas. Still, our mission is to build a system that can withstand it all. This creates a fundamental imbalance between the attacker of a system and the defender. This is also what makes the world of cryptography so exciting.

1.4 Professional Paranoia

To work in this field, you have to become devious yourself. You have to think like a malicious attacker to find weaknesses in your own work. This affects the rest of your life as well. Everybody who works on practical cryptographic systems has experienced this. Once you start thinking about how to attack systems, you apply that to everything around you. You suddenly see how you could cheat the people around you, and how they could cheat you. Cryptographers are professional paranoids. It is important to separate your professional paranoia from your real-world life so as to not go completely crazy. Most of us manage to preserve some sanity . . . we think.[1] In fact, we think that this practical paranoia can be a lot of fun. Developing this mindset will help you observe things about systems and your environment that most other people don't notice.

Paranoia is very useful in this work. Suppose you work on an electronic payment system. There are several parties involved in this system: the customer, the merchant, the customer's bank, and the merchant's bank. It can be very difficult to figure out what the threats are, so we use the paranoia model. For each participant, we assume that everybody else is part of a big conspiracy to defraud this one participant. And we also assume that the attacker might have any number of other goals, such as compromising the privacy of a participant's transactions or denying a participant's access to the system at a critical time. If your cryptographic system can survive the paranoia model, it has at least a fighting chance of surviving in the real world.

We will interchangeably refer to professional paranoia and the paranoia model as the security mindset.

[1] But remember: the fact that *you* are not paranoid doesn't mean *they* are not out to get you or compromise your system.

1.4.1 Broader Benefits

Once you develop a sense of professional paranoia, you will never look at systems the same way. This mindset will benefit you throughout your career, regardless of whether you become a cryptographer or not. Even if you don't become a cryptographer, you may someday find yourself working on the design, implementation, or evaluation of new computer software or hardware systems. If you have the security mindset, then you will be constantly thinking about what an attacker might try to do to your system. This will nicely position you to identify potential security problems with these systems early. You may not always be able to fix all of the security problems by yourself, but that's all right. The most important thing is to realize that a security problem might exist. Once you do that, it becomes a straightforward task to find others to help you fix the problem. But without the security mindset, you might never realize that your system has security problems and, therefore, you obviously can't protect against those problems in a principled way.

Technologies also change very rapidly. This means that some hot security mechanisms of today may be outdated in 10 or 15 years. But if you can learn how to think about security issues and have an appreciation for adversaries, then you can take that security mindset with you for the rest of your life and apply it to new technologies as they evolve.

1.4.2 Discussing Attacks

Professional paranoia is an essential tool of the trade. With any new system you encounter, the first thing you think of is how you can break it. The sooner you find a weak spot, the sooner you learn more about the new system. Nothing is worse than working on a system for years, only to have somebody come up and say: ''But how about if I attack it this way . . . ?'' You really don't want to experience that ''Oops'' moment.

In this field, we make a very strict distinction between attacking somebody's work and attacking somebody personally. Any work is fair game. If somebody proposes something, it is an automatic invitation to attack it. If you break one of our systems, we will applaud the attack and tell everybody about it.[2] We constantly look for weaknesses in any system because that is the only way to learn how to make more secure systems. This is one thing you will have to learn: an attack on your work is not an attack on you. Also, when you attack a system, always be sure to criticize the system, not the designers. Personal attacks in cryptography will get you the same negative response as anywhere else.

But be aware that this acceptance of attacks may not extend to everyone working on a system—particularly if they are not familiar with the field

[2] Depending on the attack, we might kick ourselves for not finding the weakness ourselves, but that is a different issue.

of cryptography and computer security. Without experience in the security community, it is very easy for people to take criticism of their work as a personal attack, with all the resulting problems. It is therefore important to develop a diplomatic approach, even if it makes it initially difficult to get the message across. Being too vague and saying something like ''There might be some issues with the security aspects'' may not be productive, since it may get a noncommittal response like ''Oh, we'll fix it,'' even if the basic design is fundamentally flawed. Experience has shown us that the best way to get the message across technically is to be specific and say something like ''If you do this and this, then an attacker could do this,'' but such a statement may be felt as harsh by the recipient. Instead, you could begin by asking, ''Have you thought about what might happen if someone did this?'' You could then ease the designers of the system into a discussion of the attack itself. You might also consider complimenting them on the remaining strengths of their system, observe the challenges to building secure systems, and offer to help them fix their security problems if possible.

So the next time someone attacks the security of your system, try not to take it personally. And make sure that when you attack a system, you only focus on the technology, you don't criticize the people behind it, and you are sensitive to the fact that the designers may not be familiar with the culture of constructive criticism in the security community.

1.5 Threat Model

Every system can be attacked. There is no such thing as perfect security. The whole point of a security system is to provide access to some people and not to others. In the end, you will always have to trust some people in some way, and these people may still be able to attack your system.

It is very important to know what you are trying to protect, and against whom you wish to protect it. What are the assets of value? What are the threats? These sound like simple questions, but it turns out to be a much harder problem than you'd think. Since there's really no such thing as perfect security, when we say that a system is ''secure,'' what we are really saying is that it provides a sufficient level of security for our assets of interest against certain classes of threats. We need to assess the security of a system under the designated threat model.

Most companies protect their LAN with a firewall, but many of the really harmful attacks are performed by insiders, and a firewall does not protect against insiders at all. It doesn't matter how good your firewall is; it won't protect against a malicious employee. This is a mismatch in the threat model.

Another example is SET. SET is a protocol for online shopping with a credit card. One of its features is that it encrypts the credit card number so that an eavesdropper cannot copy it. That is a good idea. A second feature—that not even the merchant is shown the customer's credit-card number—works less well.

The second property fails because some merchants use the credit card number to look up customer records or to charge surcharges. Entire commerce systems have been based on the assumption that the merchant has access to the customer's credit card number. And then SET tries to take this access away. When Niels worked with SET in the past, there was an option for sending the credit card number twice—once encrypted to the bank, and once encrypted to the merchant so that the merchant would get it too. (We have not verified whether this is still the case.)

But even with this option, SET doesn't solve the whole problem. Most credit card numbers that are stolen are not intercepted while in transit between the consumer and the merchant. They are stolen from the merchant's database. SET only protects the information while it is in transit.

SET makes another, more serious, mistake. Several years ago Niels's bank in the Netherlands offered a SET-enabled credit card. The improved security for online purchases was one of the major selling points. But this turned out to be a bogus argument. It is quite safe to order online with a normal credit card. Your credit card number is not a secret. You give it to every salesperson you buy something from. The real secret is your signature. That is what authorizes the transaction. If a merchant leaks your credit card number, then you might get spurious charges, but as long as there is no handwritten signature (or PIN code) there is no indication of acceptance of the transaction, and therefore no legal basis for the charge. In most jurisdictions you simply complain and get your money back. There might be some inconvenience involved in getting a new credit card with a different number, but that is the extent of the user's exposure. With SET, the situation is different. SET uses a digital signature (explained in Chapter 12) by the user to authorize the transaction. That is obviously more secure than using just a credit card number. But think about it. Now the user is liable for any transaction performed by the SET software on his PC. This opens the user up to huge liabilities. What if a virus infects his PC and subverts the SET software? The software might sign the wrong transaction, and cause the user to lose money.

So from the user's point of view, SET offers *worse* security than a plain credit card. Plain credit cards are safe for online shopping because the user can always get his money back from a fraudulent transaction. Using SET increases the user's exposure. So although the overall payment system is better secured, SET transfers the residual risk from the merchant and/or bank to the user. It changes the user's threat model from ''It will only cost me money if they forge my signature well enough'' to ''It will only cost me money if they forge my signature well enough, or if a clever virus infects my PC.''

Threat models are important. Whenever you start on a cryptographic security project, sit down and think about what your assets are and against which threats you wish to protect them. A mistake in your threat analysis can render an entire project meaningless. We won't talk a lot about threat analysis in this book, as we are discussing the limited area of cryptography here, but in any real system you should never forget the threat analysis for each of the participants.

1.6 Cryptography Is Not the Solution

Cryptography is not the solution to your security problems. It might be part of the solution, or it might be part of the problem. In some situations, cryptography starts out by making the problem worse, and it isn't at all clear that using cryptography is an improvement. The correct use of cryptography must therefore be carefully considered. Our previous discussion of SET is an example of this.

Suppose you have a secret file on your computer that you don't want others to read. You could just protect the file system from unauthorized access. Or you could encrypt the file and protect the key. The file is now encrypted, and human nature being what it is, you might not protect the file very well. You might store it on a USB stick and not worry if that USB stick is lost or stolen. But where can you store the key? A good key is too long to remember. Some programs store the key on the disk—the very place the secret file was stored in the first place. But an attack that could recover the secret file in the first situation can now recover the key, which in turn can be used to decrypt the file. Further, we have introduced a new point of attack: if the encryption system is insecure or the amount of randomness in the key is too low, then the attacker could break the encryption system itself. Ultimately, the overall security has been reduced. Therefore, simply encrypting the file is not the entire solution. It might be part of the solution, but by itself it can create additional issues that need to be solved.

Cryptography has many uses. It is a crucial part of many good security systems. It can also make systems weaker when used in inappropriate ways. In many situations, it provides only a feeling of security, but no actual security. It is tempting to stop there, since that is what most users want: to *feel* secure. Using cryptography in this manner can also make a system comply with certain standards and regulations, even if the resulting system isn't actually secure. In situations like this (which are all too common), any voodoo that the customer believes in would provide the same feeling of security and would work just as well.

1.7 Cryptography Is Very Difficult

Cryptography is fiendishly difficult. Even seasoned experts design systems that are broken a few years later. This is common enough that we are not surprised when it happens. The weakest-link property and the adversarial setting conspire to make life for a cryptographer—or any security engineer—very hard.

Another significant problem is the lack of testing. There is no known way of testing whether a system is secure. In the security and cryptography research community, for example, what we try to do is publish our systems and then get other experts to look at them. Note that the second part is not automatic; there are many published systems that nobody has even glanced at after they were published, and the conference and journal review process alone isn't sufficient to preemptively identify all potential security issues with a system prior to publication. Even with many seasoned eyes looking at the system, security deficiencies may not be uncovered for years.

There are some small areas of cryptography that we as a community understand rather well. This doesn't mean they are simple; it just means that we have been working on them for a few decades now, and we think we know the critical issues. This book is mostly about those areas. What we have tried to do in this book is to collect the information that we have about designing and building practical cryptographic systems, and bring it all together in one place.

For some reason, many people still seem to think that cryptography is easy. It is not. This book will help you understand the challenges to cryptography engineering and help propel you on the road to overcoming those challenges. But don't go out and build a new cryptographic voting machine or other critical security system right away. Instead, take what you learn here and work with others—especially seasoned cryptography experts—to design and analyze your new system. Even we, despite our years of experience in cryptography and security, ask other cryptography and security experts to review the systems that we design.

1.8 Cryptography Is the Easy Part

Even though cryptography itself is difficult, it is still one of the easy parts of a security system. Like a lock, a cryptographic component has fairly well-defined boundaries and requirements. An entire security system is much more difficult to clearly define, since it involves many more aspects. Issues like the organizational procedures used to grant access and the procedures used to check that the other procedures are being followed are much harder to deal with, as the situation is always changing. Another huge problem in computer security is the quality of much software. Security software cannot be effective if the software on the machine contains numerous bugs that lead to security holes.

Cryptography is the easy part, because there are people who know how to do a reasonably good job. There are experts for hire who will design a cryptographic system for you. They are not cheap, and they are often a pain to work with. They insist on changing other parts of the system to achieve the desired security properties. Still, for all practical purposes, cryptography poses problems that we know how to solve, and this book will give you a sense for how to go about solving them.

The rest of the security system contains problems we don't know how to solve. Key management and key storage is crucial to any cryptographic system, but most computers have no secure place to store a key. Poor software quality is another problem. Network security is even harder. And when you add users to the mix, the problem becomes harder still.

1.9 Generic Attacks

It is also important to realize that some security problems can't be solved. There are black box or generic attacks against certain types of systems. A classic example of this is the analog hole for digital rights management (DRM) systems. These DRM systems try to control the copying of digital materials, such as a picture, song, movie, or book. But no technology—cryptography or otherwise—can protect against a generic attack outside the system. For example, an attacker could take a photo of a computer screen to create a copy of the picture, or use a microphone to re-record the song.

It is important to identify what the generic attacks against a system are. Otherwise, you might spend a lot of time trying to fix an unfixable problem. Similarly, when someone claims that they've secured a system against a generic attack, you know to be skeptical.

1.10 Security and Other Design Criteria

Security is never the only design criterion for a system. Instead, security is but one of many criteria.

1.10.1 Security Versus Performance

The bridge over the Firth of Forth in Scotland has to be seen to be believed. A 19th-century engineering marvel, it is mind-numbingly large (and therefore expensive) compared to the trains that cross it. It is so incredibly over-engineered it is hard to believe your eyes. Yet the designers did the right thing. They were confronted with a problem they had not solved successfully before: building a large steel bridge. They did an astoundingly good job. They succeeded spectacularly; their bridge is still in use today over a century later. That's what good engineering looks like.

Over the years, bridge designers have learned how to build such bridges much more cheaply and efficiently. But the first priority is always to get a bridge that is safe and that works. Efficiency, in the form of reducing cost, is a secondary issue.

We have largely reversed these priorities in the computer industry. The primary design objective all too often includes very strict efficiency demands. The first priority is always speed, even in areas where speed is not important. Here speed might be the speed of the system itself, or it might be the speed with which the system can be brought to market. This leads to security cost-cutting. The result is generally a system that is somewhat efficient, yet is not sufficiently secure.

There is another side to the Firth of Forth bridge story. In 1878, Thomas Bouch completed the then-longest bridge in the world across the Firth of Tay at Dundee. Bouch used a new design combining cast iron and wrought iron, and the bridge was considered to be an engineering marvel. On the night of December 28, 1879, less than two years later, the bridge collapsed in a heavy storm as a train with 75 people on board crossed the bridge. All perished. It was the major engineering disaster of the time.[3] So when the Firth of Forth bridge was designed a few years later, the designers put in a lot more steel, not only to make the bridge safe but also to make it *look* safe to the public.

We all know that engineers will sometimes get a design wrong, especially when they do something new. And when they get it wrong, bad things can happen. But here is a good lesson from Victorian engineers: if it fails, back off and become more conservative. The computer industry has largely forgotten this lesson. When we have very serious security failures in our computer systems, and we have them all too frequently, it is very easy to just plod along, accepting it as if it were fate. We rarely go back to the drawing board and design something more conservative. We just keep throwing a few patches out and hoping this will solve the problem.

By now, it will be quite clear to you that we will choose security over efficiency any time. How much CPU time are we willing to spend on security? Almost all of it. We wouldn't care if 90% of our CPU cycles were spent on a reliable security system if the alternative was a faster but insecure system. The lack of computer security is a real hindrance to us, and to most users. That is

[3] William McGonagall wrote a famous poem about the Tay Bridge disaster, ending with the lines *For the stronger we our houses do build/The less chance we have of being killed.* This advice is still highly relevant today.

why people still have to send pieces of paper around with signatures, and why they have to worry about viruses and other attacks on our computers. Digital crooks of the future will know much more and be much better equipped, and computer security will become a larger and larger problem. We are still only seeing the beginnings of the digital crime wave. We need to secure our computers much better.

There are of course many ways of achieving security. But as Bruce extensively documented in *Secrets and Lies*, good security is always a mixture of prevention, detection, and response [114]. The role for cryptography is primarily in the prevention part, which has to be very good to ensure that the detection and response parts (which can and should include manual intervention) are not overwhelmed. Cryptography can, however, be used to provide more secure detection mechanisms, such as strong cryptographic audit logs. Cryptography is what this book is about, so we'll concentrate on that aspect.

Yes, yes, we know, 90% still sounds like a lot. But there is some consolation. Remember first that we are willing to spend 90% of our CPU on security if the alternative is an insecure system. Fortunately, in many cases the costs of security can be hidden from the user. We can only type around 10 characters per second—on a good day—and even the slow machines of a decade ago had no trouble keeping up with that. Today's machines are over a thousand times faster. If we use 90% of the CPU for security, the computer will appear one-tenth as fast. That is about the speed that computers were five years ago. And those computers were more than fast enough for us to get our work done. We may not always have to spend so many cycles on security. But we're willing to, and that's the point.

There are only a few situations in which we have to wait on the computer. These include waiting for Web pages, printing data, starting certain programs, booting the machine, etc. A good security system would not slow down any of these activities. Modern computers are so fast that it is hard to figure out how to use the cycles in a useful manner. Sure, we can use alpha-blending on screen images, 3D animations, or even voice recognition. But the number-crunching parts of these applications do not perform any security-related actions, so they would not be slowed down by a security system. It is the rest of the system, which is already as fast as it can possibly get on a human time scale, that will have the overhead. And we don't care if it goes at one-tenth the speed if it increases security. Most of the time, you wouldn't even notice the overhead. Even in situations where the overhead would be significant, that is just the cost of doing business.

It will be clear by now that our priorities are security first, second, and third, and performance somewhere way down the list. Of course, we still want the system to be as efficient as possible, but not at the expense of security. We understand that this design philosophy is not always possible in the real world. Often the realities of the marketplace trump the need for security.

Systems can rarely be developed from scratch, and often need to be secured incrementally or after deployment. Systems need to be backward-compatible with existing, insecure, systems. The three of us have designed many security systems under these constraints, and we can tell you that it's practically impossible to build a good security system that way. The design philosophy of this book is security first and security foremost. It's one we'd like to see adopted more in commercial systems.

1.10.2 Security Versus Features

Complexity is the worst enemy of security, and it almost always comes in the form of features or options.

Here is the basic argument. Imagine a computer program with 20 different options, each of which can be either on or off. That is more than a million different configurations. To get the program to work, you only need to test the most common combination of options. To make the program secure, you must evaluate each of the million possible configurations that the program can have, and check that each configuration is secure against every possible form of attack. That is impossible to do. And most programs have considerably more than 20 options. The best way to have confidence in building something secure is to keep it simple.

A simple system is not necessarily a small system. You can build large systems that are still fairly simple. Complexity is a measure of how many things interact at any one point. If the effect of an option is limited to a small part of the program, then it cannot interact with an option whose effect is limited to another part of the program. To make a large, simple system you have to provide a very clear and simple interface between different parts of the system. Programmers call this modularization. This is all basic software engineering. A good simple interface isolates the rest of the system from the details of a module. And that should include any options or features of the module.

One of the things we have tried to do in this book is define simple interfaces for cryptographic primitives. No features, no options, no special cases, no extra things to remember, just the simplest definition we could come up with. Some of these definitions are new; we developed them while writing the book. They have helped us shape our thinking about good security systems, and we hope they will help you, too.

1.10.3 Security Versus Evolving Systems

One of the other biggest problems for security is that the full system continues to evolve even after the underlying security mechanisms are put in place. This means that the designer of the security mechanism needs not only to exhibit professional paranoia and consider a wide range of attackers and attack goals, but also to anticipate and prepare for future uses of the system. This can also create enormous challenges, and is an issue that systems designers need to keep in mind.

1.11 Further Reading

Anyone interested in cryptography should read Kahn's *The Codebreakers* [67]. This is a history of cryptography, from ancient times to the 20th century. The stories provide many examples of the problems engineers of cryptographic systems face. Another good historical text, and a pleasurable read, is *The Code Book* [120].

In some ways, the book you're holding is a sequel to Bruce's first book, *Applied Cryptography* [112]. *Applied Cryptography* covers a much broader range of subjects, and includes the specifications of all the algorithms it discusses. However, it does not go into the engineering details that we talk about in this book.

For facts and precise results, you can't beat the *Handbook of Applied Cryptography*, by Menezes, van Oorschot, and Vanstone [90]. It is an encyclopedia of cryptography and an extremely useful reference book; but just like an encyclopedia, it's hardly a book to learn the field from.

If you're interested in the theory of cryptography, an excellent sequence of texts is *Foundations of Cryptography*, by Goldreich [55, 56]. Another excellent text is *Introduction to Modern Cryptography*, by Katz and Lindell [68]. There are also numerous excellent university course notes available online, such as the course notes from Bellare and Rogaway [10].

Bruce's previous book *Secrets and Lies* [114] is a good explanation of computer security in general, and how cryptography fits into that larger picture. And there's no better book on security engineering than Ross Anderson's *Security Engineering* [2]. Both are essential to understand the context of cryptography.

There are a number of good online resources for staying abreast of recent issues in cryptography and computer security. We suggest subscribing to Bruce's Crypto-Gram newsletter, `http://www.schneier.com/crypto-gram.html`, and reading Bruce's blog, `http://www.schneier.com/blog/`.

1.12 Exercises for Professional Paranoia

They say that one of the best ways to learn a foreign language is to immerse yourself in it. If you want to learn French, move to France. This book is designed to immerse you in the language and mindset of cryptography and computer security. The following exercises will help immerse you further. They will force you to think about security on a regular basis, such as when you're reading news articles, talking with friends about current events, or reading the description of a new product on Slashdot. Thinking about security will no longer be a chore relegated to the time when you are specifically tasked with thinking about security. You may even start thinking about security while you're out walking your dog, in the shower, or at a movie. In short, you will be developing the professional paranoia mindset and will start thinking like a security professional.

It is also extremely important for a computer security practitioner (and, actually, all computer scientists) to be aware of the broader contextual issues surrounding technology. Technologies don't exist in isolation. Rather, they are one small aspect of a larger ecosystem consisting of people, economics, ethics, cultural differences, politics, law, and so on. These exercises will also give you an opportunity to discuss and explore these bigger picture issues as they relate to security.

We suggest that you regularly return to the exercises below. Try to do these exercises as often as possible. For example, you might do these exercises every week for a month straight, or after you finish every few chapters in this book, whichever is more frequent. The exercises might seem laborious and tedious at first. But if you're dedicated to this practice, you will soon find yourself doing these exercises automatically whenever you encounter a security-related news article or see a new product. This is the professional paranoia mindset. Further, if you continue to do these exercises as you read this book, you will notice that your ability to evaluate the technical properties of systems will mature over time.

We also recommend doing the exercises with a friend or, if you are in a class, with a classmate as part of group instruction. Discussing security issues with others can be very enlightening—you will soon realize firsthand that security is incredibly subtle and that it is very easy to overlook critical weaknesses.

Obviously, if you're not taking a class and doing the formal exercises, then you may choose to conduct these exercises in your head rather than actually producing written reports. Still, we suggest producing a written report at least once; doing so will force you to really think through the relevant issues completely.

1.12.1 Current Event Exercises

For these exercises, you should critically analyze some event currently in the news. The event you choose should somehow relate to computer security. Maybe improved computer security mechanisms would have thwarted the event. Maybe the event motivates the design of new security mechanisms or policies.

The current events retrospective that you write should be short, concise, very thoughtful, and well written. Assume a general audience. Your goal should be to write an article that will help the reader learn about and understand the computer security field and how it fits into the broader context.

You should summarize the current event, discuss why the current event arose, reflect on what could have been done differently prior to the event arising (to perhaps prevent, deter, or alter the consequences of the event), describe the broader issues surrounding the current event (such as ethical issues or societal issues), and propose possible reactions to the current event (e.g., how the public, policy makers, corporations, the media, or others should respond).

1.12.2 Security Review Exercises

These exercises deal with developing your security mindset in the context of real products or systems. Your goal with the security reviews is to evaluate the potential security and privacy issues of new technologies, evaluate the severity of those issues, and discuss how to address those security and privacy issues. These reviews should reflect deeply on the technology that you're discussing, and should therefore be significantly longer than your current event exercises.

Each security review should contain:

- Summary of the technology that you're evaluating. You may choose to evaluate a specific product (like a recently introduced wireless implantable drug pump) or a class of products with some common goal (like the set of all implantable medical devices). This summary should be at a high level, around one or two paragraphs in length. State the aspects of the technology that are relevant to your observations in the following bullets.

  For these exercises, it is acceptable to make assumptions about how the products work. However, if you do make assumptions about a product, then you should make it clear that you are doing so, and you should explicitly state what those assumptions are.

  Being able to clearly summarize a product (even with explicitly stated assumptions) is very important. If you don't understand the technology well enough to provide a crisp and clear summary, then you probably don't understand the technology well enough to evaluate its security and privacy.

- State at least two assets and, for each asset, a corresponding security goal. Explain why the security goals are important. You should produce around one or two sentences per asset/goal.

- State at least two possible threats, where a threat is defined as an action by an adversary aimed at compromising an asset. Give an example adversary for each threat. You should have around one or two sentences per threat/adversary.

- State at least two potential weaknesses. Again, justify your answer using one or two sentences per weakness. For the purposes of these exercises, you don't need to fully verify whether these potential weaknesses are also actual weaknesses.

- State potential defenses. Describe potential defenses that the system could use or might already be using to address the potential weaknesses you identified in the previous bullet.

- Evaluate the risks associated with the assets, threats, and potential weaknesses that you describe. Informally, how serious do you think these combinations of assets, threats, and potential weaknesses are?

- Conclusions. Provide some thoughtful reflections on your answers above. Also discuss relevant ''bigger picture'' issues (ethics, likelihood the technology will evolve, and so on).

Some examples of past security reviews are online at `http://www.schneier.com/ce.html`.

1.13 General Exercises

Exercise 1.1 Create an attack tree for stealing a car. For this and the other attack tree exercises, you can present your attack tree as a figure (like Figure 1.1), or you can present your attack tree as a list numbered in outline form (e.g., 1, 1.1, 1.2, 1.2.1, 1.2.2, 1.3, ...).

Exercise 1.2 Create an attack tree for getting into a gym without paying.

Exercise 1.3 Create an attack tree for getting food from a restaurant without paying.

Exercise 1.4 Create an attack tree for learning someone's online banking account name and password.

Exercise 1.5 Create an attack tree for reading someone else's e-mail.

Exercise 1.6 Create an attack tree for preventing someone from being able to read his own e-mail.

Exercise 1.7 Create an attack tree for sending e-mail as someone else. Here, the attacker's goal is to convince an e-mail recipient that an e-mail she receives is from someone else (say, Bob), when in fact Bob never sent that e-mail.

Exercise 1.8 Find a new product or system that was announced or released within the last three months. Conduct a security review of that product or system as described in Section 1.12. Pick one of the assets that you identified and construct an attack tree for compromising that asset.

Exercise 1.9 Provide a concrete example, selected from media reports or your personal experiences, in which attackers compromised a system by exploiting something other than the weakest link. Describe the system, describe what you view the weakest link of the system to be and why, and describe how the system was compromised.

Exercise 1.10 Describe a concrete example, excluding the ones given in this chapter, where improving the security of a system against one type of attack can increase the likelihood of other attacks.

CHAPTER

2

Introduction to Cryptography

This chapter introduces basic cryptographic concepts and provides background information you will need for the rest of the book.

2.1 Encryption

Encryption is the original goal of cryptography. The generic setting is shown in Figure 2.1. Alice and Bob want to communicate with each other. (The use of personal names, particularly Alice, Bob, and Eve, is a tradition in cryptography.) However, in general, communication channels are assumed not to be secure. Eve is eavesdropping on the channel. Any message *m* that Alice sends to Bob is also received by Eve. (The same holds for messages sent by Bob to Alice, but that is the same problem, except with Alice and Bob reversed. As long as we can protect Alice's messages, the same solution will work for Bob's messages, so we concentrate on Alice's messages.) How can Alice and Bob communicate without Eve learning everything?

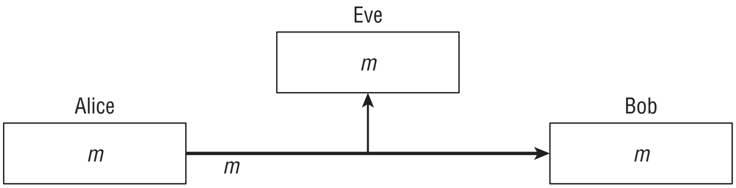

Figure 2.1: How can Alice and Bob communicate securely?

To prevent Eve from understanding the conversation that Alice and Bob are having, they use encryption as shown in Figure 2.2. Alice and Bob first agree on a secret key K_e. They have to do this via some communication channel that Eve cannot eavesdrop on. Perhaps Alice mails a copy of the key to Bob, or something similar. We will return to the exchange of keys later.

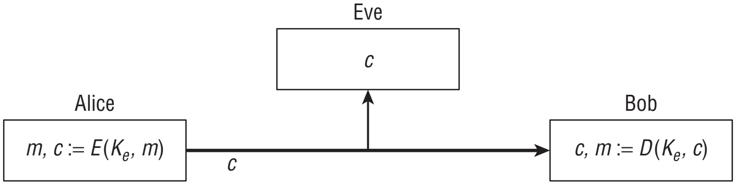

Figure 2.2: Generic setting for encryption

When Alice wants to send a message m, she first encrypts it using an encryption function. We write the encryption function as $E(K_e, m)$ and we call the result the *ciphertext* c. (The original message m is called the *plaintext*.) Instead of sending m to Bob, Alice sends the ciphertext $c := E(K_e, m)$. When Bob receives c, he can decrypt it using the decryption function $D(K_e, c)$ to get the original plaintext m that Alice wanted to send to him.

But Eve does not know the key K_e, so when she receives the ciphertext c she cannot decrypt it. A good encryption function makes it impossible to find the plaintext m from the ciphertext c without knowing the key. Actually, a good encryption function should provide even more privacy than that. An attacker shouldn't be able to learn any information about m, except possibly its length and the time it was sent.

This setting has obvious applications for transmitting e-mails, but it also applies to storage. Storing information can be thought of in terms of transmitting a message in time, rather than in space. In that situation Alice and Bob are often the same person at different points in time, so the same solution applies.

2.1.1 Kerckhoffs' Principle

Bob needs two things to decrypt the ciphertext. He must know the decryption algorithm D, and the key K_e. An important rule is Kerckhoffs' principle: the security of the encryption scheme must depend only on the secrecy of the key K_e, and not on the secrecy of the algorithm.

There are very good reasons for this rule. Algorithms are hard to change. They are built into software or hardware, which can be difficult to update. In practical situations, the same algorithm is used for a long time. That is just a fact of life. And it is hard enough to keep a simple key secret. Keeping the algorithm secret is far more difficult (and therefore more expensive). Nobody builds a cryptographic system for just two users. Every participant in the system (and there could be millions) uses the same algorithm. Eve would only have to get the algorithm from one of them, and one of them is bound to be easy to subvert. Or she could just steal a laptop with the algorithm on it. And remember our paranoia model? Eve might very well be one of the other users of the system, or even one of its designers.

There are also good reasons why algorithms should be published. From experience, we know that it is very easy to make a small mistake and create a cryptographic algorithm that is weak. If the algorithm isn't public, nobody will find this fault until the attacker tries to attack it. The attacker can then use the flaw to break the system. We have analyzed quite a number of secret encryption algorithms, and *all* of them had weaknesses. This is why there is a healthy distrust of proprietary, confidential, or otherwise secret algorithms. Don't be fooled by the old ''Well, if we keep the algorithm secret too, it will only increase security'' assurance. That is wrong. The potential increase in security is small, and the potential decrease in security is huge. The lesson is simple: don't trust secret algorithms.

2.2 Authentication

Alice and Bob have another problem in Figure 2.1. Eve can do more than just listen in on the message. Eve could change the message in some way. This requires Eve to have a bit more control over the communication channel, but that is not at all an impossibility. For example, in Figure 2.3, Alice tries to send the message m, but Eve interferes with the communication channel. Instead of receiving m, Bob receives a different message m'. We assume that Eve also learns the contents of the message m that Alice tried to send. Other things that Eve could do are delete a message so that Bob never receives it, insert new messages that she invents, record a message and then send it to Bob later, or change the order of the messages.

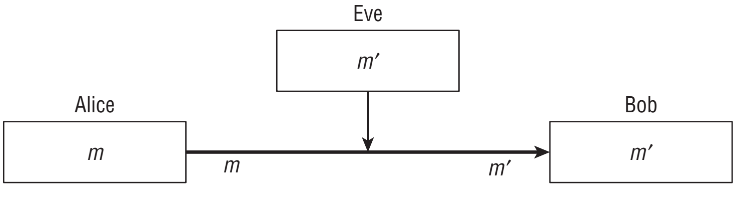

Figure 2.3: How does Bob know who sent the message?

Consider the point in the process where Bob has just received a message. Why should Bob believe the message came from Alice? He has no reason to think it did. And if he doesn't know who sent the message, then the message is pretty useless.

To resolve this problem, we introduce authentication. Like encryption, authentication uses a secret key that Alice and Bob both know. We'll call the authentication key K_a to distinguish it from the encryption key K_e. Figure 2.4 shows the process of authenticating a message m. When Alice sends the message m, she computes a *message authentication code*, or MAC. Alice computes the MAC a as $a := h(K_a, m)$, where h is the MAC function and K_a is the authentication key. Alice now sends both m and a to Bob. When Bob receives m and a, he recomputes what a should have been, using the key K_a, and checks that the a he receives is correct.

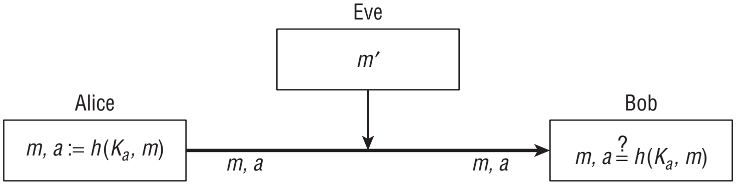

Figure 2.4: Generic setting for authentication

Now Eve wants to modify the message m to a different message m'. If she simply replaces m with m', Bob will still compute $h(K_a, m')$ and compare it to a. But a good MAC function will not give the same result for two different messages, so Bob will recognize that the message is not correct. Given that the message is wrong in one way or another, Bob will just discard the message.

If we assume that Eve does not know the authentication key K_a, the only way Eve can get a message and a valid MAC is to listen to Alice when she sends messages to Bob. This still allows Eve to try some mischief. Eve can record messages and their MACs, and then replay them by sending them to Bob at any later time.

Pure authentication is only a partial solution. Eve can still delete messages that Alice sends. She can also repeat old messages or change the message order. Therefore, authentication is almost always combined with a numbering scheme to number the messages sequentially. If m contains such a message number, then Bob is not fooled by Eve when she replays old messages. Bob will simply see that the message has a correct MAC but the sequence number is that of an old message, so he will discard it.

Authentication in combination with message numbering solves most of the problem. Eve can still stop Alice and Bob from communicating, or delay messages by first deleting them and then sending them to Bob at a later time. If the messages aren't also encrypted, then Eve can selectively delete or delay messages based on their content. But deleting or delaying messages is about the extent of what she can do.

The best way to look at it is to consider the case where Alice sends a sequence of messages $m_1, m_2, m_3, \ldots$. Bob only accepts messages with a proper MAC and whose message number is strictly greater[1] than the message number of the last message he accepted. So Bob receives a sequence of messages that is a *subsequence* of the sequence that Alice sent. A subsequence is simply the same sequence with zero or more messages deleted.

This is the extent to which cryptography can help in this situation. Bob will receive a subsequence of the messages that Alice sent, but other than deleting certain messages or stopping all communications, Eve cannot manipulate the message traffic. To avoid the loss of information, Alice and Bob will often use a scheme of resending messages that were lost, but that is more application-specific, and not part of the cryptography.

Of course, in many situations Alice and Bob will want to use both encryption and authentication. We will discuss this combination in great detail later. Never confuse the two concepts. Encrypting a message doesn't stop manipulation of its contents, and authenticating a message doesn't keep the message secret. One of the classical mistakes in cryptography is to think that encrypting a message also stops Eve from changing it. It doesn't.

2.3 Public-Key Encryption

To use encryption as we discussed in Section 2.1, Alice and Bob must share the key K_e. How did they get far enough along to share a key? Alice couldn't just send the key to Bob over the communication channel, as Eve could read the key too. The problem of distributing and managing keys is one of the really difficult parts of cryptography, for which we have only partial solutions.

Alice and Bob could have exchanged the key when they met last month for a drink. But if Alice and Bob are part of a group of 20 friends that like to communicate with each other, then each member of the group would have to exchange a total of 19 keys. All in all, the group would have to exchange a total of 190 keys. This is already very complex, and the problem grows with the number of people Alice communicates with.

Establishing cryptographic keys is an age-old problem, and one important contribution to the solution is public-key cryptography. We will first discuss public-key encryption, shown in Figure 2.5. We left Eve out of this diagram; from now on, just assume that all communications are always accessible to an enemy like Eve. Apart from Eve's absence, this figure is very similar to Figure 2.2. The major difference is that Alice and Bob no longer use the same

[1] ''Strictly greater'' means ''greater and not equal to.''

key, but instead use different keys. This is the significant idea behind public-key cryptography—the key to encrypt a message is different from the key to decrypt that message.

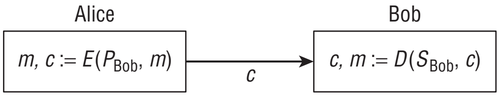
Alice
Bob
$m, c := E(P_{\text{Bob}}, m)$
c
$c, m := D(S_{\text{Bob}}, c)$

Figure 2.5: Generic setting for public-key encryption

To set things up, Bob first generates a pair of keys $(S_{\text{Bob}}, P_{\text{Bob}})$ using a special algorithm. The two keys are the secret key S_{Bob} and the public key P_{Bob}. Bob then does a surprising thing: he publishes P_{Bob} as his public key. This act makes Bob's public key P_{Bob} universally accessible to everyone, including both Alice and Eve. (Why else would it be called a public key?)

When Alice wants to send a message to Bob, she first obtains Bob's public key. She might obtain the public key from a public directory, or perhaps she obtains the public key from someone else she trusts. Alice encrypts the message m with the public key P_{Bob} to get the ciphertext c, and sends c to Bob. Bob uses his secret key S_{Bob} and the decryption algorithm to decrypt the message and get the message m.

For this to work, the key-pair generation algorithm, encryption algorithm, and decryption algorithm have to ensure that the decryption actually yields the original message. In other words: $D(S_{\text{Bob}}, E(P_{\text{Bob}}, m)) = m$ must hold for all possible messages m. We'll examine this in more detail later.

Not only are the two keys that Alice and Bob use different, but the encryption and decryption algorithms can also be very different. All public-key encryption schemes depend heavily on mathematics. One obvious requirement is that it should not be possible to compute the secret key from the corresponding public key, but there are many more requirements as well.

This type of encryption is called asymmetric-key encryption, or public-key encryption, as opposed to the symmetric-key encryption or secret-key encryption we discussed earlier.

Public-key cryptography makes the problem of distributing keys a lot simpler. Now Bob only has to distribute a single public key that everybody can use. Alice publishes her public key in the same way, and now Alice and Bob can communicate securely. Even in large groups, each group member only has to publish a single public key, which is quite manageable.

So why do we bother with secret-key encryption if public-key encryption is so much easier? Because public-key encryption is much less efficient, by several orders of magnitude. Using it for everything is simply too expensive. In practical systems that use public-key cryptography, you almost always see a mixture of public-key and secret-key algorithms. The public-key algorithms are used to establish a secret key, which in turn is used to encrypt the actual data. This combines the flexibility of public-key cryptography with the efficiency of symmetric-key cryptography.

2.4 Digital Signatures

Digital signatures are the public-key equivalent of message authentication codes. The generic setting is shown in Figure 2.6. This time, it is Alice who uses a key generation algorithm to generate a key pair $(S_{\text{Alice}}, P_{\text{Alice}})$ and publishes her public key P_{Alice}. When she wants to send a signed message m to Bob, she computes a signature $s := \sigma(S_{\text{Alice}}, m)$. She sends m and s to Bob. Bob uses a verification algorithm $v(P_{\text{Alice}}, m, s)$ that uses Alice's public key to verify the signature. The signature works just like a MAC, except that Bob can verify it with the public key, whereas the secret key is required to create a new signature.

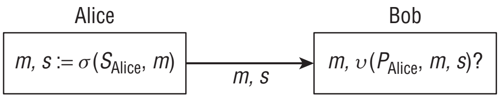
Alice
Bob
$m, s := \sigma(S_{\text{Alice}}, m)$
m, s
$m, v(P_{\text{Alice}}, m, s)?$

Figure 2.6: Generic setting for digital signature

Bob only needs to have Alice's public key to verify that the message came from Alice. Interestingly enough, anybody else can get Alice's public key and verify that the message came from Alice. This is why we generally call s a *digital signature*. In a sense, Alice signs the message. If there is ever a dispute, Bob can take m and s to a judge and prove that Alice signed the message.

This is all very nice in theory, and it works too . . . in theory. In real life, digital signatures have a number of limitations that are important to realize. The main problem is that Alice doesn't compute the signature herself; instead, she has her computer compute the signature. The digital signature is therefore no proof that Alice approved the message, or even saw it on her computer screen. Given the ease with which viruses take over computers, the digital signature actually proves very little in this scenario. Nonetheless, when used appropriately, digital signatures are extremely useful.

2.5 PKI

Public-key cryptography makes key management simpler, but Alice still has to find Bob's public key. How can she be sure it is Bob's key, and not somebody else's? Maybe Eve created a key pair and published the key while impersonating Bob. The general solution is to use a PKI, or *public key infrastructure*.

The main idea is to have a central authority called the *certificate authority*, or CA. Each user takes his public key to the CA and identifies himself to the CA. The CA then signs the user's public key using a digital signature. The signed message, or *certificate*, states: ''I, the CA, have verified that public key P_{Bob} belongs to Bob.'' The certificate will often include an expiration date and other useful information.

Using certificates, it is much easier for Alice to find Bob's key. We will assume that Alice has the CA's public key, and has verified that this is the correct key. Alice can now retrieve Bob's key from a database, or Bob can e-mail his key to Alice. Alice can verify the certificate on the key, using the CA's public key that she already has. This certificate ensures that she has the correct key to communicate with Bob. Similarly, Bob can find Alice's public key and be sure that he is communicating with the right person.

In a PKI, each participant only has to have the CA certify his public key, and know the CA's public key so that he can verify the certificates of other participants. This is far less work than exchanging keys with every party he communicates with. That's the great advantage of a PKI: register once, use everywhere.

For practical reasons, a PKI is often set up with multiple levels of CAs. There is a top-level CA, called the root, which issues certificates on the keys of lower-level CAs, which in turn certify the user keys. The system still behaves in the same way, but now Alice has to check two certificates to verify Bob's key.

A PKI is not the ultimate solution; there are still many problems. First of all, the CA must be trusted by everybody. In some situations, that's easy. In a company, the HR department knows all employees, and can take on the role of CA. But there is no entity in the world that is trusted by everybody. The idea that a single PKI can handle the whole world does not seem viable.

The second problem is one of liability. What if the CA issues a false certificate, or the secret key of the CA is stolen? Alice would be trusting a false certificate, and she might lose a lot of money because of that. Who pays? Is the CA is willing to back it up with some kind of insurance? This requires a far more extensive business relationship between Alice and the CA.

There are many companies at the moment that are trying to be the world's CA. VeriSign is probably the best-known one. However, VeriSign explicitly limits its own liability in case it fails to perform its function properly. In most cases, the liability is limited to $100. That is probably less than we paid for our last order of books: transactions which were secured using certificates signed by VeriSign. That wasn't a problem because payment by credit card is safe for the consumer. However, we won't be buying our next car using a certificate that VeriSign only backs with a $100 guarantee.

2.6 Attacks

Having described the most important functions used in cryptography, we will now talk about some attacks. We will focus on attacks against encryption schemes here. There are many types of attacks, each with its own severity.

2.6.1 The Ciphertext-Only Model

A *ciphertext-only attack* is what most people mean when talking about breaking an encryption system. This is the situation in which Alice and Bob are encrypting their data, and all you as the attacker get to see is the ciphertext. Trying to decrypt a message if you only know the ciphertext is called a ciphertext-only attack. This is the most difficult type of attack, because you have the least amount of information.

2.6.2 The Known-Plaintext Model

A *known-plaintext attack* is one in which you know both the plaintext and the ciphertext. The most obvious goal is to find the decryption key. At first this looks very implausible: how could you know the plaintext? It turns out that there are many situations in which you get to know the plaintext of a communication. Sometimes there are messages that are easy to predict. For example: Alice is away on holiday and has an e-mail autoresponder that sends an ''I'm away on holiday'' reply to every incoming e-mail. You get an exact copy of this message by sending an e-mail to Alice and reading the reply. When Bob sends an e-mail to Alice, the autoresponder also replies, this time encrypted. Now you have the ciphertext and the plaintext of a message. If you can find the key, you can decrypt all other messages that Alice and Bob exchange with the same key. The latter part is important and bears repeating: You use the knowledge of some plaintext-ciphertext pairs to learn the key, and then use knowledge of the key to decrypt other ciphertexts.

Another typical situation is where Alice sends the same message to many people, including you. You now have the plaintext and the ciphertexts of the copy she sent to everybody else.

Maybe Alice and Bob are sending drafts of a press release to each other. Once the press release is published, you know the plaintext and the ciphertext.

Even if you don't know the entire plaintext, you often know part of it. E-mails will have a predictable start, or a fixed signature at the end. The header of an IP packet is highly predictable. Such predictable data leads to a partially known plaintext, and we classify this under known-plaintext attacks.

A known-plaintext attack is more powerful than a ciphertext-only attack. You, as the attacker, get more information than in the ciphertext-only case. Extra information can only help you.

2.6.3 The Chosen-Plaintext Model

The next level of control is to let you choose the plaintext. This is a more powerful type of attack than a known-plaintext attack. Now you get to select specially prepared plaintexts, chosen to make it easy to attack the system. You can choose any number of plaintexts and get the corresponding ciphertexts. Again, this is not unrealistic in practice. There are quite a large number of situations in which an attacker can choose the data that is being encrypted. Quite often Alice will get information from some outside source (e.g., one that can be influenced by the attacker) and then forward that information to Bob in encrypted form. For example, the attacker might send Alice an e-mail that she knows Alice will forward to Bob.

Chosen-plaintext attacks are not unreasonable in any way. A good encryption algorithm has no trouble withstanding a chosen-plaintext attack. Be very skeptical if anyone ever tries to convince you that a chosen-plaintext attack is not relevant to their system.

There are two variations on this attack. In the offline attack, you prepare a list of all the plaintexts you want to have encrypted before you get the ciphertexts. In the online attack, you can choose new plaintexts depending on the ciphertexts you've already received. Most of the time this distinction can be ignored. We will normally talk about the online version of the attack, which is the more powerful of the two.

2.6.4 The Chosen-Ciphertext Model

The term *chosen-ciphertext* is a misnomer. It should really be called a chosen ciphertext and plaintext attack, but that is too long. In a chosen-plaintext attack, you get to choose plaintext values. In a chosen-ciphertext attack, you get to choose both plaintext values and ciphertext values. For every plaintext that you choose, you get the corresponding ciphertext, and for any ciphertext you choose, you get the corresponding plaintext.

Obviously, the chosen-ciphertext attack is more powerful than a chosen-plaintext attack, as the attacker has more freedom. The goal still is to recover the key. With the key, you can decrypt other ciphertexts. Again, any reasonable encryption scheme has no trouble surviving a chosen ciphertext attack.

2.6.5 The Distinguishing Attack Goal

The attacks described above recover the plaintext or the decryption key. There are attacks that do not recover the key, but let you decrypt a specific other message. There are also attacks that do not recover a message, but reveal some partial information about the message. For example, given 10 chosen plaintexts, their corresponding ciphertexts, and an 11th ciphertext, it may be possible to learn whether the least significant bit of the 11th plaintext is a 1 or a

0 even if it's not possible to learn the corresponding decryption key. Even this sort of information can be very valuable to an attacker. There are too many forms of attack to list here, and new forms of attack are thought up all the time. So what should we defend against?

We wish to defend against a *distinguishing attack*. A distinguishing attack is any nontrivial method that detects a difference between the ideal encryption scheme and the actual one. This covers all the attacks we have discussed so far, as well as any yet-to-be-discovered attacks. Of course, we will have to define what the ideal scheme is. This probably all sounds very confusing right now, since we haven't defined what an ideal scheme is yet. We will begin to clarify this in the next chapter.

Isn't this all rather far-fetched? Well, no. Our experience shows that you really want your building blocks to be perfect. Some encryption functions have imperfections that cause them to fail the distinguishing attack definition, but other than that they are perfectly satisfactory encryption functions. Every time you use them, you have to check that these imperfections do not lead to any problems. In a system with multiple building blocks, you also have to check whether any combination of imperfections leads to problems. This quickly becomes unworkable, and in practice we have found actual systems that exhibit weaknesses due to known imperfections in their building blocks.

2.6.6 Other Types of Attack

So far we have mostly talked about attacking encryption functions. You can also define attacks for other cryptographic functions, such as authentication, digital signatures, etc. We will discuss these as they arise.

Even for encryption functions, we only discussed the basic attack models in which an attacker knows or chooses plaintexts or ciphertexts. Sometimes the attacker also knows when the ciphertexts were generated, or how fast the encryption or decryption operations were. Timing information and ciphertext length can reveal private information about encrypted messages. Attacks that make use of this type of additional information are called *information leakage* or *side-channel* attacks.

2.7 Under the Hood

Let's now look under the hood at two generic attack techniques.

2.7.1 Birthday Attacks

Birthday attacks are named after the birthday paradox. If you have 23 people in a room, the chance that two of them will have the same birthday exceeds 50%. That is a surprisingly large probability, given that there are 365 possible birthdays.

So what is a birthday attack? It is an attack that depends on the fact that duplicate values, also called *collisions*, appear much faster than you would expect. Suppose a system for secure financial transactions uses a fresh 64-bit authentication key for each transaction. (For simplicity, we assume that no encryption is used.) There are 2^{64} ($=18 \cdot 10^{18}$, or eighteen billion billion) possible key values, so this should be quite difficult to break, right? Wrong! After seeing about 2^{32} ($=4 \cdot 10^{9}$, or four billion) transactions, the attacker can expect that two transactions use the same key. Suppose the first authenticated message is always the same ''Are you ready to receive a transaction?'' message. If two transactions use the same authentication key, then the MAC values on their first messages will also be the same, which is easy to detect for the attacker. By knowing that the two keys are the same, the attacker can now insert the messages from the older transaction into the newer transaction while it is going on. As they are authenticated by the correct key, these bogus messages will be accepted, which is a clear break of the financial transaction system.

In general, if an element can take on N different values, then you can expect the first collision after choosing about $\sqrt{N}$ random elements. We're leaving out the exact details here, but $\sqrt{N}$ is fairly close. For the birthday paradox, we have $N = 365$ and $\sqrt{N} \approx 19$. The number of people required before the chance of a duplicate birthday exceeds 50% is in fact 23, but $\sqrt{N}$ is close enough for our purposes and is the approximation that cryptographers often use. One way of looking at this is that if you choose k elements, then there are $k(k-1)/2$ pairs of elements, each of which has a $1/N$ chance of being a pair of equal values. So the chance of finding a collision is close to $k(k-1)/2N$. When $k \approx \sqrt{N}$, this chance is close to 50%.[2]

Most of the time we talk about n-bit values. As there are 2^n possible values, you need almost $\sqrt{2^n} = 2^{n/2}$ elements in the set before you expect a collision. We will often talk about this as the $2^{n/2}$ bound, or the birthday bound.

2.7.2 Meet-in-the-Middle Attacks

Meet-in-the-middle attacks are the cousins of birthday attacks. (Together we call them *collision attacks*.) They are more common and more useful. Instead of waiting for a key to repeat, you can build a table of keys that you have chosen yourself.

Let's go back to our previous example of the financial transaction system that uses a fresh 64-bit key to authenticate each transaction. By using a meet-in-the-middle attack, the attacker can break the system even further. Here is how he does it: he chooses 2^{32} different 64-bit keys at random. For each of these

[2] These are only approximations, but good enough for our purposes.

keys, he computes the MAC on the ''Are you ready to receive a transaction?'' message, and stores both the MAC result and the key in a table. Then he eavesdrops on each transaction and checks if the MAC of the first message appears in his table. If the MAC does appear in the table, then there is a very good chance that the authentication key for that transaction is the same key as the attacker used to compute that table entry, and that key value is stored right alongside the MAC value in the table. Now that the attacker knows the authentication key, he can insert arbitrary messages of his choosing into the transaction. (The birthday attack only allowed him to insert messages from an old transaction.)

How many transactions does the attacker need to listen to? Well, he has precomputed the MAC on 1 in 2^{32} of all the possible keys, so any time the system chooses a key, there is a 1 in 2^{32} chance of choosing one that he can recognize. So after about 2^{32} transactions, he can expect a transaction that uses a key he precomputed the MAC for. The total workload for the attacker is about 2^{32} steps in the precomputation plus listening in to 2^{32} transactions, which is a lot less work than trying all 2^{64} possible keys.

The difference between the birthday attack and the meet-in-the-middle attack is that in a birthday attack, you wait for a single value to occur twice within the same set of elements. In a meet-in-the-middle attack, you have two sets, and wait for an overlap between the two sets. In both cases, you can expect to find the first result at around the same number of elements.

A meet-in-the-middle attack is more flexible than a birthday attack. Let's look at it in a more abstract way. Suppose we have N possible values. The first set has P elements, the second has Q elements. There are PQ pairs of elements, and each pair has a chance of $1/N$ of matching. We expect a collision as soon as PQ/N is close to 1. The most efficient choice is $P \approx Q \approx \sqrt{N}$. This is exactly the birthday bound again. The meet-in-the-middle attack provides extra flexibility. Sometimes it is easier to get elements for one of the sets than it is to get elements for the other set. The only requirement is that PQ be close to N. You could choose $P \approx N^{1/3}$ and $Q \approx N^{2/3}$. In the example above, the attacker might make a list of 2^{40} possible MAC values for the first message, and expect to find the first authentication key after listening to only 2^{24} transactions.

When we do a theoretical analysis of how easy a system is to attack, we often use the $\sqrt{N}$ size for both sets, because this generally minimizes the number of steps the attacker has to perform. It also requires a more detailed analysis to find out whether the elements of one set might be harder to get than the elements of another set. If you ever want to perform a meet-in-the-middle attack in real life, you should carefully choose the sizes of the sets to ensure $PQ \approx N$ at the least possible cost.

2.8 Security Level

With enough effort, any practical cryptographic system can be attacked successfully. The real question is how much work it takes to break a system. An easy way to quantify the workload of an attack is to compare it to an exhaustive search. An *exhaustive search attack* is one that tries all possible values for some target object, like the key. If an attack requires 2^{235} steps of work, then this corresponds to an exhaustive search for a 235-bit value.

We always talk about an attacker needing a certain number of steps, but haven't yet specified what a step is. This is partly laziness, but it also simplifies the analysis. When attacking an encryption function, computing a single encryption of a given message with a given key can be a single step. Sometimes a step is merely looking something up in a table. It varies. But in all situations, a step can be executed by a computer in a very short time. Sometimes it can be done in one clock cycle, sometimes it needs a million clock cycles, but in terms of the workloads that cryptographic attacks require, a single factor of a million is not terribly important. The ease of using a step-based analysis far outweighs the built-in inaccuracies. You can always do a more detailed analysis to find out how much work a step is. For a quick estimate, we always assume that a single step requires a single clock cycle.

Any system designed today really needs at least a 128-bit security level. That means that any attack will require at least 2^{128} steps. A new system designed today is, if successful, quite likely to still be in operation 30 years from now, and should provide at least 20 years of confidentiality for the data after the point at which it was last used. So we should aim to provide security for the next 50 years. That is a rather tall order, but there has been some work done to extrapolate Moore's law and apply it to cryptography. A security level of 128 bits is sufficient [85]. One could potentially argue for 100 bits, or even 110 bits, but cryptographic primitives are often engineered around powers of two, so we'll use 128 bits.

This concept of security level is only approximate. We only measure the amount of work the attacker has to do, and ignore things like memory or interactions with the fielded system. Dealing only with the attacker's workload is hard enough; complicating the model would make the security analysis much harder still, and greatly increase the chance of overlooking a vital point. As the cost for using a simple and conservative approach is relatively low, we use the simple concept of security level. The level of security is, however, a function of the access of the adversary—is the adversary restricted to the known plaintext model or can she operate under the chosen plaintext model, and how many encrypted messages can she see as part of her attack?

2.9 Performance

Security does not come for free. While cryptographers try to make cryptographic algorithms as efficient as possible, these algorithms are sometimes perceived as being too slow. Creating custom cryptography for efficiency can be very risky. If you deviate from the beaten path in security, you have to do an enormous amount of analysis work to make sure you don't accidentally end up creating a weak system. Such analysis requires experienced cryptographers. For most systems, it is much cheaper to buy a faster computer than to go to the trouble and expense of designing and implementing a more efficient security system.

For most systems, the performance of the cryptography is not a problem. Modern CPUs are so fast that they can keep up with almost any data stream they handle. For example, encrypting a 100 Mb/s data link with the AES algorithm requires only 20% of the cycles on a 1 GHz Pentium III CPU. (Less in real life, as you never get to transfer 100 Mb/s over such a link, due to the overhead of the communication protocol.)

There are, however, some situations in which cryptography creates a performance bottleneck. A good example is Web servers that use a very large number of SSL connections. The initialization of an SSL connection uses public-key cryptography and requires a large amount of computing power on the server side. Instead of developing a custom SSL-replacement that is more efficient for the server, it is far cheaper and safer to buy hardware accelerators to handle the existing SSL protocol.

Recently we ran across a good argument to convince people to choose security over performance. ''There are already enough insecure fast systems; we don't need another one.'' This is very true. Half-measures in security cost nearly as much as doing it well, but provide very little practical security. We firmly believe that if you're going to implement any security, you should do it well.

2.10 Complexity

The more complex a system, the more likely it has security problems. Indeed, we like to say that complexity is the worst enemy of security. This is a simple statement, but it took us a while to really understand it.

Part of the problem is the test-and-fix development process used all too frequently: build something, test for errors, go back and fix the errors, test to find more errors, etc. Test, fix, repeat. This goes on until company finances or other factors dictate that the product be shipped. Sure, the result is something that works reasonably well, as long as it is used only for the things it was tested for.

This might be good enough for functionality, but it is wholly inadequate for security systems.

The problem with the test-and-fix method is that testing only shows the presence of errors, and really only those errors the testers were looking for. Security systems have to work even when under attack by clever, malicious people. The system cannot be tested for all the situations to which the attackers will expose the system. Testing can only test for functionality; security is the absence of functionality. The attacker should not be able to achieve a certain property irrespective of what he does, yet testing cannot show the absence of functionality. The system has to be secure from the start.

Consider the following analogy. Suppose you write a medium-sized application in a popular programming language. You fix the syntax errors until it compiles the first time. Then, without further testing, you put it in a box and ship it to the customer. Nobody would expect to get a functional product that way.

Yet this is exactly what is normally done for security systems. They're impossible to test because nobody knows what to test for. By definition, an attacker wins by finding any aspect that wasn't tested. And if there is any bug, the product is defective. So the only way to get a secure system is to build a very robust system from the ground up. This requires a simple system.

The only way we know of making a system simple is to modularize it. We all know this from software development. But this time we cannot afford any bugs at all, so we have to be quite ruthless in the modularization. This leads us to another rule: correctness must be a local property. In other words, one part of the system should behave correctly regardless of how the rest of the system works. No, we don't want to hear ''This won't be a problem because this other part of the system will never let this happen.'' The other part may have a bug, or may change in some future version. Each part of the system is responsible for its own functionality.

2.11 Exercises

Exercise 2.1 Consider Kerckhoffs' principle. Provide at least two arguments in favor of Kerckhoffs' principle and at least two arguments against Kerckhoffs' principle. Then state and argue your view of the validity of Kerckhoffs' principle.

Exercise 2.2 Suppose Alice and Bob are sending e-mails to each other over the Internet. They're sending these e-mails from their laptops, which are connected to free wireless networks provided by their favorite coffee shops.

- List all the parties who might be able to attack this system and what they might be able to accomplish.
- Describe how Alice and Bob might be able to defend against each of the attacks you identified above. Be as specific as possible.

Exercise 2.3 Consider a group of 30 people who wish to establish pair-wise secure communications using symmetric-key cryptography. How many keys need to be exchanged in total?

Exercise 2.4 Suppose Bob receives a message signed using a digital signature scheme with Alice's secret signing key. Does this prove that Alice saw the message in question and chose to sign it?

Exercise 2.5 Suppose that PKIs, as we describe in Section 2.5, do not exist. Suppose Alice obtains a public key *P* that purportedly belongs to Bob. How can Alice develop confidence that *P* really belongs to Bob? Consider this question in each of the following scenarios:

- Alice can talk with Bob over the phone.
- Alice can talk with someone else she trusts over the phone (let's call him Charlie), and Charlie has already verified that *P* belongs to Bob.
- Alice can communicate with Charlie via e-mail, and Charlie has already verified that *P* belongs to Bob.

Explicitly state any additional assumptions you need to make.

Exercise 2.6 Suppose a chosen-ciphertext attacker cannot recover the secret decryption key for an encryption scheme. Does this mean the encryption scheme is secure?

Exercise 2.7 Consider a symmetric-key cryptosystem in which cryptographic keys are randomly selected from the set of all n-bit strings. Approximately what should n be in order to provide 128 bits of security against a birthday attack?

Part II

Message Security

In This Part

CHAPTER

3

Block Ciphers

Block ciphers are one of the fundamental building blocks for cryptographic systems. There is a lot of literature on block ciphers, and they are among the best-understood parts of cryptography. They are, however, building blocks. For most applications, you probably don't want to use a block cipher directly. Instead, you'll want to use a block cipher in what is called a ''mode of operation,'' which we'll discuss in subsequent chapters. This chapter is designed to give you a firmer understanding of block ciphers: what they are, how cryptographers view them, and how to choose between different options.

3.1 What Is a Block Cipher?

A *block cipher* is an encryption function for fixed-size blocks of data. The current generation of block ciphers has a block size of 128 bits (16 bytes). These block ciphers encrypt a 128-bit plaintext and generate a 128-bit ciphertext as the result. The block cipher is reversible; there is a decryption function that takes the 128-bit ciphertext and decrypts it to the original 128-bit plaintext. The plaintext and ciphertext are always the same size, and we call this the block size of the block cipher.

To encrypt with a block cipher, we need a secret key. Without a secret key, there is no way to hide the message. Like the plaintext and ciphertext, the key is also a string of bits. Common key sizes are 128 and 256 bits. We often write $E(K, p)$ or $E_K(p)$ for the encryption of plaintext p with key K and $D(K, c)$ or $D_K(c)$ for the decryption of ciphertext c with key K.

Block ciphers are used for many purposes, most notably to encrypt information. For security purposes, however, one rarely uses a block cipher directly. Instead, one should use a *block cipher mode*, which we will discuss in Chapter 4.

When using block ciphers, as with any encryption task, we always follow Kerckhoffs' principle and assume that the algorithms for encryption and decryption are publicly known. Some people have a hard time accepting this, and they want to keep the algorithms secret. Don't ever trust a secret block cipher (or any other secret cryptographic primitive).

It is sometimes useful to look at a block cipher as a very big key-dependent table. For any fixed key, you could compute a lookup table that maps the plaintext to the ciphertext. This table would be huge. For a block cipher with 32-bit block size, the table would be 16 GB; for a 64-bit block size, it would be 150 million TB; and for a 128-bit block size it would be $5 \cdot 10^{39}$ bytes, a number so large there is not even a proper name for it. Of course, it is not practical to build such a table in reality, but this is a useful conceptual model. We also know that the block cipher is reversible. In other words, no two entries of the table are the same, or else the decryption function could not possibly decrypt the ciphertext to a unique plaintext. This big table will therefore contain every possible ciphertext value exactly once. This is what mathematicians call a permutation: the table is merely a list of all the possible elements where the order has been rearranged. A block cipher with a block size of k bits specifies a permutation on k-bit values for each of the key values.

As a point of clarification, since it is often confused, a block cipher does not permute the bits of the input plaintext. Rather, it takes all the 2^k possible k-bit inputs and maps each to a unique k-bit output. As a toy example, if $k = 8$, an input 00000001 might encrypt to 0100000 under a given key but it might also encrypt to 11011110 under a different key, depending on the design of the block cipher.

3.2 Types of Attack

Given the definition of a block cipher, the definition of a secure block cipher seems simple enough: it is a block cipher that keeps the plaintext secret. Although this certainly is one of the requirements, it is not sufficient. This definition only requires that the block cipher be secure against ciphertext-only attacks, in which the attacker gets to see only the ciphertext of a message. There are a few published attacks of this type [74, 121], but they are rare against well-known and established block ciphers. Most published attacks are of the chosen-plaintext type. (See Section 2.6 for an overview of attack types.) All of these attack types apply to block ciphers, and there are a few more that are specific to block ciphers.

The first one is the related-key attack. First introduced by Eli Biham in 1993 [13], a related-key attack assumes that the attacker has access to several encryption functions. These functions all have an unknown key, but their keys have a relationship that the attacker knows. This sounds very strange, but it turns out that this type of attack is useful against real systems [70]. There are real-world systems that use different keys with a known relationship. At least one proprietary system changes the key for every message by incrementing the key by one. Consecutive messages are therefore encrypted with consecutively numbered keys. It turns out that key relationships like this can be used to attack some block ciphers.

There are even more esoteric attack types. When we designed the Twofish block cipher (Section 3.5.4), we introduced the concept of a chosen-key attack, in which the attacker specifies some part of the key and then performs a related-key attack on the rest of the key [115].[1]

Why would we even consider far-fetched attack types like related-key attacks and chosen-key attacks? We have several reasons. First, we have seen actual systems in which a related-key attack on the block cipher was possible, so these attacks are not that far-fetched at all. In fact, we have even seen standardized protocols that required implementations to key a block cipher with two related keys—one key K that is chosen at random and another key K' that is equal to K plus a fixed constant.

Second, block ciphers are very useful building blocks. But, as building blocks, they tend to get abused in every imaginable way. One standard technique of constructing a hash function from a block cipher is the Davies-Meyer construction [128]. In a Davies-Meyer hash function, the attacker suddenly gets to choose the key of the block cipher, which allows related-key and chosen-key attacks. We talk about hash functions in Chapter 5, but won't go into the details of the Davies-Meyer construction in this book. It is safe to say, however, that any definition of block-cipher security that ignores these attack types, or any other attack type, is incomplete.

The block cipher is a module that should have a simple interface. The simplest interface is to ensure that it has all the properties that anyone could reasonably expect the block cipher to have. Allowing imperfections in the block cipher just adds a lot of complexity, in the form of cross-dependencies, to any system using the cipher. In short, we want to over-engineer our block ciphers for security. The challenge is to define the properties that one reasonably expects from a block cipher.

[1] Later analysis showed that this attack does not work on Twofish [50], but it might be successful against other block ciphers.

3.3 The Ideal Block Cipher

It is actually very hard to define what a block cipher is. It is something that you know when you see it—but can't quite define. The theoretical community has crystallized some of these properties into specific definitions, like pseudorandomness and super-pseudorandomness [6, 86, 94]. The block cipher community itself, however, uses a much broader definition, covering things like weak keys and chosen-key attacks. Here we take the approach of trying to help you understand what the block cipher primitives community believes a block cipher to be. We call this an ''ideal'' block cipher.

What would the ideal block cipher look like? It should be a random permutation. We should be more precise: for each key value, we want the block cipher to be a random permutation, and the different permutations for the different key values should be chosen independently. As we mentioned in Section 3.1, you can think of a 128-bit block cipher (a single permutation on 128-bit values) as a huge lookup table of 2^{128} elements of 128 bits each. The ideal block cipher consists of one of these tables for each key value, with each table chosen randomly from the set of all possible permutations.

Strictly speaking, this definition of the ideal block cipher is incomplete, as the exact choice of the tables has not been specified. As soon as we specify the tables, however, the ideal cipher is fixed and no longer random. To formalize the definition, we cannot talk about a single ideal block cipher, but have to treat the ideal block cipher as a uniform probability distribution over the set of all possible block ciphers. Any time that you use the ideal block cipher, you will have to talk in terms of probabilities. This is a mathematician's delight, but the added complexity would make our explanations far more complicated—so we will keep the informal but simpler concept of a randomly chosen block cipher. We also stress that an ideal block cipher is not something that can be obtained in practice; it is an abstract concept that we use when discussing security.

3.4 Definition of Block Cipher Security

As noted above, there are formal definitions of security for block ciphers in the literature. For our purposes we can use a simpler but informal definition.

Definition 1 *A secure block cipher is one for which no attack exists.*

This is a bit of a tautology. So now we have to define an attack on a block cipher.

Definition 2 *An attack on a block cipher is a non-generic method of distinguishing the block cipher from an ideal block cipher.*

What do we mean by distinguishing a block cipher from an ideal block cipher? Given a block cipher X, we compare it to an ideal block cipher with the same block size and the same key size. A *distinguisher* is an algorithm that is given a black-box function that computes either the block cipher X or an ideal block cipher. (A black-box function is a function that can be evaluated, but the distinguisher algorithm does not know the internal workings of the function in the black box.) Both the encryption and decryption functions are available, and the distinguisher algorithm is free to choose any key for each of the encryptions and decryptions it performs. The distinguisher's task is to figure out whether the black-box function implements the block cipher X or the ideal cipher. It doesn't have to be a perfect distinguisher, as long as it provides the correct answer significantly more often than the wrong answer.

There are, of course, generic (and trivial) solutions to this. We could encrypt the plaintext 0 with the key 0 and see if the result matches what we expect to get from block cipher X. This is a distinguisher, but to make it an attack, the distinguisher has to be non-generic. This is where it becomes difficult to define block cipher security. We cannot formalize the notion of ''generic'' and ''non-generic.'' It is a bit like obscenity: we know it when we see it.[2] A distinguisher is generic if we can find a similar distinguisher for almost any block cipher. In the above case, the distinguisher is generic because we can construct one just like it for any block cipher. This ''attack'' would even allow us to distinguish between two ideal block ciphers. Of course, there's no practical reason for wanting to distinguish between two ideal block ciphers. Rather, this attack is generic because we could use it to distinguish between two ideal block ciphers if we wanted to. The attack doesn't exploit any internal property of the block cipher itself.

We can also create a more advanced generic distinguisher. Encrypt the plaintext 0 with all keys in the range $1, \ldots, 2^{32}$ and count how often each value for the first 32 bits of the ciphertext occurs. Suppose we find that for a cipher X the value t occurs 5 times instead of the expected one time. This is a property that is unlikely to hold for the ideal cipher, and would allow us to distinguish X from an ideal cipher. This is still a generic distinguisher, as we can easily construct something similar for any cipher X. (It is in fact extremely unlikely that a cipher does not have a suitable value for t.) This attack is generic since, the way it is described, it is applicable to all block ciphers and doesn't exploit a specific weakness of X. Such a distinguisher would even allow us to distinguish between two ideal ciphers.

Things become more complicated if we design a distinguisher as follows: We make a list of 1000 different statistics that we can compute about a cipher. We compute each of these for cipher X, and build the distinguisher from the

[2] In 1964, U.S. Supreme Court judge Potter Stewart used these words to define obscenity: ''I shall not today attempt further to define the kinds of material . . . but I know it when I see it.''

statistic that gives the most significant result. We expect to find a statistic with a significance level of about 1 in 1000. We can of course apply the same technique to find distinguishers for any particular cipher, so this is a generic attack, but the generic nature now depends not only on the distinguisher itself, but also on how the distinguisher was found. That's why nobody has been able to formalize a definition of generic attacks and block cipher security. We would love to give you a clean definition of block cipher security, but the cryptographic community does not yet know enough about cryptography to be able to do this in full generality. Instead, existing formal definitions often limit the capability of an attacker. For example, existing formal definitions might not allow chosen-key attacks. While these assumptions can hold in some cases, we try to build block ciphers that are much stronger.

We must not forget to limit the amount of computation allowed in the distinguisher. We could have done this explicitly in the definition, but that would have complicated it even further. If the block cipher has an explicit security level of n bits, then a successful distinguisher should be more efficient than an exhaustive search on n-bit values. If no explicit design strength is given, the design strength equals the key size. This formulation is rather roundabout for a reason. It is tempting to just say that the distinguisher has to work in less than 2^n steps. This is certainly true, but some types of distinguishers give you only a probabilistic result that is more like a partial key search. The attack could have a trade-off between the amount of work and the probability of distinguishing the cipher from the ideal cipher. For example: an exhaustive search of half the key space requires 2^{n-1} work and provides the right answer 75% of the time. (If the attacker finds the key, he knows the answer. If he doesn't find the key, he still has a 50% chance of guessing right simply by guessing at random. Overall, his chances of getting the right answer are therefore $0.5 + 0.5 \cdot 0.5 = 0.75$.) By comparing the distinguisher to such partial key-space searches, we take this natural trade-off into account, and stop such partial key searches from being classified as an attack.

Our definition of block cipher security covers all possible forms of attack. Ciphertext only, known plaintext, (adaptively) chosen plaintext, related key, and all other types of attack all implement a non-generic distinguisher. That is why we will use this informal definition in this book. It also captures the essence of professional paranoia that we talked about in Chapter 1; we want to capture anything that could possibly be considered a non-generic attack.

So why spend multiple pages on defining what a secure block cipher is? This definition is very important because it defines a simple and clean interface between the block cipher and the rest of the system. This sort of modularization is a hallmark of good design. In security systems, where complexity is one of our main enemies, good modularization is even more important than in most other areas. Once a block cipher satisfies our security definition, you can treat it as if it were an ideal cipher. After all, if it does not behave as an ideal cipher in the system, then you have found a distinguisher for the cipher, which means the cipher is not secure according to our definition. If you use a secure block cipher, you no longer have to remember any particularities or imperfections; the cipher will have all the properties that you expect a block cipher to have. This makes the design of larger systems easier.

Of course, some ciphers that don't meet our stringent definition might be ''good enough'' in practice or for a specific application as currently defined, but why take the risk? Even if the weaknesses of a particular block cipher under our definition are highly theoretical—such as requiring an unrealistic amount of work to exploit and thus not being very vulnerable to compromise in practice—a block cipher that meets our definition is much more attractive.

3.4.1 Parity of a Permutation

Unfortunately, we have one more complication. As we discussed in Section 3.1, encryption under a single key corresponds to a lookup in a permutation table. Think about constructing this table in two steps. First you initialize the table with the identity mapping by giving the element at index *i* the value *i*. Then you create the permutation that you want by repeatedly swapping two elements in the table. It turns out there are two types of permutations: those that can be constructed from an even number of swaps (called the even permutations) and those that can be constructed from an odd number of swaps (called the odd permutations). It should not surprise you that half of all permutations are even, and the other half are odd.

Most modern block ciphers have a 128-bit block size, but they operate on 32-bit words. They build the encryption function from many 32-bit operations. This has proved to be a very successful method, but it has one side effect. It is rather hard to build an odd permutation from small operations; as a result, virtually all block ciphers only generate even permutations.

This gives us a simple distinguisher for nearly any block cipher, one which we call the *parity attack*. For a given key, extract the permutation by encrypting all possible plaintexts. If the permutation is odd, we know that we have an ideal block cipher, because the real block cipher never generates an odd permutation. If the permutation is even, we claim to have a real block cipher. This distinguisher will be right 75% of the time. It will produce the wrong answer only if it is given an ideal cipher that produces an even permutation. The success rate can be improved by repeating the work for other key values.

This attack has no practical significance whatsoever. To find the parity of a permutation, you have to compute all but one of the plaintext/ciphertext pairs of the encryption function. (The last one is trivial to deduce: the sole remaining plaintext maps to the sole remaining ciphertext.) You should never allow that many plaintext/ciphertext queries to a block cipher in a real system, because other types of attacks start to hurt much sooner. In particular, once the attacker knows most of the plaintext/ciphertext pairs, he no longer needs a key to decrypt the message, but can simply use a lookup table created from those pairs.

We could declare the parity attack to be generic by definition, but that seems disingenuous, since the even parity of block ciphers is a peculiar artifact of their designs. Rather, we prefer to change the definition of the ideal block cipher, and limit it to randomly chosen *even* permutations.

Definition 3 *An ideal block cipher implements an independently chosen random even permutation for each of the key values.*

It is a pity to complicate our ''ideal'' cipher in this way, but the only alternative is to disqualify nearly all known block ciphers. For the overwhelming majority of applications, the restriction to even permutations is insignificant. As long as we never allow all plaintext/ciphertext pairs to be computed, even and odd permutations are indistinguishable.

If you ever have a block cipher that *can* generate odd permutations, you should revert to the original definition of the ideal cipher. In practice, parity attacks have more effect on the formal definition of security than on real-world systems, so you can probably forget about this whole issue of parity.

This discussion also serves as another example of how cryptographers think. It is more important to exhibit professional paranoia and consider a superset of realistic attacks, and then pare away the unrealistic ones, than to start with only realistic attacks and try to find new ones.

3.5 Real Block Ciphers

There are hundreds of block ciphers that have been proposed over the years. It is very easy to design a new block cipher. It is fiendishly hard to design a *good* new block cipher. We're not merely talking about security; that a block cipher has to be secure goes without saying. Building a secure block cipher is a challenge unto itself. But it becomes even more difficult to create a block cipher that is efficient in a wide variety of different applications. (We previously said that we'd give up performance for security. We would. But when possible, we still prefer both.)

Designing block ciphers can be fun and educational, but one shouldn't use an unknown cipher in a real system. The cryptographic community doesn't trust a cipher until it has been reviewed thoroughly by other experts. A basic prerequisite is that the cipher has been published, but this is not enough. There are so many ciphers out there that few get any effective peer review. You are much better off using one of the well-known ciphers that already has been reviewed for you.

Virtually all block ciphers consist of several repetitions of a weak block cipher, known as a *round*. Several of these weak rounds in sequence make a strong block cipher. This structure is easy to design and implement, and is also a great help in the analysis. Most attacks on block ciphers begin by attacking versions with a reduced number of rounds. As the attacks improve, more and more rounds can be attacked.

We will discuss several block ciphers in more detail, but we won't define them exhaustively. The full specifications can be found in the references or on the Internet. We will instead concentrate on the overall structure and the properties of each cipher.

3.5.1 DES

The venerable workhorse of cryptography, the Data Encryption Standard (DES) [96] has finally outlived its usefulness. Its restricted key size of 56 bits and small block size of 64 bits make it unsuitable for today's fast computers and large amounts of data. It survives in the form of 3DES [99], which is a block cipher built from three DES encryptions in sequence—encrypt with DES with one 56-bit key, decrypt with a second 56-bit key, and then encrypt again either with the first key or a third 56-bit key. This solves the most immediate problem of the small key size, but there is no known fix for the small block size. DES is not a particularly fast cipher by current standards, and 3DES is one-third the speed of DES. You will still find DES in many systems, but we do not recommend using either DES or 3DES in new designs. It is, however, a classic design worth studying in its own right.

Figure 3.1 gives an overview of a single round of DES. This is a line diagram of the DES computations; you will commonly find diagrams like this in cryptographic literature. Each box computes a particular function, and the lines show which value is used where. There are a few standard conventions. The XOR or exclusive-or operation, sometimes called bitwise addition or addition without carry, is shown in formulas as a $\oplus$ operator and in figures as a large version of the $\oplus$ operator. You might also find drawings that include integer additions, which often are drawn to look like the $\boxplus$ operator.

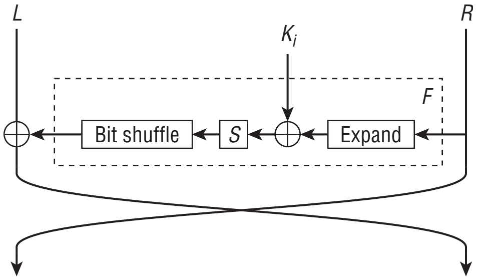

Figure 3.1: Structure of a single round of DES

DES has a 64-bit plaintext, which is split into two 32-bit halves L and R. This splitting is done by rearranging the bits in a semi-ordered fashion. Nobody seems to know why the designers bothered to rearrange the bits of the plaintext—it has no cryptographic effect—but that's how DES is defined. A similar swapping of bits is implemented at the end of the encryption to create the 64-bit ciphertext from the two halves L and R.

DES consists of 16 rounds numbered 1 through 16. Each round i uses a separate 48-bit round key K_i. Each round key is formed by selecting 48 bits from the 56-bit key, and this selection is different for each round key.[3] The algorithm that derives these round keys from the main block cipher key is called the key schedule.

Round i transforms the (L, R) pair into a new (L, R) pair under control of a round key K_i. Most of the work is done by the round function F, shown in the dashed box. As shown in the figure, the R value is first processed by an expand function, which duplicates a number of bits to produce 48 bits of output from the 32-bit input. The 48-bit result is xored with the 48-bit round key K_i. The result of this is used in the S-box tables. An S-box (the term derives from *substitution box*) is basically just a lookup table that is publicly known. As you cannot build a lookup table with 48 input bits, the S-boxes consist of eight small lookup tables, each of which maps 6 bits to 4 bits. This brings the result size back to 32 bits. These 32 bits are then swapped around by the bit shuffle function before being xored into the left value L. Finally, the values of L and R are swapped. This entire computation is repeated 16 times for a single DES encryption.

The basic structure of DES is called the Feistel construction [47]. It is a really elegant idea. Each round consists of xoring L with $F(K_i, R)$ for some function F, and then swapping L and R. The beauty of the construction is that decryption requires exactly the same set of operations as encryption. You need to swap L and R, and you need to xor L with $F(K_i, R)$. This makes it much easier to implement the encryption and decryption functions together. It also means that you only have to analyze one of the two functions, as they are almost identical. A final trick used in most Feistel ciphers is to leave out the swap after the last round, which makes the encryption and decryption functions identical except for the order of the round keys. This is particularly nice for hardware implementations, as they can use the same circuit to compute both encryptions and decryptions.

The different parts of the DES cipher have different functions. The Feistel structure makes the cipher design simpler and ensures that the two halves L and R are mixed together. Xoring the key material ensures that the key and data are mixed, which is the whole point of a cipher. The S-boxes provide nonlinearity. Without them, the cipher could be written as a bunch of binary additions, which would allow a very easy mathematical attack based on linear

[3] There is some structure to this selection, which you can find in the DES specifications [96].

algebra. Finally, the combination of the S-box, expand, and bit shuffle functions provide diffusion. They ensure that if one bit is changed in the input of F, more than one bit is changed in the output. In the next round there will be more bit changes, and even more in the round after that, etc. Without good diffusion, a small change in the plaintext would lead to a small change in the ciphertext, which would be very easy to detect.

DES has a number of properties that disqualify it according to our security definition. Each of the round keys consists purely of some of the bits selected from the cipher key. If the cipher key is 0, then all the round keys are 0 as well. In particular, all the round keys are identical. Remember that the only difference between encryption and decryption is the order of the round keys. But all round keys are zero here. So encryption with the 0 key is the same function as decryption with the 0 key. This is a very easy property to detect, and as an ideal block cipher does not have this property, it leads to an easy and efficient distinguishing attack.[4]

DES also has a complementation property that ensures that

$$E(\overline{K}, \overline{P}) = \overline{E(K, P)}$$

for all keys K and plaintexts P, where $\overline{X}$ is the value obtained by complementing all the bits in X. In other words, if you encrypt the complement of the plaintext with the complement of the key, you get the complement of the (original) ciphertext.

This is rather easy to see. Look at the figure and think about what happens if you flip all the bits in L, R, and K_i. The expand function merely copies bits around, so all the output bits are also flipped. The XOR with the key K_i has both inputs flipped, so the output remains the same. The input to the S-boxes remains the same, the output of the S-boxes remains the same, so the final XOR has one input that is flipped and one input that is the same. The new L value, soon to be swapped to the R position, is therefore also flipped. In other words, if you complement L and R at the beginning of the round and complement K_i as well, then the output is the complement of what you had originally. This property passes through the entire cipher.

The ideal block cipher would not have this curious property. More importantly, this particular property can lead to attacks on systems that use DES.

In short, DES does not pass muster anymore. The above properties disqualify DES according to our security definition. But even ignoring the properties above, the DES key length is wholly inadequate. There have already been several successful attempts to find a DES key by simple exhaustive search [44].

3DES has a larger key, but it inherits both the weak keys and the complementation property from DES, each of which is enough to disqualify the

[4] There are three other keys that have this property; together, they are called the weak keys of DES.

cipher by our standards. It is also severely limited by its 64-bit block size, which imposes severe restrictions on the amount of data we can encrypt with a single key. (See Section 4.8 for details.) Sometimes you have to use 3DES in a design for legacy reasons, but be very careful with it because of its small block size and because it does not behave like an ideal block cipher.

3.5.2 AES

The Advanced Encryption Standard (AES) is the U.S. government standard created to replace DES. Instead of designing or commissioning a cipher, the U.S. National Institute of Standards and Technology (NIST) asked for proposals from the cryptographic community. A total of 15 proposals were submitted [98]. Five ciphers were selected as finalists [100], after which Rijndael was selected to become AES.[5] AES became a standard in 2001.

AES uses a different structure than DES. It is not a Feistel cipher. Figure 3.2 shows a single round of AES. The subsequent rounds are similar. The plaintext comes in as 16 bytes (128 bits) at the very top. The first operation is to XOR the plaintext with 16 bytes of round key. This is shown by the ⊕ operators; the key bytes come into the side of the XORs. Each of the 16 bytes is then used as an index into an S-box table that maps 8-bit inputs to 8-bit outputs. The S-boxes are all identical. The bytes are then rearranged in a specific order that looks a bit messy but has a simple structure. Finally, the bytes are mixed in groups of four using a linear mixing function. The term linear just means that each output bit of the mixing function is the XOR of several of the input bits.

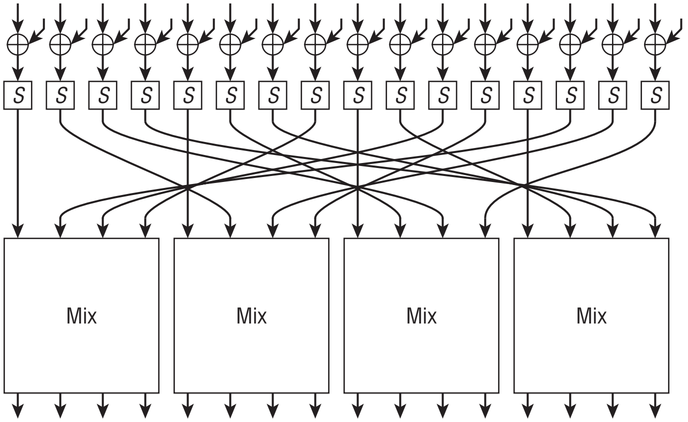

Figure 3.2: Structure of a single round of AES

[5] There has been some confusion about the correct pronunciation of ''Rijndael.'' Don't worry; it's hard to pronounce unless you speak Dutch, so just relax and pronounce it any way you like, or just call it ''AES.''

This completes a single round. A full encryption consists of 10–14 rounds, depending on the size of the key. AES is defined for 128-, 192-, and 256-bit keys, and uses 10 rounds for 128-bit keys, 12 rounds for 192-bit keys, and 14 rounds for 256-bit keys. Like DES, there is a key schedule that generates the necessary round keys, but the key schedule uses a very different structure.

The AES structure has advantages and disadvantages. Each step consists of a number of operations that can be performed in parallel. This parallelism makes high-speed implementations easy. On the other hand, the decryption operation is significantly different from the encryption operation. You need the inverse lookup table of the S-box, and the inverse mixing operation is different from the original mixing operation.

We can recognize some of the same functional blocks as in DES. The XORs add key material to the data, the S-boxes provide nonlinearity, and the byte shuffle and mixing functions provide diffusion. AES is a very clean design with clearly separated tasks for each part of the cipher.

AES has always been a fairly aggressively designed cipher. In the original presentation, the AES designers showed an attack on 6 rounds. This means that the designers knew of an attack if AES was defined to have only 6 rounds. The authors therefore chose 10–14 rounds for the full cipher, depending on the key size [27].

During the AES selection process, the attacks were improved to handle 7 rounds for 128-bit keys, 8 rounds for 192-bit keys, and 9 rounds for 256-bit keys [49]. This still left a 3 to 5 round security margin. From a different perspective: for 128-bit keys, the best attack we knew when Rijndael was selected as AES covered 70% of the cipher. In other words, the selection of Rijndael as AES relied on the assumption that future attacks would not give large improvements.

Will AES stand the test of time? It is, as always, impossible to predict the future, but sometimes it helps to look at the past. Until recently, the best-analyzed ciphers were DES, FEAL, and IDEA. In all cases, the attacks were significantly improved many years after the initial publication. Since then, the field has progressed, but it still takes a leap of faith to think we know it all and that no significant improvements in attacks will be found.

In fact, at the time of this writing we are starting to see some pretty amazing breakthroughs in the cryptanalysis of AES [14, 15, 16]. One attack can break the full 12 rounds of AES with 192-bit keys using four related keys and 2^{176} operations, and another attack can break the full 14 rounds of AES with 256-bit keys using four related keys and 2^{119} operations [15]. Another attack can break 10 of the 14 rounds of AES with 256-bit keys using two related keys and only 2^{45} operations [14].

These are huge results. They mean we now know AES does not meet our definition of security for a block cipher. The attacks against the full 192- and 256-bit versions of AES are theoretical—not practical—so we aren't ready to lose any sleep over them just yet. But they are attacks under our definition, so 192- and 256-bit AES have theoretically been broken. And even better attacks might be discovered over time.

The community is still trying to come to grips with what these results mean for the use of AES in a real system. Given all that we know today, using AES still seems like a reasonable decision. It is the U.S. government standard, which means a great deal. Using the standard avoids a number of discussions and problems. But it is important to realize it is still possible that future cryptanalytic advances may uncover even more serious weaknesses. If you are developing a system or standardizing a protocol, we recommend building in some flexibility or extensibility in case you need to replace AES with another block cipher in the future. We will come back to this in Section 3.5.6.

3.5.3 Serpent

Serpent was another AES finalist [1]. It is built like a tank. Easily the most conservative of all the AES submissions, Serpent is in many ways the opposite of AES. Whereas AES puts emphasis on elegance and efficiency, Serpent is designed for security all the way. The best attack we know of covers only 12 of the 32 rounds [38]. The disadvantage of Serpent is that it is about one-third the speed of AES. It can also be difficult to implement efficiently, as the S-boxes have to be converted to a Boolean formula suitable for the underlying CPU.

In some ways, Serpent has a similar structure to AES. It consists of 32 rounds. Each round consists of XORing in a 128-bit round key, applying a linear mixing function to the 128 bits, and then applying 32 four-bit S-boxes in parallel. In each round, the 32 S-boxes are identical, but there are eight different S-boxes that are used each in turn in a round.

Serpent has an especially nice software implementation trick. A straightforward implementation would be very slow, as each round requires 32 S-box lookups and there are 32 rounds. In total there are 1024 S-box lookups, and doing those one by one would be very slow. The trick is to rewrite the S-boxes as Boolean formulas. Each of the four output bits is written as a Boolean formula of the four input bits. The CPU then evaluates this Boolean formula directly, using AND, OR, and XOR instructions. The trick is that a 32-bit CPU can evaluate 32 S-boxes in parallel, as each bit position in the registers computes the same function, albeit on different input data. This style of implementation is called a *bitslice implementation*. Serpent is specifically designed to be implemented in this way. The mixing phase is relatively easy to compute in a bitslice implementation.

If Serpent had been as fast as Rijndael (now AES), it would almost certainly have been chosen as AES because of its conservative design. But speed is always a relative thing. When measured per encrypted byte, Serpent is nearly as fast as DES and much faster than 3DES. It is only when Serpent is compared to the other AES finalists that it seems slow.

3.5.4 Twofish

Twofish was an AES finalist as well. It can be seen as a compromise between AES and Serpent. It is nearly as fast as AES, but it has a larger security margin. The best attack we know of is on 8 of the 16 rounds. The biggest disadvantage of Twofish is that it can be rather expensive to change the encryption key, as Twofish is best implemented with a lot of precomputation on the key.

Twofish uses the same Feistel structure as DES. An overview is given in Figure 3.3.[6] Twofish splits the 128-bit plaintext into four 32-bit values, and most operations are on 32-bit values. You can see the Feistel structure of Twofish, with *F* being the round function. The round function consists of two copies of the *g* function, a function called the PHT, and a key addition. The result of the *F* function is XORed into the right half (the two vertical lines on the right). The boxes with $\lll$ or $\ggg$ symbols in them denote rotations of the 32-bit value by the specified number of bit positions.

Each *g* function consists of four S-boxes followed by a linear mixing function that is very similar to the AES mixing function. The S-boxes are somewhat different. In contrast to all the other block ciphers we have seen in this book, these S-boxes are not constant; rather, their contents depend on the key. There is an algorithm that computes the S-box tables from the key material. The motivation for this design is that key-dependent S-boxes are much harder for an attacker to analyze. This is also why Twofish implementations often do precomputations for each key. They precompute the S-boxes and store the result in memory.

The PHT function mixes the two results of the *g* functions using 32-bit addition operations. The last part of the *F* function is where the key material is added. Note that addition is shown as $\boxplus$ and exclusive or as $\oplus$.

Twofish also uses *whitening*. At both the start and the end of the cipher, additional key material is added to the data. This makes the cipher harder to attack for most types of attacks, and it costs very little.

As with the other ciphers, Twofish has a key schedule to derive the round keys and the two additional keys at the start and end from the actual cipher key.

[6] There is a reason why this figure is so much larger and detailed than the others. Two of us were on the Twofish design team, so we could lift this figure straight from our Twofish book [115].

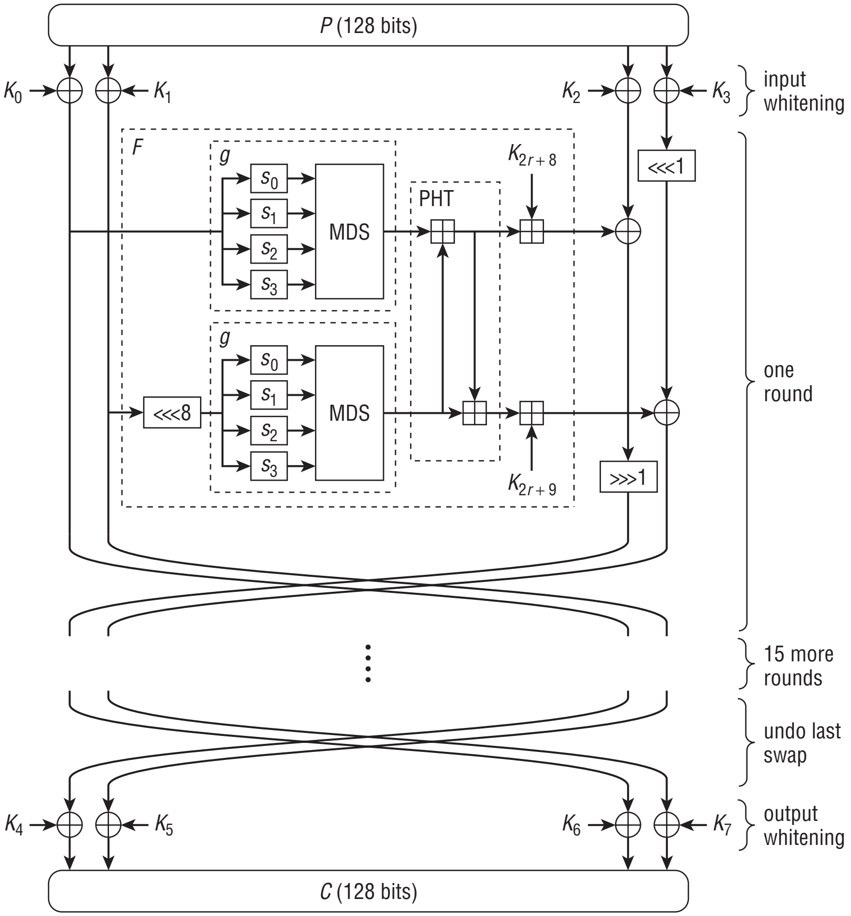

Figure 3.3: Structure of Twofish
© 1999, Niels Ferguson, John Wiley and Sons. Used with permission.

3.5.5 Other AES Finalists

We have discussed three of the five AES finalists in some detail. There were two more: RC6 [108] and MARS [22].

RC6 is an interesting design that uses 32-bit multiplications in the cipher. During the AES competition, the best attack broke a 17-round version of RC6, compared to 20 rounds of the full RC6. MARS is a design with a nonuniform structure. It uses a large number of different operations and is therefore more expensive to implement than the other AES finalists.

Both RC6 and MARS were selected as AES finalists for a reason: They are both probably good block ciphers. Details about their internal operations are in their respective specifications.

3.5.6 Which Block Cipher Should I Choose?

The recent cryptanalytic advances against AES make this a tough choice. Despite these cryptanalytic advances, AES is still what we recommend. It is fast. All known attacks are theoretical, not practical. Even though AES is now broken academically, these breaks do not imply a significant security degradation of real systems in practice. It is also the official standard, sanctioned by the U.S. government. And everybody else is using it. They used to say ''Nobody gets fired for buying IBM.'' Similarly, nobody will fire you for choosing AES.

AES has other advantages. It is relatively easy to use and implement. All cryptography libraries support it, and customers like it, because it is ''the standard.''

There are probably circumstances in which 3DES still is the best solution. If you have to be backward-compatible, or are locked into a 64-bit block size by other parts of the system, then 3DES is still your best choice. However, keep in mind that 3DES has some unique properties that cause it to not satisfy our security criteria; and be especially careful with the small 64-bit block size.

If you are really concerned about future cryptanalytic advances, you could always double encrypt—first with AES and then with Serpent or Twofish. If you do this, remember to use different, independent keys for each block cipher. Or use AES with an increased number of rounds—say, 16 rounds for AES with 128-bit keys, 20 rounds for AES with 192-bit keys, and 28 rounds for AES with 256-bit keys.

Further, remember that the recent cryptanalytic advances against AES are only just coming out as we finalize this book. It is too early to tell exactly how the community will respond. Keep an eye out for a general consensus or shift in direction from the community. Perhaps NIST will issue some specific recommendations for how to address the recent discoveries against AES. If NIST makes recommendations on how to respond to the new attacks against AES, or if there is a clear shift in the consensus of the community, no one will fault you for following those recommendations or that shift.

We also need to return to one other issue with AES. We haven't talked much about side-channel or timing attacks yet (we'll talk about these in Sections 8.5 and 15.3). It turns out that even though there are no known practical attacks against the mathematics of AES, it is possible to implement AES poorly. For example, it is possible to implement AES such that the time it takes to perform an operation depends on its inputs—on some inputs it will take more time and on other inputs it will take less time. If an attacker can measure the time a system takes to perform an AES operation, she might be able to learn bits of the key. If you use AES, you should be careful to use a constant-time implementation, or to otherwise conceal the timing information from an attacker.

3.5.7 What Key Size Should I Use?

All of the AES finalists (Rijndael, Serpent, Twofish, RC6, and MARS), and hence AES, support keys of 128, 192, and 256 bits. For almost all applications, a 128-bit security level is enough. However, to achieve 128 bits of security, we suggest keys longer than 128 bits.

A 128-bit key would be great, except for one problem: collision attacks. Time and time again, we find systems that can be attacked—at least theoretically, if not practically—by a birthday attack or a meet-in-the-middle attack. We know these attacks exist. Sometimes designers just ignore them, and sometimes they think they are safe, but somebody finds a new, clever way of using them. Most block cipher modes allow meet-in-the-middle attacks of some form. We've had enough of this race, so here is our recommendation: For a security level of n bits, every cryptographic value should be at least $2n$ bits long.

Following this recommendation makes any type of collision attack useless. In real life, it is hard to keep strictly to this rule. For 128-bit security, we really want to use a block cipher with a block size of 256 bits, but all the common block ciphers have a block size of 128 bits. This is more serious than it sounds. There are quite a number of collision attacks on block cipher modes, which we will learn about later.

Still, at least we can use the large keys that all AES candidate block ciphers support. Therefore: use 256-bit keys! We are not saying that 128-bit keys are insecure per se; we are saying that 256-bit keys provide a better safety margin, assuming that the block cipher is secure.

Note that we advocate the use of 256-bit keys for systems with a design strength of 128 bits. In other words, these systems are designed to withstand attackers that can perform 2^{128} operations in their attack. Just remember to use the design strength (128 bits), not the key length of 256 bits, for sizing the rest of the system.

Finally, let's come back to the recent cryptanalytic results against AES. These results show that AES with 192- and 256-bit keys are not secure. Moreover, the attacks against AES with 192- and 256-bit keys exploit weaknesses in the AES key schedule algorithm. This is why the known attacks against AES with 256-bit keys are more efficient than the attacks against AES with 192-bit keys. This is also why we don't yet know of attacks against AES with 128-bit keys. So, while in general we'd prefer a block cipher with 256-bit keys over a block cipher with 128-bit keys, assuming the block cipher is secure, the situation is a bit more murky for AES. To emphasize our desire for 128 bits of security, and thus our quest for a secure block cipher with 256-bit keys, we will use AES with 256-bit keys throughout the rest of this book. But once there is a clear consensus of how to respond to the new cryptanalytic results against AES, we will likely replace AES with another block cipher with 256-bit keys.

3.6 Exercises

Exercise 3.1 How much space would be required to store a table for an entire idealized block cipher that operates on 64-bit blocks and that has 80-bit keys?

Exercise 3.2 How many rounds are in DES? How many bits are in a DES key? What is the DES block size? How does 3DES work as a function of DES?

Exercise 3.3 What are the possible lengths for AES keys? For each key length, how many rounds are in AES? What is the AES block size?

Exercise 3.4 Under what situations might you choose 3DES over AES? Under what situations might you chose AES over 3DES?

Exercise 3.5 Suppose you have a processor that can perform a single DES encryption or decryption operation in 2^{-26} seconds. Suppose you also have a large number of plaintext-ciphertext pairs for DES under a single, unknown key. How many hours would it take, on average, to find that DES key, using an exhaustive search approach and a single processor? How many hours would it take, on average, to find that DES key, using an exhaustive search approach and a collection of 2^{14} processors?

Exercise 3.6 Consider a new block cipher, DES2, that consists only of two rounds of the DES block cipher. DES2 has the same block and key size as DES. For this question you should consider the DES *F* function as a black box that takes two inputs, a 32-bit data segment and a 48-bit round key, and that produces a 32-bit output.

Suppose you have a large number of plaintext-ciphertext pairs for DES2 under a single, unknown key. Give an algorithm for recovering the 48-bit round key for round 1 and the 48-bit round key for round 2. Your algorithm should require fewer operations than an exhaustive search for an entire 56-bit DES key. Can your algorithm be converted into a distinguishing attack against DES2?

Exercise 3.7 Describe an example system that uses DES but is insecure because of the DES complementation property. Specifically, describe the system, and then present an attack against that system; the attack should utilize the DES complementation property.

Exercise 3.8 Familiarize yourself with a cryptographic software development package for your computer. A popular open source package is OpenSSL, though there are numerous other alternatives.

Using an existing cryptography library, decrypt the following ciphertext (in hex)

```
53 9B 33 3B 39 70 6D 14 90 28 CF E1 D9 D4 A4 07
```

with the following 256-bit key (also in hex)

```
80 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01
```

using AES.

Exercise 3.9 Using an existing cryptography library, encrypt the following plaintext (in hex)

```
29 6C 93 FD F4 99 AA EB 41 94 BA BC 2E 63 56 1D
```

with the following 256-bit key (also in hex)

```
80 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01
```

using AES.

Exercise 3.10 Write a program that experimentally demonstrates the complementation property for DES. This program should take as input a key *K* and a plaintext *P* and demonstrate that the DES complementation property holds for this key and plaintext. You may use an existing cryptography library for this exercise.

CHAPTER

4

Block Cipher Modes

Block ciphers encrypt only fixed-size blocks. If you want to encrypt something that isn't exactly one block long, you have to use a *block cipher mode*. That's another name for an encryption function built using a block cipher.

Before proceeding with this chapter, we have one word of warning. The encryption modes that we talk about in this chapter prevent an eavesdropper from reading the traffic. They do not provide any authentication, so an attacker can still change the message—sometimes in any way she wants. Many people find this surprising, but this is simple to see. The decryption function of an encryption mode simply decrypts the data. It might produce nonsense, but it still decrypts a (modified) ciphertext to some (modified and possibly nonsensical) plaintext. You should not rely on the fact that nonsensical messages do no harm. That involves relying on other parts of the system, which all too often leads to grief. Furthermore, for some encryption schemes, the modified ciphertexts may not decrypt to garbage; some modes allow targeted plaintext changes and many data formats can be manipulated even with locally randomizing changes.

In almost all situations, the damage that modified messages can do is far greater than the damage of leaking the plaintext. Therefore, you should always combine encryption with authentication. The modes we discuss here should be combined with a separate authentication function, which we discuss in Chapter 6.

4.1 Padding

In general, a block cipher mode is a way to encrypt a plaintext P to a ciphertext C, where the plaintext and ciphertext are of an arbitrary length. Most modes require that the length of the plaintext P be an exact multiple of the block size. This requires some padding. There are many ways to pad the plaintext, but the most important rule is that the padding must be reversible. It must be possible to uniquely determine the original message from a padded message.

We sometimes see a very simple padding rule that consists of appending zeros until the length is suitable. This is not a good idea. It is not reversible, as the plaintext p and $p \parallel 0$ have the same padded form. (We use the operator $\parallel$ to denote concatenation.)

Throughout this book, we will only consider plaintexts that are an integral number of bytes long. Some cryptographic primitives are specified for odd sizes where the last byte is not fully used. We have never found this generalization useful, and it often is a hindrance. Many implementations do not allow for these odd sizes in any case, so all our sizes will be in bytes.

It would be nice to have a padding rule that does not make the plaintext any longer if it already has a suitable length. This is not possible to achieve for all situations. You can show that at least some messages that are already of a suitable length must be lengthened by any reversible padding scheme, and in practice all padding rules add a minimum of one byte to the length of the plaintext.

So how do we pad a plaintext? Let P be the plaintext and let $\ell(P)$ be the length of P in bytes. Let b be the block size of the block cipher in bytes. We suggest using one of two simple padding schemes:

1. Append a single byte with value 128, and then as many zero bytes as required to make the overall length a multiple of b. The number of zero bytes added is in the range $0, \ldots, b-1$.
2. Determine the number of padding bytes required. This is a number n which satisfies $1 \leq n \leq b$ and $n + \ell(P)$ is a multiple of b. Pad the plaintext by appending n bytes, each with value n.

Either padding scheme works just fine. There are no cryptographic ramifications to padding. Any padding scheme is acceptable, as long as it is reversible. The two we gave are just the simplest ones. You could also include the length of P at the beginning, and then P, and then pad to a block boundary. This assumes that you know the length of P when you start processing the data, which you might not.

Once the padded length is a multiple of the block size, we cut the padded plaintext into blocks. The plaintext P is thereby turned into a sequence of blocks $P_1, \ldots, P_k$. The number of blocks k can be computed as $\lceil (\ell(P)+1)/b \rceil$, where $\lceil \cdots \rceil$ denotes the ceiling function that rounds a number upward to the next integer. For most the rest of this chapter we will simply assume that the plaintext P consists of an integral number of blocks $P_1, \ldots, P_k$.

After decrypting the ciphertext using one of the block cipher modes we will discuss, the padding has to be removed. The code that removes the padding should also check that the padding was correctly applied. Each of the padding bytes has to be verified to ensure it has the correct value. An erroneous padding should be treated in the same manner as an authentication failure.

4.2 ECB

The simplest method to encrypt a longer plaintext is known as the *electronic codebook* mode, or ECB. This is defined by

$$C_i = E(K, P_i) \qquad \text{for } i = 1, \ldots, k$$

This is quite simple: you just encrypt each block of the message separately. Of course, things cannot be so simple, or we would not have allocated an entire chapter to the discussion of block cipher modes. Do not ever use ECB for anything. It has serious weaknesses, and is only included here so that we can warn you away from it.

What is the trouble with ECB? If two plaintext blocks are the same, then the corresponding ciphertext blocks will be identical, and that is visible to the attacker. Depending on the structure of the message, this can leak quite a lot of information to the attacker.

There are many situations in which large blocks of text are repeated. For example, this chapter contains the words ''ciphertext block'' many times. If two of the occurrences happen to line up on a block boundary, then a plaintext block value will be repeated. In most Unicode strings, every other byte is a zero, which greatly increases the chance of a repeated block value. Many file formats will have large blocks of only zeros, which result in repeated block values. In general, this property of ECB makes it too weak to use.

4.3 CBC

The *cipher block chaining* (CBC) mode is one of the most widely used block cipher modes. The problems of ECB are avoided by XORing each plaintext block with the previous ciphertext block. The standard formulation of CBC is as follows:

$$C_i = E(K, P_i \oplus C_{i-1}) \qquad \text{for } i = 1, \ldots, k$$

The problems of ECB are avoided by ''randomizing'' the plaintext using the previous ciphertext block. Equal plaintext blocks will typically encrypt to different ciphertext blocks, significantly reducing the information available to an attacker.

We are still left with the question of which value to use for C_0. This value is called the *initialization vector*, or IV. We discuss strategies for picking the IV below.

4.3.1 Fixed IV

You should not use a fixed IV, as that introduces the ECB problem for the first block of each message. If two different messages start with the same plaintext block, their encryptions will start with the same ciphertext blocks. In real life, messages often start with similar or identical blocks, and we do not want the attacker to be able to detect this.

4.3.2 Counter IV

An alternative idea we sometimes see is to use a counter for the IV. Use IV = 0 for the first message, IV = 1 for the second message, etc. Again, this is not a very good idea. As we mentioned, many real-life messages start in similar ways. If the first blocks of the messages have simple differences, then the simple IV counter could very well cancel the differences in the XOR, and generate identical ciphertext blocks again. For example: the values 0 and 1 differ in exactly one bit. If the leading plaintext blocks of the first two messages also differ in only this bit (which happens much more often than you might expect), then the leading ciphertext blocks of the two messages will be identical. The attacker can promptly draw conclusions about the differences between the two messages, something a secure encryption scheme should not allow.

4.3.3 Random IV

The problems with ECB and fixed-IV or counter-IV CBC both stem from the fact that plaintext messages are highly nonrandom. Very often they have a fixed value header, or a very predictable structure. A chosen plaintext attacker could even exert control over the structure of the plaintext. In CBC, the ciphertext blocks are used to ''randomize'' the plaintext blocks, but for the first block we have to use the IV. This suggests that we should choose a random IV.

This leads to another problem. The recipient of the message needs to know the IV. The standard solution is to choose a random IV and to send it as a first block before the rest of the encrypted message. The resulting encryption procedure is as follows:

$$\begin{aligned} C_0 &:= \text{random block value} \\ C_i &:= E(K, P_i \oplus C_{i-1}) \qquad \text{for } i = 1, \ldots, k \end{aligned}$$

with the understanding that the (padded) plaintext $P_1, \ldots, P_k$ is encrypted as $C_0, \ldots, C_k$. Note that the ciphertext starts at C_0 and not C_1; the ciphertext is one block longer than the plaintext. The corresponding decryption procedure is easy to derive:

$$P_i := D(K, C_i) \oplus C_{i-1} \qquad \text{for } i = 1, \ldots, k$$

The principal disadvantage of a random IV is that the ciphertext is one block longer than the plaintext. For short messages, this results in a significant message expansion, which is always undesirable.

4.3.4 Nonce-Generated IV

Here is another solution to the IV problem. The solution consists of two steps. First, each message that is to be encrypted with this key is given a unique number called a *nonce*. The term is a contraction of ''*n*umber used *once*.'' The critical property of a nonce is that it is unique. You should never use the same nonce twice with the same key. Typically, the nonce is a message number of some sort, possibly combined with some other information. Message numbers are already available in most systems, as they are used to keep the messages in their correct order, detect duplicate messages, etc. The nonce does not have to be secret, but it can be used only once.

The IV necessary for CBC encryption is generated by encrypting the nonce.

In a typical scenario, the sender numbers the messages consecutively and includes the message number in each transmission. The following steps should be used to send a message:

1. Assign a message number to this message. Typically, the message number is provided by a counter that starts at 0. Note that the counter should never be allowed to wrap around back to 0, as that would destroy the uniqueness property.
2. Use the message number to construct a unique nonce. For a given key, the nonce should be unique in the entire system, not just on this computer. For example, if the same key is used to encrypt traffic in two directions, then the nonce should consist of the message number plus an indication of which direction this message is being sent in. The nonce should be as large as a single block of the block cipher.

3. Encrypt the nonce with the block cipher to generate the IV.
4. Encrypt the message in CBC mode using this IV.
5. Add enough information to the ciphertext to ensure that the receiver can reconstruct the nonce. Typically this involves adding the message number just in front of the ciphertext, or using a reliable transport method for communicating the ciphertext, in which case the message number might be implicit. The IV value itself (C_0 in our equations) does not have to be sent.

The extra information that needs to be included in the message is usually much smaller than in the random IV case. For most systems, a message counter of 32–48 bits is sufficient, compared to a 128-bit random IV overhead for the random IV solution. Most practical communications systems need a message counter anyway, or use a reliable transport with an implicit counter, so the generated IV solution adds no message overhead.

If the attacker has complete control over the nonce, then the nonce should be encrypted with a separate key when generating the IV. Any practical system would need to ensure nonce uniqueness anyway, however, and hence would not allow arbitrary nonce choices. So in most situations we would use the same key to encrypt the nonce as we use to encrypt the message itself.

4.4 OFB

So far the modes have all taken the message and encrypted it by applying the block cipher to the message blocks in some way. *Output feedback* mode, or OFB, is different in that the message itself is never used as an input to the block cipher. Instead, the block cipher is used to generate a pseudorandom stream of bytes (called the *key stream*), which in turn is XORed with the plaintext to generate the ciphertext. An encryption scheme that generates such a random key stream is called a *stream cipher*. Some people seem to think that stream ciphers are bad in some way. Not at all! Stream ciphers are extremely useful, and do their work very well. They just require a bit of care in their use. Abuse of a stream cipher, mostly in the form of reusing a nonce, can very easily lead to an insecure system. A mode like CBC is more robust in the sense that even when abused it still does a pretty good job. Still, the advantages of stream ciphers often outweigh their disadvantages.

OFB is defined by:

$$\begin{aligned} K_0 &:= \mathrm{IV} \\ K_i &:= E(K, K_{i-1}) \qquad \text{for } i = 1, \ldots, k \\ C_i &:= P_i \oplus K_i \end{aligned}$$

Here too, there is an IV K_0 which is used to generate the key stream $K_1, \ldots, K_k$ by repeatedly encrypting the value. The key stream is then xored with the plaintext to generate the ciphertext.

The IV value has to be random, and as with CBC it can either be chosen randomly and transmitted with the ciphertext (see Section 4.3.3), or it can be generated from a nonce (see Section 4.3.4).

One advantage of OFB is that decryption is exactly the same operation as encryption, which saves on implementation effort. Especially useful is that you only need to use the encryption function of the block cipher, so you don't even have to implement the decryption function.

A second advantage is that you don't need any padding. If you think of the key stream as a sequence of bytes, then you can use as many bytes as your message is long. In other words, if the last plaintext block is only partially full, then you only send the ciphertext bytes that correspond to actual plaintext bytes. The lack of padding reduces the overhead, which is especially important with small messages.

OFB also demonstrates the one danger of using a stream cipher. If you *ever* use the same IV for two different messages, they will be encrypted with the same key stream. This is very bad indeed. Let us suppose that two messages are the plaintexts P and P', and they have been encrypted using the same key stream to the ciphertexts C and C' respectively. The attacker can now compute $C_i \oplus C'_i = P_i \oplus K_i \oplus P'_i \oplus K_i = P_i \oplus P'_i$. In other words, the attacker can compute the difference between the two plaintexts. Suppose the attacker already knows one of the plaintexts. (This does happen very often in real life.) Then it is trivial for her to compute the other plaintext. There are even well-known attacks that recover information about two unknown plaintexts from the difference between them [66].

OFB has one further problem: if you are unlucky, you will repeat a key block value, after which the sequence of key blocks simply repeats. In a single large message, you might be unlucky and get into a cycle of key block values. Or the IV for one message might be the same as a key block halfway through the second message, in which case the two messages use the same key stream for part of their plaintexts. In either case, you end up encrypting different message blocks with the same key block, which is not a secure encryption scheme.

You need to encrypt quite a lot of data before this becomes plausible. It is basically a collision attack between the key stream blocks and the initial starting points, so you are talking about encrypting at least 2^{64} blocks of data before you expect such a collision. This is an example of why a block cipher with 128-bit blocks may only provide 64 bits of security. If you limit the amount of data that you encrypt with each key, you can limit the probability of repeating a key block value. Unfortunately, the risk always remains, and if you are unlucky, you could lose the confidentiality of an entire message.

4.5 CTR

Counter mode, generally known by the three-letter abbreviation CTR, is another block cipher encryption mode. Although it has been around for ages, it was not standardized as an official DES mode [95], and therefore has often been overlooked in textbooks. It has recently been standardized by NIST [40]. Like OFB, counter mode is a stream cipher mode. It is defined by:

$$\begin{aligned} K_i &:= E(K, \text{Nonce} \parallel i) \qquad \text{for } i = 1, \ldots, k \\ C_i &:= P_i \oplus K_i \end{aligned}$$

Like any stream cipher, you must supply a unique nonce of some form. Most systems build the nonce from a message number and some additional data to ensure the nonce's uniqueness.

CTR uses a remarkably simple method to generate the key stream. It concatenates the nonce with the counter value, and encrypts it to form a single block of the key stream. This requires that the counter and the nonce fit in a single block, but with modern 128-bit block sizes, this is rarely a problem. Obviously, the nonce must be smaller than a single block, as there needs to be room for the counter value i. A typical setup might use a 48-bit message number, 16 bits of additional nonce data, and 64 bits for the counter i. This limits the system to encrypting 2^{48} different messages using a single key, and limits each message to 2^{68} bytes.

As with OFB mode, you must make absolutely sure never to reuse a single key/nonce combination. This is a disadvantage that is often mentioned for CTR, but CBC has exactly the same problem. If you use the same IV twice, you start leaking data about the plaintexts. CBC is a bit more robust, as it is more likely to limit the amount of information leaked. But any leakage of information violates our requirements, and in a modularized design you cannot count on the rest of the system to limit the damage if you only leak a little bit of information. So both in the case of CBC and CTR you have to ensure that the nonce or IV is unique.

The real question is whether you can ensure that the nonce is unique. If there's any doubt, you should use a mode like random IV CBC mode, where the IV is generated randomly and outside of the application developer's control. But if you can guarantee that the nonce will be unique, then CTR mode is very easy to use. You only need to implement the encryption function of the block cipher, and the CTR encryption and decryption functions are identical. It is very easy to access arbitrary parts of the plaintext, as any block of the key stream can be computed immediately. For high-speed applications, the computation of the key stream can be parallelized to an arbitrary degree. Furthermore, the security of CTR mode is trivially linked to the security of the block cipher. Any weakness of CTR encryption mode immediately implies a chosen plaintext attack on the block cipher. The logical converse of this is that if there is no attack on the block cipher, then there is no attack on CTR mode (other than the traffic analysis and information leakage we will discuss shortly).

4.6 Combined Encryption and Authentication

All of the modes we have discussed so far date back to the 1970s and early 1980s. In the last few years, some new block cipher modes have been proposed. NIST recently chose to standardize two, called CCM [41] and GCM [43]. These modes provide both authentication and encryption. We will discuss these modes in Chapter 7, after we discuss authentication.

4.7 Which Mode Should I Use?

We have discussed several modes, but there are really only two modes we would consider using: CBC and CTR. We've already explained that ECB is not secure enough. OFB is a good mode, but CTR is better in some respects and doesn't suffer from the short cycle problem. There is no reason to choose OFB over CTR.

So, should you use CBC or CTR? In the first edition of this book, we recommended CTR. However, we are always learning more, and we now recommend CBC with random IV. Why the change? We have seen too many applications that are insecure because they do not generate the nonce correctly. CTR is a very good mode, but only if the application can guarantee that the nonce is unique, even when the system is under attack. That turns out to be a major source of problems and security weaknesses. CBC with random IV has some disadvantages (the ciphertext is larger, the plaintext needs padding, and the system needs a random number generator), but it is robust and stands up well to abuse. Nonce generation turns out to be a really hard problem in many systems, so we do not recommend exposing to application developers any mode that uses nonces. That is even true of CBC with nonce-generated IV. So if you're developing an application and need to use an encryption mode, play it safe and use random IV CBC mode.

Always keep in mind that an encryption mode only provides confidentiality. That is, the attacker cannot find any information about the data you are communicating, other than the fact *that* you are communicating, *when* you are communicating, *how much* you are communicating, and *whom* you are communicating with. Analyzing these sorts of external information is called *traffic analysis*.[1]

Also recall that the encryption modes in this chapter are only designed to provide confidentiality against eavesdroppers; they do not stop the attacker from changing the data. We will come back to protecting both confidentiality and authenticity in Chapter 7.

4.8 Information Leakage

We now come to the dark secret of block cipher modes. All block cipher modes leak some information.

For this discussion, we will assume that we have a perfect block cipher. But even with a perfect block cipher, the ciphertexts that the encryption modes produce reveal information about the plaintexts. This has to do with equalities and inequalities of ciphertext and plaintext blocks.

Let's start with ECB. If two plaintext blocks are equal ($P_i = P_j$), then the two ciphertext blocks are equal, too ($C_i = C_j$). For random plaintexts, this will happen very rarely, but most plaintext is not random but highly structured. Thus, equal plaintext blocks occur far more frequently than random, and the equal ciphertext blocks reveal this structure. That is why we dismissed ECB.

What about CBC mode? Equal plaintext blocks do not lead to equal ciphertext blocks, as each plaintext block is first XORed with the previous ciphertext block before it is encrypted. Think of all the ciphertext blocks as random values; after all, they were produced by a block cipher that produces a random output for any given input. But what if we have two ciphertext blocks that are equal? We have

$$\begin{aligned} C_i &= C_j && \\ E(K, P_i \oplus C_{i-1}) &= E(K, P_j \oplus C_{j-1}) && \text{from the CBC specifications} \\ P_i \oplus C_{i-1} &= P_j \oplus C_{j-1} && \text{decrypt both sides} \\ P_i \oplus P_j &= C_{i-1} \oplus C_{j-1} && \text{basic algebra} \end{aligned}$$

The last equation gives the difference between two plaintext blocks as the XOR of two ciphertext blocks, which we assume the attacker knows. This is certainly not something you would expect from a perfect message encryption system. And if the plaintext is something with a lot of redundancy, such as plain English text, it probably contains enough information to recover both plaintext blocks.

A similar situation occurs when two ciphertexts are unequal. Knowing that $C_i \neq C_j$ implies that $P_i \oplus P_j \neq C_{i-1} \oplus C_{j-1}$, so each unequal pair of ciphertexts leads to an inequality formula between the plaintext blocks.

[1] Traffic analysis can provide very useful information to an attacker. Preventing traffic analysis is possible, but generally too expensive in terms of bandwidth for anyone but the military.

CTR has similar properties. With this encryption mode we know that the K_i blocks are all different, because they are encryptions of a nonce and counter value. All the plaintext values of the encryption are different, so all the ciphertext values (which form the key blocks) are different. Given two ciphertexts C_i and C_j, you know that $P_i \oplus P_j \neq C_i \oplus C_j$ because otherwise the two key stream blocks would have had to be equal. In other words, CTR mode provides a plaintext inequality for each pair of ciphertext blocks.

There are no problems with collisions in CTR. Two key blocks are never equal, and equal plaintext blocks or equal ciphertext blocks lead to nothing. The only thing that makes CTR deviate from the absolute ideal stream cipher is the absence of key block collisions.

OFB is worse than either CBC or CTR. As long as there are no collisions on the key stream blocks, OFB leaks the same amount of information as CTR. But if there is ever a collision of two key stream blocks, then all subsequent key stream blocks also produce a collision. This is a disaster from a security point of view, and one reason why CTR is preferable to OFB.

4.8.1 Chances of a Collision

So what are the chances that two ciphertext blocks are equal? Let's say we encrypt M blocks in total. It doesn't matter whether this is done in a few large messages, or in a large number of small messages. All that counts is the total number of blocks. A good rough estimate is that there are $M(M-1)/2$ pairs of blocks, and each pair has a chance of 2^{-n} of being equal, where n is the block size of the block cipher. So the expected number of equal ciphertext blocks is $M(M-1)/2^{n+1}$, which gets close to unity when $M \approx 2^{n/2}$. In other words, when you encrypt about $2^{n/2}$ blocks, you can expect to get two ciphertext blocks that are equal.[2] With a block size of $n = 128$ bits, we can expect the first duplicate ciphertext block value after about 2^{64} blocks. This is the birthday paradox we explained in Section 2.7.1. Now, 2^{64} blocks is a lot of data, but don't forget that we are designing systems with a lifetime of 30 years. Maybe people will want to process something close to 2^{64} blocks of data in the future.

Smaller data sets are also at risk. If we process 2^{40} blocks (about 16 TB of data) then there is a 2^{-48} chance of having a ciphertext block collision. That is a really small probability. But look at it from the attacker's point of view. For a particular key that is being used, he collects 2^{40} blocks and checks for duplicate blocks. Because the chance of finding one is small, he has to repeat this whole job for about 2^{48} different keys. The total amount of work before he

[2] The actual number of blocks you can encrypt before you expect the first duplicate is closer to $\sqrt{\pi 2^{n-1}} = 2^{n/2}\sqrt{\pi/2}$, but the theory behind the analysis is much harder and we don't need that level of precision here.

finds a collision is $2^{40} \cdot 2^{48} = 2^{88}$, which is much less than our design strength of 128 bits.

Let's concentrate on CBC and CTR. In CTR you get a plaintext inequality for every pair of blocks. In CBC you get an inequality if the two ciphertext blocks are unequal, and an equality if the blocks are equal. Obviously an equality provides much more information about the plaintext to the attacker than an inequality does, so CTR leaks less information.

4.8.2 How to Deal With Leakage

So how do we achieve our goal of a 128-bit security level? Basically, we don't, but we get as close as we can. There is no easy way of achieving a 128-bit security level with a block cipher whose block size is 128 bits. This is why we want to have block ciphers with 256-bit blocks, but there are no widely studied proposals of such a block cipher out there, so that is a dead end. What we can do is get close to our design security level, and limit the damage.

CTR leaks very little data. Suppose we encrypt 2^{64} blocks of data and produce a ciphertext C. For any possible plaintext P that is 2^{64} blocks long, the attacker can compute the key stream that would have to be used for this P to be encrypted to C. There is roughly a 50% chance that the resulting key stream will contain a collision. We know that CTR mode never produces collisions, so if a collision occurs, that particular plaintext P can be ruled out. This means that the attacker can rule out approximately half of all possible plaintexts. This corresponds to leaking a single bit of information to the attacker. Even revealing a single bit of information can sometimes be problematic. But leaking a single bit of information for 2^{64} blocks is not much. If we restrict ourselves to encrypting only 2^{48} blocks, then the attacker can rule out approximately 2^{-32} of all plaintexts, which is even less information. In a practical setting, such a small leakage is insignificant when taken in the context of the attack requirements. So although CTR encryption is not perfect, we can limit the damage to an extremely small leak by not encrypting too much information with a single key. It would be reasonable to limit the cipher mode to 2^{60} blocks, which allows you to encrypt 2^{64} bytes but restricts the leakage to a small fraction of a bit.

When using CBC mode you should be a bit more restrictive. If a collision occurs in CBC mode, you leak 128 bits of information about the plaintext. It is a good policy to keep the probability of such a collision low. We suggest limiting CBC encryption to 2^{32} blocks or so. That leaves a residual risk of 2^{-64} that you will leak 128 bits, which is probably harmless for most applications, but certainly far from our desired security level.

Just a reminder; these limits are on the total amount of information encrypted using a single key. It does not matter whether the data is encrypted in one very large message, or as a large number of smaller messages.

This is not a satisfactory state of affairs, but it is the situation we face. The best you can do at this point is use CTR or CBC and limit the amount of data you process with any one key. We will talk later about key negotiation protocols. It is quite easy to set up a fresh key when the old key is nearing its usage limit. Assuming you already use a key negotiation protocol to set up the encryption key, having to refresh a key is not particularly difficult. It is a complication, but a justifiable one.

4.8.3 About Our Math

Readers with a mathematical background may be horrified at our blithe use of probabilities without checking whether the probabilities are independent. They are right, of course, when arguing from a purely mathematical standpoint. But just like physicists, cryptographers use math in a way that they have found useful. Cryptographic values typically behave very randomly. After all, cryptographers go to great length to absolutely destroy all patterns, as any pattern leads to an attack. Experience shows that this style of dealing with probabilities leads to quite accurate results. Mathematicians are welcome to work through the details and figure out the exact results for themselves, but we prefer the rougher approximations for their simplicity.

4.9 Exercises

Exercise 4.1 Let P be a plaintext and let $\ell(P)$ be the length of P in bytes. Let b be the block size of the block cipher in bytes. Explain why the following is *not* a good padding scheme: Determine the minimum number of padding bytes necessary in order to pad the plaintext to a block boundary. This is a number n which satisfies $0 \leq n \leq b - 1$ and $n + \ell(P)$ is a multiple of b. Pad the plaintext by appending n bytes, each with value n.

Exercise 4.2 Compare the security and performance advantages and disadvantages of each variant of CBC mode covered in this chapter: a fixed IV, a counter IV, a random IV, and a nonce-generated IV.

Exercise 4.3 Suppose you, as an attacker, observe the following 32-byte ciphertext C (in hex)

```
46 64 DC 06 97 BB FE 69 33 07 15 07 9B A6 C2 3D
2B 84 DE 4F 90 8D 7D 34 AA CE 96 8B 64 F3 DF 75
```

and the following 32-byte ciphertext C' (also in hex)

```
51 7E CC 05 C3 BD EA 3B 33 57 0E 1B D8 97 D5 30
7B D0 91 6B 8D 82 6B 35 B7 8B BB 8D 74 E2 C7 3B.
```

Suppose you know these ciphertexts were generated using CTR mode with the same nonce. The nonce is implicit, so it is not included in the ciphertext. You also know that the plaintext P corresponding to C is

```
43 72 79 70 74 6F 67 72 61 70 68 79 20 43 72 79
70 74 6F 67 72 61 70 68 79 20 43 72 79 70 74 6F.
```

What information, if any, can you infer about the plaintext P' corresponding to C'?

Exercise 4.4 The ciphertext (in hex)

```
87 F3 48 FF 79 B8 11 AF 38 57 D6 71 8E 5F 0F 91
7C 3D 26 F7 73 77 63 5A 5E 43 E9 B5 CC 5D 05 92
6E 26 FF C5 22 0D C7 D4 05 F1 70 86 70 E6 E0 17
```

was generated with the 256-bit AES key (also in hex)

```
80 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01
```

using CBC mode with a random IV. The IV is included at the beginning of the ciphertext. Decrypt this ciphertext. You may use an existing cryptography library for this exercise.

Exercise 4.5 Encrypt the plaintext

```
62 6C 6F 63 6B 20 63 69 70 68 65 72 73 20 20 20
68 61 73 68 20 66 75 6E 63 74 69 6F 6E 73 20 78
62 6C 6F 63 6B 20 63 69 70 68 65 72 73 20 20 20
```

using AES in ECB mode and the key

```
80 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01.
```

You may use an existing cryptography library for this exercise.

Exercise 4.6 Let P_1, P_2 be a message that is two blocks long, and let P'_1 be a message that is one block long. Let C_0, C_1, C_2 be the encryption of P_1, P_2 using CBC mode with a random IV and a random key, and let C'_0, C'_1 be the encryption of P'_1 using CBC mode with a random IV and the same key. Suppose an attacker knows P_1, P_2 and suppose the attacker intercepted and thus know C_0, C_1, C_2 and C'_0, C'_1. Further suppose that, by random chance, $C'_1 = C_2$. Show that the attacker can compute P'_1.

CHAPTER

5

Hash Functions

A *hash function* is a function that takes as input an arbitrarily long string of bits (or bytes) and produces a fixed-size result. A typical use of a hash function is digital signatures. Given a message m, you could sign the message itself. However, the public-key operations of most digital signature schemes are fairly expensive in computational terms. So instead of signing m itself, you apply a hash function h and sign $h(m)$ instead. The result of h is typically between 128 and 1024 bits, compared to multiple thousands or millions of bits for the message m itself. Signing $h(m)$ is therefore much faster than signing m directly. For this construction to be secure, it must be infeasible to construct two messages m_1 and m_2 that hash to the same value. We'll discuss the details of the security properties of hash functions below.

Hash functions are sometimes called *message digest* functions, and the hash result is also known as the *digest*, or the *fingerprint*. We prefer the more common name *hash function*, as hash functions have many other uses besides digesting messages. We must warn you about one possible confusion: the term ''hash function'' is also used for the mapping function used in accessing hash tables, a data structure used in many algorithms. These so-called hash functions have similar properties to cryptographic hash functions, but there is a huge difference between the two. The hash functions we use in cryptography have specific security properties. The hash-table mapping-function has far weaker requirements. Be careful not to confuse the two. When we talk about hash functions in this book, we always mean cryptographic hash functions.

Hash functions have many applications in cryptography. They make a great glue between different parts of a cryptographic system. Many times when you have a variable-sized value, you can use a hash function to map it to a fixed-size value. Hash functions can be used in cryptographic pseudorandom number generators to generate several keys from a single shared secret. And they have a one-way property that isolates different parts of a system, ensuring that even if an attacker learns one value, she doesn't learn the others.

Even though hash functions are used in almost every system, we as a community currently know less about hash functions than we do about block ciphers. Until recently, much less research had been done on hash functions than block ciphers, and there were not very many practical proposals to choose from. This situation is changing. NIST is now in the process of selecting a new hash function standard, to be called SHA-3. The SHA-3 hash function selection process is proving to be very similar to the process that selected the AES as the new block cipher standard.

5.1 Security of Hash Functions

As we mentioned above, a hash function maps an input m to a fixed-size output $h(m)$. Typical output sizes are 128–1024 bits. There might be a limit on the input length, but for all practical purposes the input can be arbitrarily long. There are several requirements for a hash function. The simplest one is that it must be a one-way function: given a message m it is easy to compute $h(m)$, but given a value x it is not possible to find an m such that $h(m) = x$. In other words, a one-way function is a function that can be computed but that cannot be inverted—hence its name.

Of the many properties that a good hash function should have, the one that is mentioned most often is *collision resistance*. A collision is two different inputs m_1 and m_2 for which $h(m_1) = h(m_2)$. Of course, every hash function has an infinite number of these collisions. (There are an infinite number of possible input values, and only a finite number of possible output values.) Thus, a hash function is never collision-free. The collision-resistance requirement merely states that, although collisions exist, they cannot be found.

Collision resistance is the property that makes hash functions suitable for use in signature schemes. However, there are collision-resistant hash functions that are utterly unsuitable for many other applications, such as key derivation, one-way functions, etc. In practice, cryptographic designers expect a hash function to be a random mapping. Therefore, we require that a hash function be indistinguishable from a random mapping. Any other definition leads to a situation in which the designer can no longer treat the hash function as an idealized black box, but instead has to consider how the hash function properties interact with the system around it. (We number our definitions globally throughout this book.)

Definition 4 *The ideal hash function behaves like a random mapping from all possible input values to the set of all possible output values.*

Like our definition of the ideal block cipher (Section 3.3), this is an incomplete definition. Strictly speaking, there is no such thing as a random mapping; you can only talk about a probability distribution over all possible mappings. However, for our purposes this definition is good enough.

We can now define what an attack on a hash function is.

Definition 5 *An attack on a hash function is a non-generic method of distinguishing the hash function from an ideal hash function.*

Here the ideal hash function must obviously have the same output size as the hash function we are attacking. As with the block ciphers, the ''non-generic'' requirement takes care of all the generic attacks. Our remarks about generic attacks on block ciphers carry over to this situation. For example, if an attack could be used to distinguish between two ideal hash functions, then it doesn't exploit any property of the hash function itself and it is a generic attack.

The one remaining question is how much work the distinguisher is allowed to perform. Unlike the block cipher, the hash function has no key, and there is no generic attack like the exhaustive key search. The one interesting parameter is the size of the output. One generic attack on a hash function is the birthday attack, which generates collisions. For a hash function with an n-bit output, this requires about $2^{n/2}$ steps. But collisions are only relevant for certain uses of hash functions. In other situations, the goal is to find a pre-image (given x, find an m with $h(m) = x$), or to find some kind of structure in the hash outputs. The generic pre-image attack requires about 2^n steps. We're not going to discuss at length here which attacks are relevant and how much work would be reasonable for the distinguisher to use for a particular style of attack. To be sensible, a distinguisher has to be more efficient than a generic attack that yields similar results. We know this is not an exact definition, but—as with block ciphers—we don't have an exact definition. If somebody claims an attack, simply ask yourself if you could get a similar or better result from a generic attack that does not rely on the specifics of the hash function. If the answer is yes, the distinguisher is useless. If the answer is no, the distinguisher is real.

As with block ciphers, we allow a reduced security level if it is specified. We can imagine a 512-bit hash function that specifies a security level of 128 bits. In that case, distinguishers are limited to 2^{128} steps.

5.2 Real Hash Functions

There are very few good hash functions out there. At this moment, you are pretty much stuck with the existing SHA family: SHA-1, SHA-224, SHA-256, SHA-384, and SHA-512. There are other published proposals, including submissions for the new SHA-3 standard, but these all need to receive more attention before we can fully trust them. Even the existing functions in the SHA family have not been analyzed nearly enough, but at least they have been standardized by NIST, and they were developed by the NSA.[1]

Almost all real-life hash functions, and all the ones we will discuss, are iterative hash functions. Iterative hash functions split the input into a sequence of fixed-size blocks $m_1, \ldots, m_k$, using a padding rule to fill out the last block. A typical block length is 512 bits, and the last block will typically contain a string representing the length of the input. The message blocks are then processed in order, using a compression function and a fixed-size intermediate state. This process starts with a fixed value H_0, and defines $H_i = h'(H_{i-1}, m_i)$. The final value H_k is the result of the hash function.

Such an iterative design has significant practical advantages. First of all, it is easy to specify and implement, compared to a function that handles variable-length inputs directly. Furthermore, this structure allows you to start computing the hash of a message as soon as you have the first part of it. So in applications where a stream of data is to be hashed, the message can be hashed on the fly without ever storing the data.

As with block ciphers, we will not spend our time explaining the various hash functions in great detail. The full specifications contain many details that are not relevant to the main goals of this book.

5.2.1 A Simple But Insecure Hash Function

Before discussing real hash functions, however, we will begin by giving an example of a trivially insecure iterative hash function. This example will help clarify the definition of a generic attack. This hash function is built from AES with a 256-bit key. Let K be a 256-bit key set to all zeros. To hash the message m, first pad it in some way and break it into 128-bit blocks $m_1, \ldots, m_k$; the details of the padding scheme aren't important here. Set H_0 to a 128-bit block of all zeros. And now compute $H_i = \text{AES}_K(H_{i-1} \oplus m_i)$. Let H_k be the result of the hash function.

Is this a secure hash function? Is it collision resistant? Before reading further, try to see if you can find a way of breaking this hash function yourself.

Now here's a non-generic attack. Pick a message m such that after padding it splits into two blocks m_1 and m_2. Let H_1 and H_2 denote the values computed as part of the hash function's internal processing; H_2 is also the output of the hash function. Now let $m'_1 = m_2 \oplus H_1$ and let $m'_2 = H_2 \oplus m_2 \oplus H_1$, and let m' be the message that splits into m'_1 and m'_2 after padding. Due to properties of the hash function's construction, m' also hashes to H_2; you can verify this in the exercises at the end of this chapter. And with very high probability, m and m' are different strings. That's right—m and m' are two distinct messages

[1] Whatever you may think about the NSA, so far the cryptography it has published has been quite decent.

that produce a collision when hashed with this hash function. To convert this into a distinguishing attack, simply try to mount this attack against the hash function. If the attack works, the hash function is the weak one we described here; otherwise, the hash function is the ideal one. This attack exploits a specific weakness in how this hash function was designed, and hence this attack is non-generic.

5.2.2 MD5

Let's now turn to some real hash function proposals, beginning with MD5. MD5 is a 128-bit hash function developed by Ron Rivest [104]. It is a further development of a hash function called MD4 [106] with additional strengthening against attacks. Its predecessor MD4 is very fast, but also broken [36]. MD5 has now been broken too. You will still hear people talk about MD5, however, and it is still in use in some real systems.

The first step in computing MD5 is to split the message into blocks of 512 bits. The last block is padded and the length of the message is included as well. MD5 has a 128-bit state that is split into four words of 32 bits each. The compression function h' has four rounds, and in each round the message block and the state are mixed. The mixing consists of a combination of addition, XOR, AND, OR, and rotation operations on 32-bit words. (For details, see [104].) Each round mixes the entire message block into the state, so each message word is in fact used four times. After the four rounds of the h' function, the input state and result are added together to produce the output of h'.

This structure of operating on 32-bit words is very efficient on 32-bit CPUs. It was pioneered by MD4, and is now a general feature of many cryptographic primitives.

For most applications, the 128-bit hash size of MD5 is insufficient. Using the birthday paradox, we can trivially find collisions on any 128-bit hash function using 2^{64} evaluations of the hash function. This would allow us to find real collisions against MD5 using only 2^{64} MD5 computations. This is insufficient for modern systems.

But the situation with MD5 is worse than that. MD5's internal structure makes it vulnerable to more efficient attacks. One of the basic ideas behind the iterative hash function design is that if h' is collision-resistant, then the hash function h built from h' is also collision-resistant. After all, any collision in h can only occur due to a collision in h'. For over a decade now it has been known that the MD5 compression function h' has collisions [30]. The collisions for h' don't immediately imply a collision for MD5. But recent cryptanalytic advances, beginning with Wang and Yu [124], have now shown that it is actually possible to find collisions for the full MD5 using much fewer than 2^{64} MD5 computations. While the existence of such efficient collision finding attacks may not immediately break all uses of MD5, it is safe to say that MD5 is very weak and should no longer be used.

5.2.3 SHA-1

The Secure Hash Algorithm was designed by the NSA and standardized by NIST [97]. The first version was just called SHA (now often called SHA-0). The NSA found a weakness with SHA-0, and developed a fix that NIST published as an improved version, called SHA-1. However, they did not release any details about the weakness. Three years later, Chabaud and Joux published a weakness of SHA-0 [25]. This is a weakness that is fixed by the improved SHA-1, so it is reasonable to assume that we now know what the problem was.

SHA-1 is a 160-bit hash function based on MD4. Because of its shared parentage, it has a number of features in common with MD5, but it is a far more conservative design. It is also slower than MD5. Unfortunately, despite its more conservative design, we now know that SHA-1 is also insecure.

SHA-1 has a 160-bit state consisting of five 32-bit words. Like MD5, it has four rounds that consist of a mixture of elementary 32-bit operations. Instead of processing each message block four times, SHA-1 uses a linear recurrence to ''stretch'' the 16 words of a message block to the 80 words it needs. This is a generalization of the MD4 technique. In MD5, each bit of the message is used four times in the mixing function. In SHA-1, the linear recurrence ensures that each message bit affects the mixing function at least a dozen times. Interestingly enough, the only change from SHA-0 to SHA-1 was the addition of a one-bit rotation to this linear recurrence.

Independent of any internal weaknesses, the main problem with SHA-1 is the 160-bit result size. Collisions against any 160-bit hash function can be generated in only 2^{80} steps, well below the security level of modern block ciphers with key sizes from 128 to 256 bits. It is also insufficient for our design security level of 128 bits. Although it took longer for SHA-1 to fall than MD5, we now know that it is possible to find collisions in SHA-1 using much less work than 2^{80} SHA-1 computations [123]. Remember that attacks always get better? It is no longer safe to trust SHA-1.

5.2.4 SHA-224, SHA-256, SHA-384, and SHA-512

In 2001, NIST published a draft standard containing three new hash functions, and in 2004 they updated this specification to include a fourth hash function [101]. These hash functions are collectively referred to as the SHA-2 family of hash functions. These have 224-, 256-, 384-, and 512-bit outputs, respectively. They are designed to be used with the 128-, 192-, and 256-bit key sizes of AES, as well as the 112-bit key size of 3DES. Their structure is very similar to SHA-1.

These hash functions are new, which is generally a red flag. However, the known weaknesses of SHA-1 are much more severe. Further, if you want more security than SHA-1 can give you, you need a hash function with a larger result. None of the published designs for larger hash functions have received much public analysis; at least the SHA-2 family has been vetted by the NSA, which generally seems to know what it is doing.

SHA-256 is much slower than SHA-1. For long messages, computing a hash with SHA-256 takes about as much time as encrypting the message with AES or Twofish, or maybe a little bit more. This is not necessarily bad, and is an artifact of its conservative design.

5.3 Weaknesses of Hash Functions

Unfortunately, all of these hash functions have some properties that disqualify them according to our security definition.

5.3.1 Length Extensions

Our greatest concern about all these hash functions is that they have a length-extension bug that leads to real problems and that could easily have been avoided. Here is the problem. A message m is split into blocks $m_1, \ldots, m_k$ and hashed to a value H. Let's now choose a message m' that splits into the block $m_1, \ldots, m_k, m_{k+1}$. Because the first k blocks of m' are identical to the k blocks of message m, the hash value $h(m)$ is merely the intermediate hash value after k blocks in the computation of $h(m')$. We get $h(m') = h'(h(m), m_{k+1})$. When using MD5 or any hash function from the SHA family, you have to choose m' carefully to include the padding and length field, but this is not a problem as the method of constructing these fields is known.

The length extension problem exists because there is no special processing at the end of the hash function computation. The result is that $h(m)$ provides direct information about the intermediate state after the first k blocks of m'.

This is certainly a surprising property for a function we want to think of as a random mapping. In fact, this property immediately disqualifies all of the mentioned hash functions, according to our security definition. All a distinguisher has to do is to construct a few suitable pairs (m, m') and check for this relationship. You certainly wouldn't find this relationship in an ideal hash function. This is a non-generic attack that exploits properties of the hash functions themselves, so this is a valid attack. The attack itself takes only a few hash computations, so it is very quick.

How could this property be harmful? Imagine a system where Alice sends a message to Bob and wants to authenticate it by sending $h(X \parallel m)$, where X is a secret known only to Bob and Alice, and m is the message. If h were an ideal hash function, this would make a decent authentication system. But with length extensions, Eve can now append text to the message m, and update the authentication code to match the new message. An authentication system that allows Eve to modify the message is, of course, of no use to us.

This issue will be resolved in SHA-3; one of the NIST requirements is that SHA-3 not have length-extension properties.

5.3.2 Partial-Message Collision

A second problem is inherent in the iterative structure of most hash functions. We'll explain the problem with a specific distinguisher.

The first step of any distinguisher is to specify the setting in which it will differentiate between the hash function and the ideal hash function. Sometimes this setting can be very simple: given the hash function, find a collision. Here we use a slightly more complicated setting. Suppose we have a system that authenticates a message m with $h(m \parallel X)$, where X is the authentication key. The attacker can choose the message m, but the system will only authenticate a single message.[2]

For a perfect hash function of size n, we expect that this construction has a security level of n bits. The attacker cannot do any better than to choose an m, get the system to authenticate it as $h(m \parallel X)$, and then search for X by exhaustive search. The attacker can do much better with an iterative hash function. She finds two strings m and m' that lead to a collision when hashed by h. This can be done using the birthday attack in only $2^{n/2}$ steps or so. She then gets the system to authenticate m, and replaces the message with m'. Remember that h is computed iteratively, so once there is a collision and the rest of the hash inputs are the same, the hash value stays the same, too. Because hashing m and m' leads to the same value, $h(m \parallel X) = h(m' \parallel X)$. Notice that this attack does not depend on X—the same m and m' would work for *all* values for X.

This is a typical example of a distinguisher. The distinguisher sets its own ''game'' (a setting in which it attempts an attack), and then attacks the system. The object is still to distinguish between the hash function and the ideal hash function, but that is easy to do here. If the attack succeeds, it is an iterative hash function; if the attack fails, it is the ideal hash function.

5.4 Fixing the Weaknesses

We want a hash function that we can treat as a random mapping, but all well-known hash functions fail this property. Will we have to check for length-extension problems in every place we use a hash function? Do we check for partial-message collisions everywhere? Are there any other weaknesses we need to check for?

[2] Most systems will only allow a limited number of messages to be authenticated; this is just an extreme case. In real life, many systems include a message number with each message, which has the same effect on this attack as allowing only a single message to be chosen.

Leaving weaknesses in the hash function is a very bad idea. We can guarantee that it will be used somewhere in a way that exposes the weakness. Even if you document the known weaknesses, they will not be checked for in real systems. Even if you could control the design process that well, you would run into a complexity problem. Suppose the hash function has three weaknesses, the block cipher two, the signature scheme four, etc. Before you know it, you will have to check hundreds of interactions among these weaknesses: a practical impossibility. We have to fix the hash function.

The new SHA-3 standard will address these weaknesses. In the meantime, we need short-term fixes.

5.4.1 Toward a Short-term Fix

Here is one potential solution. Ultimately, we'll recommend the fixes in the subsequent subsections, and this particular proposal has not received significant review within the community. But this discussion is illustrative, so we include it here.

Let h be one of the hash functions mentioned above. Instead of $m \mapsto h(m)$, one could use $m \mapsto h(h(m) \parallel m)$ as a hash function.[3] Effectively we put $h(m)$ before the message we are hashing. This ensures that the iterative hash computations immediately depend on all the bits of the message, and no partial-message or length-extension attacks can work.

Definition 6 *Let h be an iterative hash function. The hash function h_{DBL} is defined by $h_{\text{DBL}}(m) := h(h(m) \parallel m)$.*

We believe that if h is any of the newer SHA-2 family hash functions, this construction has a security level of n bits, where n is the size of the hash result.

A disadvantage of this approach is that it is slow. You have to hash the entire message twice, which takes twice as long. Another disadvantage is that this approach requires the whole message m to be buffered. You can no longer compute the hash of a stream of data as it passes by. Some applications depend on this ability, and using h_{DBL} would simply not work.

5.4.2 A More Efficient Short-term Fix

So how do we keep the full speed of the original hash function? We cheat, kind of. Instead of $h(m)$, we can use $h(h(0^b \parallel m))$ as a hash function, and claim a security level of only $n/2$ bits. Here b is the block length of the underlying compression function, so $0^b \parallel m$ equates to prepending the message with an all zero block before hashing. The cheat is that we normally expect an n-bit

[3] The notation $x \mapsto f(x)$ is a way of writing down a function without having to give it a name. For example: $x \mapsto x^2$ is a function that squares its input.

hash function to provide a security level of n bits for those situations in which a collision attack is not possible.[4] The partial-message collision attacks all rely on birthday attacks, so if we reduce the security level to $n/2$ bits, these attacks no longer fall within the claimed security level.

In most situations, reducing the security level in this way would be unacceptable, but we are lucky here. Hash functions are already designed to be used in situations where collision attacks are possible, so the hash function sizes are suitably large. If we apply this construction to SHA-256, we get a hash function with a 128-bit security level, which is exactly what we need.

Some might argue that all n-bit hash functions provide only $n/2$ bits of security. That is a valid point of view. Unfortunately, unless you are very specific about these things, people will abuse the hash function and assume it provides n bits of security. For example, people want to use SHA-256 to generate a 256-bit key for AES, assuming that it will provide a security level of 256 bits. As we explained earlier, we use 256-bit keys to achieve a 128-bit security level, so this matches perfectly with the reduced security level of our fixed version of SHA-256. This is not accidental. In both cases the gap between the size of the cryptographic value and the claimed security level is due to collision attacks. As we assume collision attacks are always possible, the different sizes and security levels will fit together nicely.

Here is a more formal definition of this fix.

Definition 7 *Let h be an iterative hash function, and let b denote the block length of the underlying compression function. The hash function h_d is defined by $h_d(m) := h(h(0^b \parallel m))$, and has a claimed security level of $\min(k, n/2)$ where k is the security level of h and n is the size of the hash result.*

We will use this construction mostly in combination with hash functions from the SHA family. For any hash function SHA-X, where X is $1, 224, 256, 384$, or 512, we define SHA$_d$-X as the function that maps m to SHA-X(SHA-$X(0^b \parallel m)$). SHA$_d$-256 is just the function $m \mapsto$ SHA-256(SHA-256($0^{512} \parallel m$)), for example.

This particular fix to the SHA family of iterative hash functions, in addition to being related to our construction in Section 5.4.1, was also described by Coron et al. [26]. It can be demonstrated that the fixed hash function h_d is at least as strong as the underlying hash function h.[5] HMAC uses a similar hash-it-again approach to protect against length-extension attacks. Prepending the message with a block of zeros makes it so that, unless something unusual happens, the

[4] Even the SHA-256 documentation claims that an n-bit hash function should require 2^n steps to find a pre-image of a given value.

[5] We're cheating a little bit here. By hashing twice, the range of the function is reduced, and birthday attacks are a little bit easier. This is a small effect, and it falls well within the margin of approximation we've used elsewhere.

first block input to the inner hash function in h_d is different than the input to the outer hash function. Both h_{DBL} and h_d eliminate the length extension bug that poses the most danger to real systems. Whether h_{DBL} in fact has a security level of n bits remains to be seen. We would trust both of them up to $n/2$ bits of security, so in practice we would use the more efficient h_d construction.

5.4.3 Another Fix

There is another fix to some of these weaknesses with the SHA-2 family of iterative hash functions: Truncate the output [26]. If a hash function produces n-bit outputs, only use the first $n - s$ of those bits as the hash value for some positive s. In fact, SHA-224 and SHA-384 both already do this; SHA-224 is roughly SHA-256 with 32 output bits dropped, and SHA-384 is roughly SHA-512 with 128 output bits dropped. For 128 bits of security, you could hash with SHA-512, drop 256 bits of the output, and return the remaining 256 bits as the result of the truncated hash function. The result would be a 256-bit hash function which, because of birthday attacks, would meet our 128-bit security design goal.

5.5 Which Hash Function Should I Choose?

Many of the submissions to NIST's SHA-3 competition have revolutionary new designs, and they address the weaknesses we've discussed here and other concerns. However, the competition is still going on and NIST has not selected a final SHA-3 algorithm. Much additional analysis is necessary in order to have sufficient confidence in the SHA-3 submissions. In the short term, we recommend using one of the newer SHA hash function family members—SHA-224, SHA-256, SHA-384, or SHA-512. Moreover, we suggest you choose a hash function from the SHA$_d$ family, or use SHA-512 and truncate the output to 256 bits. In the long term, we will very likely recommend the winner of the SHA-3 competition.

5.6 Exercises

Exercise 5.1 Use a software tool to generate two messages M and M', $M \neq M'$, that produce a collision for MD5. To generate this collision, use one of the known attacks against MD5. A link to example code for finding MD5 collisions is available at: `http://www.schneier.com/ce.html`.

Exercise 5.2 Using an existing cryptography library, write a program to compute the SHA-512 hash value of the following message in hex:

```
48 65 6C 6C 6F 2C 20 77 6F 72 6C 64 2E 20 20 20.
```

Exercise 5.3 Consider SHA-512-n, a hash function that first runs SHA-512 and then outputs only the first n bits of the result. Write a program that uses a birthday attack to find and output a collision on SHA-512-n, where n is a multiple of 8 between 8 and 48. Your program may use an existing cryptography library. Time how long your program takes when n is 8, 16, 24, 32, 40, and 48, averaged over five runs for each n. How long would you expect your program to take for SHA-512-256? For SHA-512-384? For SHA-512 itself?

Exercise 5.4 Let SHA-512-n be as in the previous exercise. Write a program that finds a message M (a pre-image) that hashes to the following value under SHA-512-8 (in hex):

```
A9.
```

Write a program that finds a message M that hashes to the following value under SHA-512-16 (in hex):

```
3D 4B.
```

Write a program that finds a message M that hashes to the following value under SHA-512-24 (in hex):

```
3A 7F 27.
```

Write a program that finds a message M that hashes to the following value under SHA-512-32 (in hex):

```
C3 C0 35 7C.
```

Time how long your programs take when n is 8, 16, 24, and 32, averaged over five runs each. Your programs may use an existing cryptography library. How long would you expect a similar program to take for SHA-512-256? For SHA-512-384? For SHA-512 itself?

Exercise 5.5 In Section 5.2.1, we claimed that m and m' both hash to H_2. Show why this claim is true.

Exercise 5.6 Pick two of the SHA-3 candidate hash function submissions and compare their performance and their security under the currently best published attacks. Information about the SHA-3 candidates is available at `http://www.schneier.com/ce.html`.

CHAPTER

6

Message Authentication Codes

A *message authentication code*, or MAC, is a construction that detects tampering with messages. Encryption prevents Eve from reading the messages but does not prevent her from manipulating the messages. This is where the MAC comes in. Like encryption, MACs use a secret key, K, known to both Alice and Bob but not to Eve. Alice sends not just the message m, but also a MAC value computed by a MAC function. Bob checks that the MAC value of the message received equals the MAC value received. If they do not match, he discards the message as unauthenticated. Eve cannot manipulate the message because without K she cannot find the correct MAC value to send with the manipulated message.

In this chapter we will only consider authentication. The mechanisms for combining encryption and authentication will be dealt with in Chapter 7.

6.1 What a MAC Does

A MAC is a function that takes two arguments, a fixed-size key K and an arbitrarily sized message m, and produces a fixed-size MAC value. We'll write the MAC function as $\text{MAC}(K, m)$. To authenticate a message, Alice sends not only the message m but also the MAC code $\text{MAC}(K, m)$, also called the tag. Suppose Bob, also with key K, receives a message m' and a tag T. Bob uses the MAC verification algorithm to verify that T is a valid MAC under key K for message m'.

We start with a look at the MAC function in isolation. Be warned that using a MAC function properly is more complicated than just applying it to the message. We'll get to those problems later on, in Section 6.7.

6.2 The Ideal MAC and MAC Security

There are various ways to define the security of a MAC. We describe here our preferred definition. This definition is based on the notion of an ideal MAC function, which is very similar to the notion of an ideal block cipher. The primary difference is that block ciphers are permutations, whereas MACs are not. This is our preferred definition because it encompasses a broad range of attacks, including weak key attacks, related-key attacks, and more.

The ideal MAC is a random mapping. Let n be the number of bits in the result of a MAC. Our definition of an ideal MAC is:

Definition 8 *An ideal MAC function is a random mapping from all possible inputs to n-bit outputs.*

Remember that, in this definition, the MAC takes two inputs, a key and a message. In practice, the key K is not known to the attacker or, more precisely, it is not fully known. There could be a weakness in the rest of the system that provides partial information about K to the attacker.

We define the security of a MAC as follows.

Definition 9 *An attack on a MAC is a non-generic method of distinguishing the MAC from an ideal MAC function.*

Cryptography is a broad field. There are more formal definitions that theoreticians use. When possible, we prefer the definition above because it is broader and more aligned with the full range of attacks one might consider. Our attack model includes some forms of attacks not captured by the conventional, formal definitions, such as related-key attacks and attacks that assume that the attacker has partial knowledge about the key. That is why we prefer our style of security definitions, which are robust even if the function is abused or used in an unusual environment.

The more restrictive standard definition is one in which an attacker selects n different messages of her choosing, and is given the MAC value for each of these messages. The attacker then has to come up with $n + 1$ messages, each with a valid MAC value.

6.3 CBC-MAC and CMAC

CBC-MAC is a classic method of turning a block cipher into a MAC. The key K is used as the block cipher key. The idea behind CBC-MAC is to encrypt the message m using CBC mode and then throw away all but the last block of ciphertext. For a message $P_1, \ldots, P_k$, the MAC is computed as:

$$\begin{aligned} H_0 &:= \mathrm{IV} \\ H_i &:= E_K(P_i \oplus H_{i-1}) \\ \mathrm{MAC} &:= H_k \end{aligned}$$

Sometimes the output of the CBC-MAC function is taken to be only part (e.g., half) of the last block. The most common definition of CBC-MAC requires the IV to be fixed at 0.

In general, one should never use the same key for both encryption and authentication. It is especially dangerous to use CBC encryption and CBC-MAC authentication with the same key. The MAC ends up being equal to the last ciphertext block. What's more, depending on when and how CBC encryption and CBC-MAC are applied, using the same key for both can lead to privacy compromises for CBC encryption and authenticity compromises for CBC-MAC.

Using CBC-MAC is a bit tricky, but it is generally considered secure when used correctly and when the underlying cipher is secure. Studying the strengths and weaknesses of CBC-MAC can be very educational. There are a number of different collision attacks on CBC-MAC that effectively limit the security to half the length of the block size [20]. Here is a simple collision attack: let M be a CBC-MAC function. If we know that $M(a) = M(b)$ then we also know that $M(a \parallel c) = M(b \parallel c)$. This is due to the structure of CBC-MAC. Let's illustrate this with a simple case: c consists of a single block. We have

$$\begin{aligned} M(a \parallel c) &= E_K(c \oplus M(a)) \\ M(b \parallel c) &= E_K(c \oplus M(b)) \end{aligned}$$

and these two must be equal, because $M(a) = M(b)$.

The attack proceeds in two stages. In the first stage, the attacker collects the MAC values of a large number of messages until a collision occurs. This takes 2^{64} steps for a 128-bit block cipher because of the birthday paradox. This provides the a and b for which $M(a) = M(b)$. If the attacker can now get the sender to authenticate $a \parallel c$, he can replace the message with $b \parallel c$ without changing the MAC value. The receiver will check the MAC and accept the bogus message $b \parallel c$. (Remember, we work in the paranoia model. It is quite acceptable for the attacker to create a message and get it authenticated by the sender. There are many situations in which this is possible.) There are many extensions to this attack that work even with the addition of length fields and padding rules [20].

This is not a generic attack, as it does not work on an ideal MAC function. Finding the collision is not the problem. That can be done for an ideal MAC function in exactly the same way. But once you have two messages a and b, for which $M(a) = M(b)$, you cannot use them to forge a MAC on a new message, whereas you can do that with CBC-MAC.

As another example attack, suppose c is one block long and $M(a \parallel c) = M(b \parallel c)$. Then $M(a \parallel d) = M(b \parallel d)$ for any block d. The actual attack is similar to the one above. First the attacker collects the MAC values of a large number of messages that end in c until a collision occurs. This provides the values of a and b. The attacker then gets the sender to authenticate $a \parallel d$. Now he can replace the message with $b \parallel d$ without changing the MAC value.

There are some nice theoretical results which argue that, in the particular proof model used, CBC-MAC provides 64 bits of security when the block size is 128 bits [6] and when the MAC is only ever applied to messages that are the same length. Unfortunately, this is short of our desired design strength, though in practice it's not immediately clear how to achieve our desired design strength with 128-bit block ciphers. CBC-MAC would be fine if we could use a block cipher with a 256-bit block size.

There are other reasons why you have to be careful how you use CBC-MAC. You cannot just CBC-MAC the message itself if you wish to authenticate messages with different lengths, as that leads to simple attacks. For example, suppose a and b are both one block long, and suppose the sender MACs a, b, and $a \parallel b$. An attacker who intercepts the MAC tags for these messages can now forge the MAC for the message $b \parallel (M(b) \oplus M(a) \oplus b)$, which the sender never sent. The forged tag for this message is equal to $M(a \parallel b)$, the tag for $a \parallel b$. You can figure out why this is true as an exercise, but the problem arises from the fact that the sender MACs messages that are different lengths.

If you wish to use CBC-MAC, you should instead do the following:

1. Construct a string s from the concatenation of l and m, where l is the length of m encoded in a fixed-length format.
2. Pad s until the length is a multiple of the block size. (See Section 4.1 for details.)
3. Apply CBC-MAC to the padded string s.
4. Output the last ciphertext block, or part of that block. Do not output any of the intermediate values.

The advantage of CBC-MAC is that it uses the same type of computations as the block cipher encryption modes. In many systems, encryption and MAC are the only two functions that are ever applied to the bulk data, so these are the two speed-critical areas. Having them use the same primitive functions makes efficient implementations easier, especially in hardware.

Still, we don't advocate the use of CBC-MAC directly, because it is difficult to use correctly. One alternate that we recommend is CMAC [42]. CMAC is based on CBC-MAC and was recently standardized by NIST. CMAC works almost exactly like CBC-MAC, except it treats the last block differently. Specifically, CMAC XORs one of two special values into the last block prior to the last block cipher encryption. These special values are derived from the CMAC key, and the specific one used by CMAC depends on whether the length of the message is a multiple of the block cipher's block length or not. The XORing of these values into the MAC disrupts the attacks that compromise CBC-MAC when used for messages of multiple lengths.

6.4 HMAC

Given that the ideal MAC is a random mapping with keys and messages as input and that we already have hash functions that (try to) behave like random mappings with messages as input, it is an obvious idea to use a hash function to build a MAC. This is exactly what HMAC does [5, 81]. The designers of HMAC were of course aware of the problems with hash functions, which we discussed in Chapter 5. For this reason, they did not define HMAC to be something simple like $\text{MAC}(K, m)$ as $h(K \parallel m)$, $h(m \parallel K)$, or even $h(K \parallel m \parallel K)$, which can create problems if you use one of the standard iterative hash functions [103].

Instead, HMAC computes $h(K \oplus a \parallel h(K \oplus b \parallel m))$, where a and b are specified constants. The message itself is only hashed once, and the output is hashed again with the key. For details, see the specifications in [5, 81]. HMAC works with any of the iterative hash functions we discussed in Chapter 5. What's more, because of HMAC's design, it's not subject to the same collision attacks that have recently undermined the security of SHA-1 [4]. This is because, in the case of HMAC, the beginning of the message to hash is based on a secret key and is not known to the attacker. This means that HMAC with SHA-1 is not as bad as straight SHA-1. But given that attacks often get better over time, we now view HMAC with SHA-1 as too risky and do not recommend its use.

The HMAC designers carefully crafted HMAC to resist attacks, and proved security bounds on the resulting construction. HMAC avoids key recovery attacks that reveal K to the attacker, and avoids attacks that can be done by the attacker without interaction with the system. However, HMAC—like CMAC—is still limited to $n/2$ bits of security, as there are generic birthday attacks against the function that make use of the internal collisions of the iterated hash function. The HMAC construction ensures that these require $2^{n/2}$ interactions with the system under attack, which is more difficult to do than performing $2^{n/2}$ computations on your own computer.

The HMAC paper [5] presents several good examples of the problems that arise when the primitives (in this case, the hash function) have unexpected properties. This is why we are so compulsive about providing simple behavioral specifications for our cryptographic primitives.

We like the HMAC construction. It is neat, efficient, and easy to implement. It is widely used with the SHA-1 hash function, and by now you will find it in a lot of libraries. Still, to achieve our 128-bit security level, we would only use it with a 256-bit hash function such as SHA-256.

6.5 GMAC

NIST recently standardized a new MAC, called GMAC [43], that is very efficient in hardware and software. GMAC was designed for 128-bit block ciphers.

GMAC is fundamentally different from CBC-MAC, CMAC, and HMAC. The GMAC authentication function takes three values as input—the key, the message to authenticate, and a nonce. Recall that a nonce is a value that is only ever used once. CBC-MAC, CMAC, and HMAC do not take a nonce as input. If a user MACs a message with a key and a nonce, the nonce will also need to be known by the recipient. The user could explicitly send the nonce to the recipient, or the nonce might be implicit, such as a packet counter that both the sender and the recipient maintain.

Given its different interface, GMAC doesn't meet our preferred definition of MAC in Section 6.2, which involves being unable to distinguish it from an ideal MAC function. Instead, we have to use the unforgeability definition mentioned at the end of that section. Namely, we consider a model in which an attacker selects n different messages of his choosing, and is given the MAC value for each of these messages. The attacker then has to come up with $n + 1$ messages, each with a valid MAC value. If an attacker can't do this, then the MAC is unforgeable.

Under the hood, GMAC uses something called an universal hash function [125]. This is very different from the types of hash functions we discussed in Chapter 5. The details of how universal hash functions work are outside our scope, but you can think of GMAC as computing a simple mathematical function of the input message. This function is much simpler than anything like SHA-1 or SHA-256. GMAC then encrypts the output of that function with a block cipher in CTR mode to get the tag. GMAC uses a function of its nonce as the IV for CTR mode.

GMAC is standardized and is a reasonable choice in many circumstances. But we also want to offer some words of warning. Like HMAC and CMAC, GMAC only provides at most 64 bits of security. Some applications may wish to use tags that are shorter than 128 bits. However, unlike HMAC and CMAC, GMAC offers diminished security for these short tag values. Suppose an application uses GMAC but truncates the tags so that they are 32 bits long. One might expect the resulting system to offer 32 bits of security, but in fact it is possible to forge the MAC after 2^{16} tries [48]. Our recommendation is to not use GMAC when you need to produce short MAC values.

Finally, requiring the system to provide a nonce can be risky because security can be undone if the system provides the same value for the nonce more than once. As we discussed in Section 4.7, real systems fail time and time again for not correctly handling nonce generation. We therefore recommend avoiding modes that expose nonces to application developers.

6.6 Which MAC to Choose?

As you may have gathered from the previous discussion, we would choose HMAC-SHA-256: the HMAC construction using SHA-256 as a hash function. We really want to use the full 256 bits of the result. Most systems use 64- or 96-bit MAC values, and even that might seem like a lot of overhead. As far as we know, there is no collision attack on the MAC value if it is used in the traditional manner, so truncating the results from HMAC-SHA-256 to 128 bits should be safe, given current knowledge in the field.

We are not particularly happy with this situation, as we believe that it should be possible to create faster MAC functions. But until suitable functions are published and analyzed, and become broadly accepted, there is not a whole lot we can do about it. GMAC is fast, but provides only at most 64 bits of security and isn't suitable when used to produce short tags. It also requires a nonce, which is a common source of security problem.

Some of the submissions for NIST's SHA-3 competition have special modes that allow them to be used to create faster MACs. But that competition is still ongoing and it is too early to say with much confidence which submissions will be deemed secure.

6.7 Using a MAC

Using a MAC properly is much more complicated than it might initially seem. We'll discuss the major problems here.

When Bob receives the value $\text{MAC}(K, m)$, he knows that somebody who knew the key K approved the message m. When using a MAC, you have to be very careful that this statement is sufficient for all the security properties that you need. For example, Eve could record a message from Alice to Bob, and then send a copy to Bob at a later time. Without some kind of special protection against these sorts of attacks, Bob would accept it as a valid message from Alice. Similar problems arise if Alice and Bob use the same key K for traffic in two directions. Eve could send the message back to Alice, who would believe that it came from Bob.

In many situations, Alice and Bob want to authenticate not only the message m, but also additional data d. This additional data includes things like the message number used to prevent replay attacks, the source and destination of the message, and so on. Quite frequently these fields are part of the header of the authenticated (and often encrypted) message. The MAC has to authenticate d as well as m. The general solution is to apply the MAC to $d \parallel m$ instead of just to m. (Here we're assuming that the mapping from d and m to $d \parallel m$ is one-to-one; otherwise, we'd need to use a better encoding.)

The next issue is best captured in the following design rule:

The Horton Principle: Authenticate what is meant, not what is said.

A MAC only authenticates a string of bytes, whereas Alice and Bob want to authenticate a message with a specific meaning. The gap between what is said (i.e., the bytes sent) and what is meant (i.e., the interpretation of the message) is important.

Suppose Alice uses the MAC to authenticate $m := a \parallel b \parallel c$, where a, b, and c are some data fields. Bob receives m, and splits it into a, b, and c. But how does Bob split m into fields? Bob must have some rules, and if those rules are not compatible with the way Alice constructed the message, Bob will get the wrong field values. This would be bad, as Bob would have received authenticated bogus data. Therefore, it is vital that Bob split m into the fields that Alice put in.

This is easy to do in simple systems. Fields have a fixed size. But soon you will find a situation in which some fields need to be variable in length, or a newer version of the software will use larger fields. Of course, a new version will need a backward compatibility mode to talk to the old software. And here is the problem. Once the field length is no longer constant, Bob is deriving it from some context, and that context could be manipulated by the attacker. For example, Alice uses the old software and the old, short field sizes. Bob uses the new software. Eve, the attacker, manipulates the communications between Alice and Bob to make Bob believe that the new protocol is in use. (Details of how this works are not important; the MAC system shouldn't depend on other parts of the system being secure.) Bob happily splits the message using the larger field sizes, and gets bogus data.

This is where the Horton Principle [122] comes in.[1] You should authenticate the meaning, not the message. This means that the MAC should authenticate not only m, but also all the information that Bob uses in parsing m into its meaning. This would typically include data like protocol identifier, protocol version number, protocol message identifier, sizes for various fields, etc. One partial solution is to not just concatenate the fields but use a data structure like XML that can be parsed without further information.

The Horton Principle is one of the reasons why authentication at lower protocol levels does not provide adequate authentication for higher-level protocols. An authentication system at the IP packet level cannot know how the e-mail program is going to interpret the data. This precludes it from checking that the context in which the message is interpreted is the same as the context in which the message was sent. The only solution is to have the e-mail program provide its own authentication of the data exchanged—in addition to the authentication on the lower levels, of course.

To recap: whenever you do authentication, always think carefully about what other information should be included in the authentication. Be sure that you code all of this information, including the message, into a string of bytes in a way that can be parsed back into the fields in a unique manner. Do not forget to apply this to the concatenation of the additional data and the message we discussed at the start of this section. If you authenticate $d \parallel m$, you had better have a fixed rule on how to split the concatenation back into d and m.

6.8 Exercises

Exercise 6.1 Describe a realistic system that uses CBC-MAC for message authentication and that is vulnerable to a length extension attack against CBC-MAC.

Exercise 6.2 Suppose c is one block long, a and b are strings that are a multiple of the block length, and $M(a \parallel c) = M(b \parallel c)$. Here M is CBC-MAC. Then $M(a \parallel d) = M(b \parallel d)$ for any block d. Explain why this claim is true.

Exercise 6.3 Suppose a and b are both one block long, and suppose the sender MACs a, b, and $a \parallel b$ with CBC-MAC. An attacker who intercepts the MAC tags for these messages can now forge the MAC for the message $b \parallel (M(b) \oplus M(a) \oplus b)$, which the sender never sent. The forged tag for this

[1] For readers who did not grow up in the U.S.: this is named after one of the characters of Dr. Seuss, who was a writer of children's books [116].

message is equal to $M(a \| b)$, the tag for $a \| b$. Justify mathematically why this is true.

Exercise 6.4 Suppose message a is one block long. Suppose that an attacker has received the MAC t for a using CBC-MAC under some random key unknown to the attacker. Explain how to forge the MAC for a two-block message of your choice. What is the two-block message that you chose? What is the tag that you chose? Why is your chosen tag a valid tag for your two-block message?

Exercise 6.5 Using an existing cryptography library, compute the MAC of the message

```
4D 41 43 73 20 61 72 65 20 76 65 72 79 20 75 73
65 66 75 6C 20 69 6E 20 63 72 79 70 74 6F 67 72
61 70 68 79 21 20 20 20 20 20 20 20 20 20 20 20
```

using CBC-MAC with AES and the 256-bit key

```
80 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01.
```

Exercise 6.6 Using an existing cryptography library, compute the MAC of the message

```
4D 41 43 73 20 61 72 65 20 76 65 72 79 20 75 73
65 66 75 6C 20 69 6E 20 63 72 79 70 74 6F 67 72
61 70 68 79 21
```

using HMAC with SHA-256 and the key

```
0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b
0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b 0b.
```

Exercise 6.7 Using an existing cryptography library, compute the MAC of the message

```
4D 41 43 73 20 61 72 65 20 76 65 72 79 20 75 73
65 66 75 6C 20 69 6E 20 63 72 79 70 74 6F 67 72
61 70 68 79 21
```

using GMAC with AES and the 256-bit key

```
80 00 00 00 00 00 00 00 00 00 00 00 00 00 00 00
00 00 00 00 00 00 00 00 00 00 00 00 00 00 00 01
```

and the nonce

```
00 00 00 00 00 00 00 00 00 00 00 01.
```

CHAPTER

7

The Secure Channel

Finally we come to the first of the real-world problems we will solve. The secure channel is probably the most common of all practical problems.

7.1 Properties of a Secure Channel

Informally, we can define the problem as creating a secure connection between Alice and Bob. We'll have to formalize this a bit before it becomes clear what we are talking about.

7.1.1 Roles

First, most connections are bi-directional. Alice sends messages to Bob, and Bob sends messages to Alice. You don't want to confuse the two streams of messages, so there must be some kind of asymmetry in the protocol. In real systems, maybe one party is the client and the other the server, or maybe it is easier to speak of the initiator (the party that initiated the secure connection) and the responder. It doesn't matter how you do it, but you have to assign the Alice and Bob roles to the two parties in question in such a way that each of them knows who is playing which role.

Of course, there is always Eve, who tries to attack the secure channel in any way possible. Eve can read all of the communications between Alice and Bob and arbitrarily manipulate these communications. In particular, Eve can delete, insert, or modify data that is being transmitted.

We always talk about transmitting messages from Alice to Bob, and most of the time our mental image is of two separate computers sending messages to each other over a network of some sort. Another very interesting application is storing data securely. If you think of storing data as transmitting it to the future, then all the discussions here apply. Alice and Bob might be the same person, and the transmission medium could be a backup tape or a USB stick. You still want to protect the medium from outside eavesdroppers and manipulations. Of course, when you send data to the future, you cannot have an interactive protocol, since the future cannot send a message back to the past.

7.1.2 Key

To implement a secure channel, we need a shared secret. In this case we will assume that Alice and Bob share a secret key *K*, but that nobody else knows this key. This is an essential property. The cryptographic primitives can never identify Alice as a person. They can at most identify the key. Thus Bob's verification algorithm will tell him something like: ''This message was sent by somebody who knows the key *K* and who played the role of Alice.'' This statement is only useful if Bob knows that knowledge of *K* is restricted, preferably to himself and Alice.

How the key is established is not our business here. We just assume the key is there. We will talk about key management in great detail in Chapter 14. The requirements for the key are as follows:

- The key *K* is known only to Alice and Bob.
- Every time the secure channel is initialized, a new value is generated for the key *K*.

The second item is also important. If the same key is used over and over again, then messages from older sessions can be replayed to Alice or Bob, and lead to much confusion. Therefore, even in situations where you have a fixed password as key, you need a key negotiation protocol between Alice and Bob to set up a suitable unique key *K*, and you must re-run this protocol every time a secure channel is established. A key such as *K* that is used for a single communication session is called a *session key*. Again, how *K* is generated will be discussed in Chapter 14.

The secure channel is designed to achieve a security level of 128 bits. Following our discussion in Section 3.5.7, we will use a 256-bit key. Thus, *K* is a 256-bit value.

7.1.3 Messages or Stream

The next question is whether we look at the communications between Alice and Bob as a sequence of discrete messages (such as e-mails) or as a continuous stream of bytes (such as streaming media). We will only consider systems that handle discrete messages. These can trivially be converted to handle a stream of bytes by cutting the data stream into separate messages and reassembling the stream at the receiver's end. In practice, almost all systems use a discrete message system at the cryptographic layer.

We also assume that the underlying transport system that conveys the messages between Alice and Bob is not reliable. Even a reliable communication protocol like TCP/IP does not form a reliable communication channel from a cryptographic point of view. After all, the attacker can easily change, remove, or insert data in a TCP stream without interrupting the flow of data. TCP is only reliable with respect to random events such as loss of packet. It does not protect against an active adversary. From our adversarial point of view, there is no such thing as a reliable communication protocol. (This is a good example of how cryptographers view the world.)

7.1.4 Security Properties

We can now formulate the security properties of the channel. Alice sends a sequence of messages $m_1, m_2, \ldots$ that are processed by the secure channel algorithms and then sent to Bob. Bob processes the received messages through the secure channel algorithms, and ends up with a sequence of messages $m'_1, m'_2, \ldots.$

The following properties must hold:

- Eve does not learn anything about the messages m_i except for their timing and size.
- Even when Eve attacks the channel by manipulating the data that is being communicated, the sequence $m'_1, m'_2, \ldots$ of messages that Bob receives is a subsequence of $m_1, m_2, \ldots$, and Bob learns exactly which subsequence he received. (A subsequence is best defined by saying that it can be constructed from the original sequence by the removal of zero or more elements.)

The first property is secrecy. Ideally, Eve should not learn *anything* about the messages. In real life, this is very hard to achieve. It is extremely hard to hide information such as the size or the timing of the messages. The known solutions require Alice to send a continuous stream of messages at the maximum bandwidth that she will ever use. If she doesn't have any messages to send, she should invent some trivial ones and send those. This might be acceptable for military applications, but it is not acceptable for most civilian applications. Given that Eve can see the size and timing of messages on a communication channel, she can find out who is communicating with whom, how much, and when. This is called *traffic analysis*. It yields a host of information, and is extremely hard to prevent. This is a well-known problem with other secure channels, like SSL/TLS, IPsec, and SSH. We will not solve it in this book, so Eve will be able to perform traffic analysis on our secure channel.

The second property ensures that Bob only gets proper messages, and that he gets them in their correct order. Ideally, we would want Bob to receive the exact sequence of messages that Alice sent. But none of the real-world communications protocols are reliable in a cryptographic sense. Eve can always delete a message in transit. As we cannot prevent the loss of messages, Bob will necessarily have to make do with getting only a subsequence of the messages. Note that the remaining messages that he does receive are in order. There are no duplicates, no modified messages, and no bogus messages sent by someone other than Alice. As a further requirement, Bob learns exactly which messages he has missed. This can be important in some applications where the interpretation of the message depends on the order in which they are received.

In most situations, Alice wants to ensure that Bob gets all the information she sent him. Most systems implement a scheme whereby Bob sends acknowledgments (either explicit or implicit) to Alice, and Alice resends any information for which she didn't receive an acknowledgment from Bob. Note that our secure channel never takes the initiative in resending a message. Alice will have to do that herself, or at least the protocol layer that makes use of the secure channel will have to do that.

So why not make the secure channel reliable by implementing the resend functionality inside the secure channel? Because that would complicate the secure channel description. We like to keep the security-critical modules simple. Message acknowledgments and resends are standard communication protocol techniques, and they can be implemented on top of our secure channel. Also, this is a book about cryptography, not about basic communication protocol techniques.

7.2 Order of Authentication and Encryption

Obviously we will apply both encryption and authentication to the message. There are three approaches [7, 82]: we can encrypt first and then authenticate the ciphertext (encrypt-then-authenticate); authenticate first and then encrypt both the message and the MAC value (authenticate-then-encrypt); or both encrypt the message and authenticate the message and then combine (such as concatenate) the two results (encrypt-and-authenticate). There is no simple answer for which method is best.

There are two main arguments in favor of encrypting first. There are theoretical results that show that, given certain specific definitions of secure encryption and authentication, the encrypt-first solution is secure, whereas the other approaches are insecure. If you look at the details, it turns out that authenticate-first is only insecure if the encryption scheme has a specific type of weakness. In practical systems, we never use encryption schemes with such weaknesses. However, these weak encryption schemes satisfy a particular formal security definition. Applying the MAC to the ciphertext of such a weak encryption scheme fixes it and makes it secure. Having these theoretical results is valuable. But these theoretical results don't always apply to real-life encryption schemes. In fact, there are similar proofs that these problems do not occur at all for stream ciphers (such as CTR mode) and CBC-mode encryption when the nonce or IV is authenticated.

The second argument in favor of encrypting first is that it is more efficient in discarding bogus messages. For normal messages, Bob has to both decrypt the message and check the authentication, irrespective of the order they were applied in. If the message is bogus (i.e., has a wrong MAC field) then Bob will discard it. With encrypt-first, the decryption is done last on the receiver side, and Bob never has to decrypt bogus messages, since he can identify and discard them before decryption. With authenticate-first, Bob has to decrypt the message before he can check the authentication. This is more work for bogus messages. The situation in which this is relevant is when Eve sends Bob a very large number of bogus messages. With encrypt-first, Bob saves the work of decrypting them, which reduces the CPU load. Under some very special circumstances, this makes a denial-of-service (DOS) attack a little bit harder, though only by a factor of at most approximately two. Further, in many real-life situations, a more effective DOS attack works by saturating the communication channel rather than by bogging down Bob's CPU.

The main argument for encrypt-and-authenticate is that the encryption and authentication processes can happen in parallel. This can increase performance in some situations. Under the encrypt-and-authenticate composition approach, an attacker can view the MAC tag of the initial message itself. This is because the MAC is not encrypted (unlike the authenticate-then-encrypt approach) and because the MAC is not of an encrypted value (unlike the encrypt-then-authenticate approach). MACs are designed to protect authenticity, not privacy. This means the MAC in an encrypt-and-authenticate approach could leak private information about the underlying message, thereby compromising the privacy of the secure channel. As with authenticate-first, there are also some underlying encryption schemes that are insecure when used in an encrypt-and-authenticate approach. With judicious choice of the underlying MAC and the underlying encryption scheme, and by including additional data like the nonce in the input to the MAC, the encrypt-and-authenticate approach can be secure.

There are two main arguments in favor of authenticating first. In the encrypt-first configuration, the MAC input and MAC value are both visible to Eve. In the authenticate-first configuration, Eve only gets to see the ciphertext and the encrypted MAC value; the MAC input (i.e., the plaintext) and actual MAC value are hidden. This makes it much harder to attack the MAC than in the encrypt-first situation. The real choice is which of the two functions is applied last. If encryption is applied last, then Eve gets to attack the encryption function without further hindrance. If the authentication function is applied last, she gets to attack the authentication function without further hindrance. In many cases, one can argue that authentication is more important than encryption. We therefore prefer to expose the encryption function to Eve's direct attacks, and protect the MAC as much as possible. Of course, these issues are moot if both the underlying encryption scheme and MAC are secure, but we take an approach of professional paranoia and would like a secure channel with some robustness even if we do not assume that.

When might authentication be more important than encryption? Imagine a situation in which a secure channel is being used. Consider how much damage Eve could do if she could read all the traffic. Then think about how much damage Eve could do if she could modify the data being communicated. In most situations, modifying data is a devastating attack, and does more damage than merely reading it.

The second argument in favor of authenticating first is the Horton Principle. You should authenticate what you mean, not what you say. Authenticating the ciphertext breaks this rule, and creates a vulnerability. The danger is that Bob might check that the ciphertext is correctly authenticated, but then decrypt the ciphertext with a different key than what Alice used to encrypt the message. Bob will get a different plaintext than Alice sent, even though the authentication checked out. This shouldn't happen, but it can. There is a particular (unusual) configuration of IPsec that has this problem [51]. This vulnerability has to be fixed. You could include the encryption key in the additional data being authenticated, but we don't like using keys for anything but their normal use. It introduces extra risks; you don't want a faulty MAC function leaking information about the encryption key. The standard solution is to derive both the encryption key and the authentication key for the secure channel from a single secure channel key, as we do in Section 7.4.1. This removes the vulnerability, but it also introduces a cross-dependency. The authentication suddenly depends on the key derivation system.

You can argue for hours which order of operations is better. All orders can result in good systems, all can result in bad systems. Each has its own advantages and disadvantages. We choose to authenticate first for the rest of this chapter. We like the simplicity of authenticate-first, and its security under our practical paranoia model.

7.3 Designing a Secure Channel: Overview

The solution consists of three components: message numbering, authentication, and encryption. We will walk through the design of one possible secure channel and, in the process, illustrate how to think about the underlying issues.

7.3.1 Message Numbers

Message numbers are vital for various reasons. They provide a source for IVs for the encryption algorithm; they allow Bob to reject replayed messages without the necessity of keeping a large database; they tell Bob which messages were lost in transit; and they ensure that Bob receives the messages in their correct order. For these reasons, the message numbers must increase monotonically (i.e., later messages have larger message numbers) and must be unique (no two messages may have the same message number).

Assigning message numbers is easy. Alice numbers the first message as 1, the second message as 2, etc. Bob keeps track of the message number of the last message he received. Any new message must have a message number that is larger than the message number of the previous message. By accepting only increasing message numbers, Bob ensures that Eve cannot replay him an old message.

For our secure channel design, we will use a 32-bit number for the message number. The first message is numbered 1. The number of messages is limited to $2^{32} - 1$. If the message number overflows, then Alice will have to stop using this key and rerun the key negotiation protocol to generate a new key. The message number *must* be unique, so we cannot allow it to wrap back to 0.

We could have used a 64-bit message number, but that has a higher overhead. (We would have to include 8 bytes of message number with each message, instead of only 4 bytes.) 32 bits is enough for most applications. Besides, the key should be changed regularly anyway.[1] You can, of course, use 40 or 48 bits if you want to; it doesn't matter much.

Why start numbering at 1 when most C programmers like to start at 0? This is a small implementation trick. If there are N numbers that could be assigned, then both Alice and Bob need to be able to keep track of $N + 1$ states. After all, the number of messages sent so far could be any of the set $\{0, \ldots, N\}$. By restricting ourselves to $2^{32} - 1$ messages, this state can be encoded in a single 32-bit number. Had we started numbering the messages at 0, then each implementation would require an additional flag to indicate that either no messages had been sent so far, or that the message number space was exhausted. Extra flags add a lot of tricky extra code that is executed very rarely. If it is rarely used, it will have been tested only a few times, and therefore there's a higher chance it won't work. In short, there is an entire area of easy mistakes that we can eliminate by starting our numbering at 1.

Throughout the rest of this chapter we'll write i for the message number.

[1] All keys should be updated at reasonable intervals. Heavily used keys should be updated more often. Restricting a key to $2^{32} - 1$ messages is quite reasonable.

7.3.2 Authentication

We need a MAC for the authentication function. As you might expect, we will use HMAC-SHA-256 with the full 256-bit result. The input to the MAC consists of the message m_i and the extra authentication data x_i. As we explained in Chapter 6, there is often some contextual data that has to be included in the authentication. This is the context data that Bob will use to interpret what the message means; it typically includes something that identifies the protocol, the protocol version number, and the negotiated field sizes. We are just specifying the secure channel here; the actual value for x_i will have to be provided by the rest of the application. From our point of view, each x_i is a string and both Alice and Bob have the same value for x_i.

Let $\ell(\cdot)$ be the function that returns the length (in bytes) of a string of data. The MAC value a is computed as

$$a_i := \textsc{mac}(i \parallel \ell(x_i) \parallel x_i \parallel m_i)$$

where i and $\ell(x_i)$ are both 32-bit unsigned integers in least-significant-byte-first format. The $\ell(x_i)$ ensures that the string $i \parallel \ell(x_i) \parallel x_i \parallel m_i$ uniquely parses into its fields. Without $\ell(x_i)$, there would be many ways to split it into i, x_i, and m_i, and as a result, the authentication would not be unambiguous. Of course, x_i should be encoded in such a way that it can be parsed into its different fields without further context information, but that is not something we can ensure at this level. The application using this secure channel will have to guarantee that.

7.3.3 Encryption

For encryption, we will use AES in CTR mode. But wait, in Section 4.7 didn't we say that CTR mode is dangerous because of the nonce? Yes, we did—sort of. We said that exposing the control of the nonce to developers is risky, and that we have seen too many applications that are insecure because they did not generate the nonce correctly. However, our secure channel handles the nonce internally—it never gives control of nonce generation to any other party. We use the message number as the unique nonce value that CTR mode needs. So our secure channel uses CTR mode. But we still wouldn't expose the generation of nonces to external systems. We recommend that you never use CTR mode directly.

We limit the size of each message to $16 \cdot 2^{32}$ bytes, which limits the block counter to 32 bits. Of course, we could use a 64-bit counter, but 32 bits is easier to implement on many platforms, and most applications don't need to process such huge messages.

The key stream consists of the bytes $k_0, k_1, \ldots$. For a message with nonce i, the key stream is defined by

$$\begin{aligned} & k_0, \ldots, k_{2^{36}-1} \\ & \qquad := E(K, 0 \parallel i \parallel 0) \parallel E(K, 1 \parallel i \parallel 0) \parallel \cdots \parallel E(K, 2^{32} - 1 \parallel i \parallel 0) \end{aligned}$$

where each plaintext block of the cipher is built from a 32-bit block number, the 32-bit message number, and 64 bits of zeros. The key stream is a very long string. We will only use the first $\ell(m_i) + 32$ bytes of the key stream. (We shouldn't have to mention that you don't have to compute the rest of the key stream ....) We concatenate m_i and a_i, and XOR these bytes with $k_0, \ldots, k_{\ell(m_i)+31}$.

7.3.4 Frame Format

We cannot just send the encrypted $m_i \parallel a_i$, because Bob needs to know the message number. The final message sent will consist of i encoded as a 32-bit integer, least significant byte first, followed by the encrypted m_i and a_i.

7.4 Design Details

We can now discuss the details of the secure channel. Again, we stress that this is not the only way to implement a secure channel, but instead an opportunity to dive into the challenges and nuances with building a secure channel. For convenience, we've defined the channel to be bi-directional, so the same key can be used for both directions. If we define the channel to be one-directional, then you can bet that somebody will use the same key for both directions and utterly destroy the security. Making the channel bi-directional reduces this risk. On the flip side, if you're using a secure channel defined by someone else, be extra careful not to use the same key in both directions.

We describe all our algorithms using a pseudocode notation that should be easy to read for anyone familiar with the conventions of programming. Program blocks are denoted both by the indent level and by paired key words such as if/fi and do/od.

7.4.1 Initialization

The first algorithm we show is the initialization of the channel data. This has two main functions: setting up the keys and setting up the message numbers. We derive four subsidiary keys from the channel key: an encryption key and an authentication key to send messages from Alice to Bob, and an encryption key and an authentication key to send messages from Bob to Alice.

```
function InitializeSecureChannel
input:  K       Key of the channel, 256 bits.
        R       Role. Specifies if this party is Alice or Bob.
output: $\mathcal{S}$       State for the secure channel.
    First compute the four keys that are needed. The four strings are ASCII strings
            without any length or zero-termination.
    KeySendEnc ← SHA_d-256(K ‖ "Enc Alice to Bob")
    KeyRecEnc ← SHA_d-256(K ‖ "Enc Bob to Alice")
    KeySendAuth ← SHA_d-256(K ‖ "Auth Alice to Bob")
    KeyRecAuth ← SHA_d-256(K ‖ "Auth Bob to Alice")
    Swap the encryption and decryption keys if this party is Bob.
    if R = "Bob" then
        swap(KeySendEnc, KeyRecEnc)
        swap(KeySendAuth, KeyRecAuth)
    fi
    Set the send and receive counters to zero. The send counter is the number of the
            last sent message. The receive counter is the number of the last received
            message.
    (MsgCntSend, MsgCntRec) ← (0, 0)
    Package the state.
    $\mathcal{S}$ ← (KeySendEnc,
            KeyRecEnc,
            KeySendAuth,
            KeyRecAuth,
            MsgCntSend,
            MsgCntRec)
    return $\mathcal{S}$
```

There is also a function to wipe the state information $\mathcal{S}$. We will not specify this in any detail. All it does is wipe the memory that $\mathcal{S}$ used to store information. It is vital that this information be wiped because the keys were stored in that area. On many systems, just deallocating the memory doesn't necessarily wipe it, so you must erase $\mathcal{S}$ when you are done with it.

7.4.2 Sending a Message

We now turn to the processing required to send a message. This algorithm takes the session state, a message to send, and additional data to be authenticated, and produces the encrypted and authenticated message ready for transmission. The recipient must have the same additional data at hand to check the authentication.

```
function SendMessage
input:  $\mathcal{S}$   Secure session state.
        $m$   Message to be sent.
        $x$   Additional data to be authenticated.
output: $t$   Data to be transmitted to the receiver.
    First check the message number and update it.
    assert MsgCntSend $< 2^{32} - 1$
    MsgCntSend $\leftarrow$ MsgCntSend $+ 1$
    $i \leftarrow$ MsgCntSend
    Compute the authentication. The values $\ell(x)$ and i are encoded in four bytes, least
            significant byte first.
    $a \leftarrow$ HMAC-SHA-256(KeySendAuth, $i \,\|\, \ell(x) \,\|\, x \,\|\, m$)
    $t \leftarrow m \,\|\, a$
    Generate the key stream. Each plaintext block of the block cipher consists of a
            four-byte counter, four bytes of i, and eight zero bytes. Integers are
            LSByte first, E is AES encryption with a 256-bit key.
    $K \leftarrow$ KeySendEnc
    $k \leftarrow E_K(0 \,\|\, i \,\|\, 0) \,\|\, E_K(1 \,\|\, i \,\|\, 0) \,\|\, \cdots$
    Form the final text. Again, i is encoded as four bytes, LSByte first.
    $t \leftarrow i \,\|\, (t \oplus \text{First-}\ell(t)\text{-bytes}(k))$
    return $t$
```

Given our earlier discussions, this is relatively straightforward. We check for exhaustion of the message counter. We cannot stress enough how important this check is. If the counter ever wraps, the entire security falls apart—and this is a mistake we've seen often. The authentication and encryption are as described in our previous discussion. Finally, we send i with the encrypted and authenticated message so that the receiver will know the message number.

Note that the session state is updated because the MsgCntSend value is modified. Again, this is vital, as the message number must be unique. In fact, almost everything in these algorithms is vital for the security.

Our secure channel uses CTR mode for encryption. If the encryption scheme requires padding, be sure to verify the contents of the padding when you decrypt.

7.4.3 Receiving a Message

The receiving algorithm requires the encrypted and authenticated message that SendMessage produced and the same additional data x to be authenticated. We assume the receiver knows x through some out-of-band means. For example, if x contains the protocol version number, then surely Bob must know this if he's participating in the protocol.

```
function ReceiveMessage
input:  $\mathcal{S}$    Secure session state.
        $t$    Text received from the transmitter.
        $x$    Additional data to be authenticated.
output: $m$    Message that was sent.
    The received message must contain at least a 4-byte message number and a 32-byte
            MAC field. This check ensures that all the future splitting operations
            will work.
    assert $\ell(t) \geq 36$
    Split t into i and the encrypted message plus authenticator. The split is well-defined
            because i is always 4 bytes long.
    $i \parallel t \leftarrow t$
    Generate the key stream, just as the sender did.
    $K \leftarrow \textsc{KeyRecEnc}$
    $k \leftarrow E_K(0 \parallel i \parallel 0) \parallel E_K(1 \parallel i \parallel 0) \parallel \cdots$
    Decrypt the message and MAC field, and split. The split is well-defined because a
            is always 32 bytes long.
    $m \parallel a \leftarrow t \oplus \textsc{First-}\ell(t)\textsc{-bytes}(k)$
    Recompute the authentication. The values $\ell(x)$ and i are encoded in four bytes,
            least significant byte first.
    $a' \leftarrow \text{HMAC-SHA-256}(\textsc{KeyRecAuth}, i \parallel \ell(x) \parallel x \parallel m)$
    if $a' \neq a$ then
        destroy $k, m$
        return AuthenticationFailure
    else if $i \leq \textsc{MsgCntRec}$ then
        destroy $k, m$
        return MessageOrderError
    fi
    $\textsc{MsgCntRec} \leftarrow i$
    return $m$
```

We have used the canonical order for the operations here. You could put the check on the message number before the decryption, but then this function would report the wrong error if i were mangled during transmission. Instead of notifying the caller that the message was mangled, it would notify the caller that the message is in the wrong order. As the caller might wish to handle the two situations differently, this routine should not give the wrong information. The reason some people like to put the check earlier is that it allows false messages to be discarded more quickly. We don't consider this to be of great importance; if you receive so many false packets that the speed of discarding them becomes significant, you already have much bigger problems.

There is one very important issue for the receiver. The ReceiveMessage function may not release any information about the key stream or the plaintext message until the authentication has been verified. If the authentication fails, a failure indication is returned, but neither the key stream nor the plaintext may be revealed. An actual implementation should wipe the memory areas used to store these elements. So why is this so important? The plaintext message reveals the key stream, because it is assumed that every attacker knows the ciphertext. The danger is that the attacker will send a fake message (with an incorrect MAC value) but still learn the key stream from the data released by the receiver. This is the paranoia model at work again. Any data released or leaked by this routine is automatically assumed to end up in possession of the attacker. By destroying the data held in k and m before returning with an error, this routine ensures that this data can never be leaked.

7.4.4 Message Order

Like the transmitter, the receiver updates the state $\mathcal{S}$ by modifying the MsgCntRec variable. The receiver ensures that the message numbers of the messages it accepts are strictly increasing. This certainly ensures that no message is accepted twice, but if the stream of messages is reordered during transmission, otherwise perfectly valid messages will be lost.

It is relatively easy to fix this, but at a cost. If you let the receiver accept messages out of order, then the application that uses the secure channel must be able to handle these out-of-order messages. Many applications cannot deal with this. Some applications are designed to handle it, but have subtle bugs (often security-relevant) when messages are reordered. In most situations, we prefer to fix the underlying transport layer and prevent accidental reordering of messages, so that the secure channel does not have to deal with this problem.

There is one situation that we know of in which the receiver allows messages to arrive out of order, and for a very good reason. This is IPsec, the IP security protocol [73] that encrypts and authenticates IP packets. As IP packets can be reordered during transport, and as all applications that use IP are very well aware of this property, IPsec maintains a replay protection window rather than just remembering the counter value of the last received message. If c is the message number of the last received message, then IPsec maintains a bitmap for the message numbers $c-31, c-30, c-29, \ldots, c-1, c$. Each bit indicates whether a message with the corresponding message number has been received. Messages with numbers smaller than $c-31$ are always refused. Messages in the range $c-31$ to $c-1$ are only accepted if the corresponding bit is 0 (and this bit is then set, of course). If the new message has a message number larger than c, then c is updated and the bitmap is shifted to maintain the invariant. Such a bitmap construction allows some limited reordering of the messages without adding too much state to the receiver.

Another option is to terminate the communications if a message is dropped. This is particularly suited when the secure channel runs on top of a reliable transport like TCP. Unless there is malicious activity, messages should arrive in order and without any loss. So if a message is dropped or arrives out of order, terminate the communications.

7.5 Alternatives

The secure channel definition we have given is not always practical; especially when implementing a secure channel in embedded hardware, it becomes relatively costly to implement SHA-256. As an alternative, there has recently been interest in creating dedicated block cipher modes for providing both privacy and authenticity at the same time.

These dedicated privacy-and-authenticity block cipher modes take a single key as input, just like CBC mode and CBC-MAC. These modes generally also take a message as input, additional data to be authenticated, and a nonce. These modes are not as simple as just using CBC mode and CBC-MAC with the same key, however. Using the same key for both a regular encryption mode and a regular MAC can lead to security problems.

The most well-known initial combined mode is OCB [109]. This mode is very efficient. Each plaintext block can be processed in parallel, which is attractive for high-speed hardware. The existence of patents has limited OCB's adoption.

Because of the patent issues surrounding OCB, and because of the need for a dedicated, single-key block cipher mode for encryption and authentication, Doug Whiting, Russ Housley, and Niels developed a mode called CCM [126]. It is a combination of CTR mode encryption and CBC-MAC authentication, but with care taken to allow for the use of the same key with both CTR mode and CBC-MAC. Compared to OCB, it requires twice as many computations to encrypt and authenticate a message, but as far as we know there are no patent issues at all with CCM. The designers know of no patents that cover CCM, and they have not applied, nor will they apply, for a patent. Jakob Jonsson provided a proof of security for CCM [65]. NIST has since standardized CCM as a block cipher mode [41].

To improve on the efficiency of CCM, Doug Whiting, John Viega, and Yoshi developed another mode called CWC [80]. CWC builds on CTR mode to provide encryption. Under the hood, CWC uses universal hashing to achieve authenticity [125]. We mentioned but did not discuss universal hashing in Chapter 6 when we introduced GMAC [43]. CWC's use of universal hashing makes CWC fully parallelizable, like OCB, but avoids the patents surrounding OCB. David McGrew and John Viega improved on CWC with a more efficient universal hashing function for hardware implementations. Their improved mode is called GCM [43]. NIST has now standardized GCM as a block cipher mode.

Just like our secure channel from earlier in this chapter, OCB, CCM, CWC, and GCM can all take two strings as input—a message to be sent and additional data to be authenticated. The GMAC message authentication scheme is actually just GCM mode where the main message is the empty string.

These modes are all reasonable choices. Because they are standardized and unencumbered by patents, we prefer CCM and GCM. Unfortunately, GCM's authentication capability shares the limitations of GMAC that we discussed in Section 6.5. Therefore, although it is possible to reduce the size of the authenticator for GCM from 128 bits to something less, we recommend not doing so. Our recommendation is to only use GCM with the full 128-bit authentication tag.

Another important point: OCB, CCM, CWC, GCM, and similar modes do not by themselves provide the full secure channel. They provide the encryption/authentication functionality, and require a key and a unique nonce for each packet. We discussed the risks of relying on external systems to correctly generate nonces in Section 4.7. It is easy, however, to adapt our secure channel algorithms to use one of these block cipher modes rather than the separate MAC and encryption functions. Instead of the four subsidiary keys generated in InitializeSecureChannel, you will need two keys, one for each direction of traffic. The nonce can be constructed by padding the message number to the correct size.

Stepping back, we observe that the secure channel is one of the most useful applications of cryptography, and it is used in almost all cryptographic systems. You can construct a secure channel from good encryption and authentication primitives, and there are also dedicated privacy-and-authenticity block cipher modes that you can build upon. There are many details to pay attention to, and all the details must of course be done correctly. A separate challenge, which we will consider later, is establishing a symmetric key.

7.6 Exercises

Exercise 7.1 In our design of a secure channel, we said that the message numbers must not repeat. What bad things can happen if the message numbers do repeat?

Exercise 7.2 Modify the algorithms for the secure channel in this chapter to use the encrypt-then-authenticate order for encryption and authentication.

Exercise 7.3 Modify the algorithms for the secure channel in this chapter to use the a dedicated, single-key mode for providing both encryption and authentication. You can use OCB, CCM, CWC, or GCM as a black box.

Exercise 7.4 Compare and contrast the advantages and disadvantages among the different orders of applying encryption and authentication when creating a secure channel.

Exercise 7.5 Find a new product or system that uses (or should use) a secure channel. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on the security and privacy issues surrounding the secure channel.

Exercise 7.6 Suppose Alice and Bob are communicating using the secure channel described in this chapter. Eve is eavesdropping on the communications. What types of traffic analysis information could Eve learn by eavesdropping on the encrypted channel? Describe a situation in which information exposure via traffic analysis is a serious privacy problem.

CHAPTER 8

Implementation Issues (I)

Now that we have come this far, we would like to talk a bit about implementation issues. Implementing cryptographic systems is sufficiently different from implementing normal programs to deserve its own treatment.

The big problem is, as always, the weakest-link property (see Section 1.2). It is very easy to screw up the security at the implementation level. In fact, implementation errors such as buffer overflows are one of the biggest security problems in real-world systems. With few exceptions, you don't hear about cryptography systems that are broken in practice. This is not because the cryptography in most systems is any good; we've reviewed enough of them to know this is not the case. It is just easier in most cases to find an implementation-related hole than it is to find a cryptographic vulnerability, and attackers are smart enough not to bother with the cryptography when there is this much easier route.

So far in this book we have restricted our discussion to cryptography, but in this chapter we will focus more on the environment in which the cryptography operates. Every part of the system affects security, and to do a really good job, the entire system must be designed from the ground up not just with security in mind, but with security as one of the primary goals. The ''system'' we're talking about is very big. It includes everything that could damage the security properties if it were to misbehave.

One major part is, as always, the operating system. But historically, none of the operating systems in widespread use was designed with security as a primary goal. And the diversity of operating systems is enormous—from the operating systems we interact with on our desktop computers to operating systems on embedded devices and phones. The logical conclusion to draw from this is that it is impossible to implement a secure system. We don't know how to do it, and we don't know anyone else who knows how to do it, either.

Real-life systems include many components that were never designed for security, and that makes it impossible to achieve the level of security that we really need. So should we just give up? Of course not. When we design a cryptographic system, we do our very best to make sure that at least our part is secure. This might sound like an odd mentality: all we care about is our little domain. But we *do* care about the other parts of the system; we just can't do anything about them in the context of this book. That is one of the reasons for writing this book: to get other people to understand the insidious nature of security, and how important it is to do it right.

Another important reason to get at least the cryptography right is one we mentioned before: attacks on the cryptography are especially damaging because they can be invisible. If the attacker succeeds in breaking your cryptography, you are unlikely to notice. This can be compared to a burglar who has a set of keys to your house. If the burglar exercises reasonable caution, how would you ever find out?

Our long-term goal is to make secure computer systems. To achieve that goal, everybody will have to do their part. This book is about making the cryptography secure. Other parts of the system will have to be made secure, too. The overall security of the system is going to be limited by the weakest link, and we will do our utmost to ensure that the weakest link will never be the cryptography.

Another important reason to do the cryptography right is that it is very difficult to switch cryptographic systems once they've been implemented. An operating system runs on a single computer. Cryptographic systems are often used in communication protocols to let many computers communicate with each other. Upgrading the operating system of a single computer is feasible, and in practice it is done relatively often. Modifying the communication protocols in a network is a nightmare, and as a result many networks still use the designs of the 1970s and 1980s. We must keep in mind that any new cryptographic system we design today, if adopted widely, is quite likely to still be in use 30 or 50 years from now. We hope that by that time the other parts of the system will have achieved a much higher level of security, and we certainly don't want cryptography to be the weakest link.

8.1 Creating Correct Programs

The core of the implementation problem is that we in the IT industry don't know how to write a correct program or module. (A ''correct'' program is one that behaves exactly according to its specifications.) There are several reasons for the difficulty we seem to have in writing correct programs.

8.1.1 Specifications

The first problem is that for most programs, there is no clear description of what they are supposed to do. If there are no specifications, then you cannot even check whether a program is correct or not. For such programs, the whole concept of correctness is undefined.

Many software projects have a document called the functional specification. In theory, this should be the specification of the program. But in practice, this document often does not exist, is incomplete, or specifies things that are irrelevant for the behavior of the program. Without clear specifications, there is no hope of getting a correct program.

There are really three stages in the specification process:

Requirements Requirements are an informal description of what the program is supposed to achieve. It is really a ''*what* can I do with it'' document, rather than a ''how exactly do I do something with it'' document. Requirements are often a bit vague and leave details out in order to concentrate on the larger picture.

Functional specification The functional specification gives a detailed and exhaustive definition of the behavior of the program. The functional specification can only specify things that you can measure on the outside of the program.

For each item in the functional specification, ask yourself whether you could create a test on the finished program that would determine whether that item was adhered to or not. The test can only use the external behavior of the program, not anything from the inside. If you can't create a test for an item, it does not belong in the functional specification.

The functional specification should be complete. That is, every piece of functionality should be specified. Anything not in the functional specification does not have to be implemented.

Another way to think of the functional specification is as the basis for testing the finished program. Any item can, and should, be tested.

Implementation design This document has many names, but it specifies how the program works internally. It contains all of the things that cannot be tested from the outside. A good implementation design will often split the program into several modules, and describe the functionality of each. In turn, these module descriptions can be seen as the requirements for the module, and the whole cycle starts all over again, this time by splitting the module itself into multiple sub-modules.

Of these three documents, the functional specification is without a doubt the most important one. This is the document against which the program will be tested when it is finished. You can sometimes get by with informal requirements, or an implementation design that is nothing but a few sketches on a whiteboard. But without functional specifications, there is no way to even describe what you have achieved in the end when the program is finished.

8.1.2 Test and Fix

The second problem in writing correct programs is the test-and-fix development method that is in almost universal use. Programmers write a program, and then test whether it behaves correctly. If it doesn't, they fix the bugs and test again. As we all know, this does not lead to a correct program. It results in a program that kind of works in the most common situations.

Back in 1972, Edsger Dijkstra commented in his Turing Award lecture that testing can only show the presence of bugs, never the absence of bugs [35]. This is very true, and ideally we would like to write programs that we can demonstrate to be correct. Unfortunately, current techniques in proving the correctness of programs are not good enough to handle day-to-day programming tasks, let alone a whole project.

Computer scientists do not know how to solve this. Maybe it will be possible in the future to prove that a program is correct. Maybe we just need a far more extensive and thorough testing infrastructure and methodology. But even without having a full solution, we can certainly do our very best with the tools we do have.

There are some simple rules about bugs that any good software engineering book includes:

- If you find a bug, first implement a test that detects the bug. Check that the bug is detected. Then fix the bug, and check that the test no longer finds the bug. And then keep running that test on every future version to make sure the bug does not reappear.

- Whenever you find a bug, think about what caused it. Are there any other places in the program where a similar bug might reside? Go check them all.

- Keep track of every bug you find. Simple statistical analysis of the bugs you have found can show you which part of the program is especially buggy, or what type of error is made most frequently, etc. Such feedback is necessary for a quality control system.

This is not even a bare minimum, but there is not a lot of methodology to draw from. There are quite a few books that discuss software quality. They don't all agree with each other. Many of them present a particular software development methodology as *the* solution, and we are always suspicious of such one-cure-does-it-all schemes. The truth is almost always somewhere in the middle.

8.1.3 Lax Attitude

The third problem is the incredibly lax attitude of many in the computer industry. Errors in programs are frequently accepted as a matter of course. If your word processor crashes and destroys a day's worth of work, everybody seems to think this is quite normal and acceptable. Often they blame the user: ''You should have saved your work more often.'' Software companies routinely ship products with known bugs in them. This wouldn't be so bad if they only sold computer games, but nowadays our work, our economy, and—more and more—our lives depend on software. If a car manufacturer finds a defect (bug) in a car after it was sold, they will recall the car and fix it. Software companies get away with disclaiming any and all liability in their software license, something they wouldn't be allowed to do if they produced any other product. This lax attitude means there are still not enough serious efforts being made at producing correct software.

8.1.4 So How Do We Proceed?

Don't ever think that all you need is a good programmer or code reviews or an ISO 9001–certified development process or extensive testing or even a combination of all of them. Reality is much more difficult. Software is too complex to be tamed by a few rules and procedures. We find it instructive to look at the best engineering quality control system in the world: the airline industry. Everybody in that industry is involved in the safety system. There are very strict rules and procedures for almost every operation. There are multiple backups in case of failures. Every nut and bolt of the airplane has to be flight-qualified before it can ever be used. Anytime a mechanic takes a screwdriver to the plane, his work is checked and signed off by a supervisor. Every modification is carefully recorded. Any accident is meticulously investigated to find all the underlying causes, which are then fixed. This fanatical pursuit of quality has a very high cost. An airplane is probably an order of magnitude more expensive than it would be if you just sent the drawings to an ordinary engineering firm. But the pursuit of quality has also been amazingly effective. Flying is an entirely routine operation today, in a machine where every failure is potentially fatal—a machine where you cannot just hit the brakes and stop when something goes wrong. One where the only safe way back to the ground is the quite delicate operation of landing on one of the rare specially prepared spots in the world. The airline industry has been amazingly effective at making flying secure. We would do well to learn all we can from them. Maybe writing correct software *would* cost an order of magnitude more than what we are used to now. But given the cost to society of the bugs in software that we see today, we are sure it would be cost-effective in the long run.

8.2 Creating Secure Software

So far, we have only talked about correct software. Just writing correct software is not good enough for a security system. The software must be secure as well.

What is the difference? Correct software has a specified functionality. If you hit button A, then B will happen. Secure software has an additional requirement: a *lack* of functionality. No matter what the attacker does, she cannot do X. This is a very fundamental difference; you can test for functionality, but not for lack of functionality. The security aspects of the software cannot be tested in any effective way, which makes writing secure software much more difficult than writing correct software. The inevitable conclusion is:

Standard implementation techniques
are entirely inadequate to create secure code.

We actually don't know how to create secure code. Software quality is a vast area that would take several books to cover. We don't know enough about it to write those books, but we *do* know the cryptography-specific issues and the problems that we see most frequently, and that is what we will discuss in the rest of this chapter.

Before we start, let us make our point of view clear: unless you are willing to put real effort into developing a secure implementation, there is little point in bothering with the cryptography. Designing cryptographic systems might be fun, but cryptography is generally only a small part of a larger system.

8.3 Keeping Secrets

Anytime you work with cryptography, you are dealing with secrets. And secrets have to be kept. This means that the software that deals with the secrets has to ensure that they don't leak out.

For the secure channel we have two types of secrets: the keys and the data. Both of these secrets are transient secrets; we don't have to store them for a long time. The data is only stored while we process each message. The keys are only stored for the duration of the secure channel. Here we will only discuss keeping transient secrets. For a discussion on storing secrets long-term, see Chapter 21.

Transient secrets are kept in memory. Unfortunately, the memory on most computers is not very secure. We will discuss each of the typical problems in turn.

8.3.1 Wiping State

A basic rule of writing security software: wipe any information as soon as you no longer need it. The longer you keep it, the higher the chance that someone will be able to access it. What's more, you should definitely wipe the data before you lose control over the underlying storage medium. For transient secrets, this involves wiping the memory locations.

This sounds easy to do, but it leads to a surprising number of problems. If you write the entire program in C, you can take care of the wiping yourself. If you write a library for others to use, you have to depend on the main program to inform you that the state is no longer needed. For example, when the communication connection is closed, the crypto library should be informed so that it can wipe the secure channel session state. The library can contain a function for this, but there's a reasonable chance that the programmer of the application won't call this function. After all, the program works perfectly well without calling this function.

In some object-oriented languages, things are a bit easier. In C++, there is a destructor function for each object, and the destructor can wipe the state. This is certainly standard practice for security-relevant code in C++. As long as the main program behaves properly and destroys all objects it no longer needs, the memory state will be wiped. The C++ language ensures that all stack-allocated objects are properly destroyed when the stack is unwound during exception handling, but the program has to ensure that all heap-allocated objects are destroyed. Calling an operating system function to exit the program might not even unwind the call stack. And you have to ensure that all sensitive data is wiped even if the program is about to exit. After all, the operating system gives no guarantees that it will wipe the data soon, and some operating systems don't even bother wiping the memory before they give it to the next application.

Even if you do all this, the computer might still frustrate your attempts. Some compilers try too hard to optimize. A typical security-relevant function performs some computations in local variables, and then tries to wipe them. You can do this in C with a call to the `memset` function. Good compilers will optimize the `memset` function to in-line code, which is more efficient. But some of them are too clever by half. They detect that the variable or array that is being wiped will never be used again, and ''optimize'' the `memset` away. It's faster, but suddenly the program does not behave the same way anymore. It is not uncommon to see code that reveals data that it happens to find in memory. If the memory is given to some library without having been wiped first, the library might leak the data to an attacker. So check the code that your compiler produces, and make sure the secrets are actually being wiped.

In a language like Java, the situation is even more complicated. All objects live on the heap, and the heap is garbage-collected. This means that the finalization function (similar to the C++ destructor) is not called until the garbage collector figures out that the object is no longer in use. There are no specifications about how often the garbage collector is run, and it is quite conceivable that secret data remains in memory for a very long time. The use of exception handling makes it hard to do the wiping by hand. If an exception is thrown, then the call-stack unwinds without any way for the programmer to insert his own code, except by writing *every* function as a big try clause. The latter solution is so ugly that it is impractical. It also has to be applied throughout the program, making it impossible to create a security library for Java that behaves properly. During exception handling, Java happily unwinds the stack, throwing away the references to the objects without cleaning up the objects themselves. Java is really bad in this respect. The best solution we've been able to come up with is to at least ensure that the finalization routines are run at program exit. The `main` method of the program uses a `try-finally` statement. The `finally` block contains some code to force a garbage collect, and to instruct the garbage collector to attempt to complete all the finalization methods. (See the functions `System.gc()` and `System.runFinalization()` for more details.) There is still no guarantee that the finalization methods will be run, but it is the best we've been able to find.

What we really need is support from the programming language itself. In C++ it is at least theoretically possible to write a program that wipes all states as soon as they are no longer needed, but many other features of the language make it a poor choice for security software. Java makes it very difficult to wipe the state. One improvement would be to declare variables as ''sensitive,'' and have the implementation guarantee that they will be wiped. Even better would be a language that always wipes all data that is no longer needed. That would avoid a lot of errors without significantly affecting efficiency.

There are other places where secret data can end up. All data is eventually loaded into a CPU register. Wiping registers is not possible in most programming languages, but on register-starved CPUs like the x86, it is very unlikely that any data will survive for any reasonable amount of time.

During a context-switch (when the operating system switches from running one program to running the next program), the values in the registers of the CPU are stored in memory where their values might linger for a long time. As far as we know, there is nothing you can do about this, apart from fixing the operating system to ensure the confidentiality of that data.

8.3.2 Swap File

Most operating systems (including all current Windows versions and all UNIX versions) use a virtual memory system to increase the number of programs that can be run in parallel. While a program is running, not all of its data is kept in memory. Some is stored in a swap file. When the program tries to access data that is not in memory, the program is interrupted. The virtual memory system reads the required data from the swap file into a piece of memory, and the program is allowed to continue. What's more, when the virtual memory system decides that it needs more free memory, it will take an arbitrary piece of memory from a program and write it to the swap file.

Of course, not all virtual memory systems or configurations keep the data secret, or encrypt it before it is written to the disk. Most software is designed for a cooperative environment, not the adversarial environment that cryptographers work in. So our problem is the following: the virtual memory system could just take some of the memory of our program and write it to the swap file on disk. The program never gets told, and does not notice. Suppose this happens to the memory in which the keys are stored. If the computer crashes—or is switched off—the data remains on the disk. Most operating systems leave the data on disk even when you shut them down properly. There may be no mechanism to wipe the swap file, so the data could linger indefinitely on disk. Who knows who will have access to this swap file in future? We really cannot afford the risk of having our secrets written to the swap file.[1]

So how do we stop the virtual memory system from writing our data to disk? On some operating systems there are system calls that you can use to inform the virtual memory system that specified parts of memory are not to be swapped out. Some operating systems support a secure swap system where the swapped-out data is cryptographically protected, but these systems might require the user to toggle the appropriate system configuration flags. If neither of these options is available on all the systems that you wish to run your application, there might not be much you can do to protect against this particular avenue of attack.

Assuming you can lock the memory and prevent it from being swapped out, which memory should be locked? All the memory that can ever hold secrets, of course. This brings up a secondary problem. Many programming environments make it very hard to know where exactly your data is being stored. Objects are often allocated on a heap, data can be statically allocated, and many local variables end up on the stack. Figuring out the details is complicated and very error-prone. Probably the best solution is to simply lock all the memory of your application. Even that is not quite as easy as it sounds, because you could lose a number of operating system services such as the automatically allocated stack. And locking all the memory makes the virtual memory system ineffective.

It shouldn't be this difficult. The proper solution is, of course, to make a virtual memory system that protects the confidentiality of the data. This is an

[1] In fact, we should never write secrets to any permanent media without encrypting them, but that is an issue we will discuss later.

operating system change, and beyond our control. Even if the next version of your operating system were to have this feature, you should carefully check that the virtual memory system does a good job of keeping secrets. And, depending on your application, you may still have to deal with the fact that your application needs to run on older systems or systems in insecure configurations.

8.3.3 Caches

Modern computers don't just have a single type of memory. They have a hierarchy of memories. At the bottom is the main memory—often gigabytes in size. But because the main memory is relatively slow, there is also a cache. This is a smaller but faster memory. The cache keeps a copy of the most recently used data from the main memory. If the CPU wants to access the data, it first checks the cache. If the data is in the cache, the CPU gets the data relatively quickly. If the data is not in the cache, it is read (relatively slowly) from main memory, and a copy is stored in the cache for future use. To make room in the cache, a copy of some other piece of data is thrown away.

This is important because caches keep copies of data, including copies of our secret data. The problem is that when we try to wipe our secrets, this wiping might not take place properly. In some systems, the modifications are only written to the cache and not to the main memory. The data will eventually be written to main memory, but only when the cache needs more room to store other data. We don't know all the details of these systems, and they change with every CPU. There is no way to know if there is some interaction between the memory allocation unit and the cache system that might result in some wipe operations escaping the write-to-main-memory part when the memory is deallocated before the cache is flushed. Manufacturers never specify how to wipe data in a guaranteed manner. At least, we have never seen any specifications like that, and as long as it is not specified, we can't trust it.

A secondary danger of caches is that under some circumstances a cache learns that a particular memory location has been modified, perhaps by the other CPU in a multi-CPU system. The cache then marks the data it has for that location as ''invalid,'' but typically the actual data is not wiped. Again, there might exist a copy of our secrets that has not been wiped.

There is very little you can do about this. It is not a great danger, because in most systems, minus physical attacks, only the OS code can access the cache mechanisms directly. And we have to trust the operating system anyway, so we could trust it with this as well. We are nevertheless concerned about these designs, because they clearly do not provide the functionality that is required to implement security systems properly.

8.3.4 Data Retention by Memory

Something that surprises many people is that simply overwriting data in memory does not delete the data. The details depend to some extent on the exact type of memory involved, but basically, if you store data in a memory location, that location slowly starts to ''learn'' the data. When you overwrite or switch off the computer, the old value is not completely lost. Depending on the circumstances, just powering the memory off and back on again can recover some or all of the old data. Other memories can ''remember'' old data if you access them using (often undocumented) test modes [57].

Several mechanisms cause this phenomenon. If the same data is stored for a time in the same location in SRAM (Static RAM), then this data becomes the preferred power-up state of that memory. A friend of ours encountered this problem with his home-built computer long ago [17]. He wrote a BIOS that used a magic value in a particular memory location to determine whether a reset was a cold reboot or a warm reboot.[2] After a while the machine refused to boot after power-up because the memory had learned the magic value, and the boot process therefore treated every reset as a warm reboot. As this did not initialize the proper variables, the boot process failed. The solution in his case was to swap some memory chips around, scrambling the magic value that the SRAM had learned. For us, it was a lesson to remember: memory retains more data than you think.

Similar processes happen in DRAM (Dynamic RAM), although they are somewhat more complicated. DRAM works by storing a small charge on a very small capacitor. The insulating material around the capacitor is stressed by the resulting field. The stress results in changes to the material, specifically causing the migration of impurities [57]. An attacker with physical control over the memory can potentially recover this data. Additionally, because of how DRAM capacitors discharge, their values may remain for seconds at room temperature if power is removed or even longer if the memory is cooled.

These are important problems. The latter class of issues was recently demonstrated in the context of cold boot attacks [59]. These researchers were able to recover secret cryptographic keys from the memories of computers after they were rebooted. These researchers were also able to physically extract the memory from one computer, put that memory in another computer, and recover the cryptographic keys. If your computer is ever compromised (e.g., stolen), you do not want the data that you had in memory to be

[2] In those days home-built machines were programmed by entering the binary form of machine language directly. This led to many errors, and the one sure way to recover from a program that crashed was to reset the machine. A cold reboot is one after power-up. A warm reboot is the sort performed when the user presses the reset button. A warm reboot does not reinitialize all the state, and therefore does not wipe the settings the user made.

compromised as well. To achieve this goal, we have to make the computer forget information.

We can only give a partial solution, which works if we make some reasonable assumptions about the memory. This solution, which we call a Boojum,[3] works for relatively small amounts of data, such as keys. Our description of Boojum has been updated slightly since the first edition of this book, and includes a defense against the cold boot attack from [59]. For Boojum, let m be the data we want to store. Instead of storing m, we generate a random string R and store both R and $h(R) \oplus m$ where h is a hash function. These two values are stored in different memory locations, preferably not too close together. One trick is to change R regularly. At regular intervals, say every 1 second, we generate a new random R', and update the memory to store $R \oplus R'$ and $h(R \oplus R') \oplus m$. This ensures that each bit of the memory is written with a sequence of random bits. To wipe the memory, you simply write a new m with the value zero.

To read information from this storage, you read both parts, hash the first, and XOR them together to get m. Writing is done by XORing the new data with $h(R)$ and storing it in the second location.

Care should be taken that the bits of R and $h(R) \oplus m$ are not adjacent on the RAM chip. Without information about how the RAM chip works, this can be difficult, but most memories store bits in a rectangular matrix of bits, with some address bits selecting the row and other address bits selecting the column. If the two pieces are stored at addresses that differ by `0x5555`, it is highly unlikely that the two will be stored adjacent on the chip. (This assumes that the memory does not use the even-indexed address bits as row number and the odd-indexed address bits as column number, but we have never seen a design like that.) An even better solution might be to choose two random addresses in a very large address space. This makes the probability that the two locations are adjacent very small, independent of the actual chip layouts of the memory.

This is only a partial solution, and a rather cumbersome one at that. It is limited to small amounts of data. But using this solution ensures that there is no physical point on the memory chip that is continually stressed or unstressed depending on the secret data. Further, as long as k bits of R are not recoverable, the attacker would have to exhaustively search for those k bits before being able to recover $h(R) \oplus m$.

There is still no guarantee that the memory will be wiped. If you read the documentation of a memory chip, there are no specifications that prevent the chip from retaining all data ever stored in it. No chip does that, of course, but it shows that we can at most achieve a heuristic security.

We have concentrated on the main memory here. The same solution will work for the cache memory, except that you cannot control the position on the chip where the data will be stored. This solution does not work for the

[3] After Lewis Carroll's *The Hunting of the Snark* [24].

CPU registers, but they are used so often for so much different data that we doubt they will pose a data retention problem. On the other hand, extension registers, such as floating point registers or MMX-style registers, are used far less frequently, so they could pose a problem.

If you have large amounts of data that need to be kept secret, then the solution of storing two copies and XORing new random strings into both copies regularly becomes too expensive. A better solution is to encrypt a large block of data and store the ciphertext in memory that potentially retains information. Only the key needs to be stored in a way that avoids data retention, for example, using a Boojum. For details, see [32].

8.3.5 Access by Others

There's yet another problem with keeping secrets on a computer: other programs on the same machine might access the data. Some operating systems allow different programs to share memory. If the other program can read your secret keys, you have a serious problem. Often the shared memory has to be set up by both programs, which reduces the risk. In other situations, the shared memory might be set up automatically as a result of loading a shared library.

Debuggers are especially dangerous. Modern operating systems often contain features designed to be used by debuggers. Various Windows versions allow you to attach a debugger to an already running process. The debugger can do many things, including reading the memory. Under UNIX, it is sometimes possible to force a core-dump of a program. The core-dump is a file that contains a memory image of the program data, including all of your secrets.

Another danger comes from especially powerful users. Called *superusers*, or *administrators*, these users can access things on the machine that normal users cannot. Under UNIX, for example, the superuser can read any part of the memory.

In general, your program cannot effectively defend itself against these types of attacks. If you are careful, you may be able to eliminate some of these problems, but often you'll find yourself limited in what can be achieved. Still, you should consider these issues on the particular platform you are working on.

8.3.6 Data Integrity

In addition to keeping secrets, we should protect the integrity of the data we are storing. We use the MAC to protect the integrity of the data during transit, but if the data can be modified in memory, we still have problems.

In this discussion, we will assume that the hardware is reliable. If the hardware is unreliable, there is very little you can do. If you are unsure about the hardware reliability, perhaps you should spend part of your time and memory simply to verify it, although that is really the operating system's job.

One thing we try to do is make sure the main memory on our machines is ECC (error-correcting code) memory.[4] If there is a single bit failure, then the error-correcting code will detect and correct the error. Without ECC memory, any bit error leads to the CPU reading the wrong data.

Why is this important? There is an enormous number of bits in a modern computer. Suppose the engineering is done really well, and each bit has only a 10^{-15} chance of failing in each second. If you have 128 MB of memory, then you have about 10^9 bits of memory, and you can expect one bit failure every 11 days. The error rate increases with the amount of memory in the machine, so it is even worse if you have 1 GB of memory, with one failure every 32 hours. Servers typically use ECC memory because they have more memory and run for longer periods of time. We like to have the same stability in all machines.

Of course, this is a hardware issue, and you typically don't get to specify the type of memory on the machine that will run the final application.

Some of the dangers that threaten data confidentiality also endanger the data integrity. Debuggers can sometimes modify your program's memory. Superusers can directly modify memory, too. Again, there is nothing you can do about it, but it is useful to be aware of the situation.

8.3.7 What to Do

Keeping a secret on a modern computer is not as easy as it sounds. There are many ways in which the secret can leak out. To be fully effective, you have to stop all of them. Unfortunately, current operating systems and programming languages do not provide the required support to stop the leakage completely. You have to do the best you can. This involves a lot of work, all of it specific to the environment you work in.

These problems also make it very difficult to create a library with the cryptographic functions in it. Keeping the secrets safe often involves modifications to the main program. And of course, the main program also handles data that should be kept confidential; otherwise, it wouldn't need the cryptography library in the first place. This is the familiar issue of security considerations affecting every part of the system.

8.4 Quality of Code

If you create an implementation for a cryptographic system, you will have to spend a great deal of time on the quality of the code. This book is not about programming, but we will say a few words about code quality here.

[4] You have to make sure that all components of the computer support ECC memory. Beware of slightly cheaper memory modules that do not store the extra information but instead recompute it on the fly. This defeats the whole purpose of ECC memory.

8.4.1 Simplicity

Complexity is the main enemy of security. Therefore, any security design should strive for simplicity. We are quite ruthless about this, even though this does not make us popular. Eliminate all the options that you can. Get rid of all those baroque features that few people use. Stay away from committee designs, because the committee process always leads to extra features or options in order to achieve compromise. In security, simplicity is king.

A typical example is our secure channel. It has no options. It doesn't allow you to encrypt the data without authenticating it, or to authenticate the data without encrypting it. People always ask for these features, but in many cases they do not realize the consequences of using partial security features. Most users are not informed enough about security to be able to select the correct security options. The best solution is to have no options and make the system secure by default. If you absolutely have to, provide a single option: secure or insecure.

Many systems also have multiple cipher suites, where the user (or someone else) can choose which cipher and which authentication function to use. If at all possible, eliminate this complexity. Choose a single mode that is secure enough for all possible applications. The computational difference between the various encryption modes is not that large, and cryptography is rarely the bottleneck for modern computers. Apart from getting rid of the complexity, it also gets rid of the danger that users might configure their application to use weak cipher suites. After all, if choosing an encryption and authentication mode is so difficult that the designer can't do it, it will be even more challenging for a user to make an informed decision.

8.4.2 Modularization

Even after you have eliminated a lot of options and features, the resulting system will still be quite complex. There is one main technique for making the complexity manageable: modularization. You divide the system into separate modules, and design, analyze, and implement each module separately.

You should already be familiar with modularization; in cryptography it becomes even more important to do it right. Earlier we talked about cryptographic primitives as modules. The module interface should be simple and straightforward. It should behave according to the reasonable expectations of a user of the module. Look closely at the interface of your modules. Often there are features or options that exist to solve some other module's problems. If possible, rip them out. Each module should solve its own problems. We have found that when module interfaces start to develop weird features, it is time to redesign the software because they are almost always a result of design deficiencies.

Modularization is so important because it is the only efficient way we have of dealing with complexity. If a particular option is restricted to a single module, it can be analyzed within the context of this module. However, if the option changes the external behavior of one module, it can affect other modules as well. If you have 20 modules, each with a single binary option that changes the module behavior, there are over a million possible configurations. You would have to analyze each of these configurations for security—an impossible task.

We have found that many options are created in the quest for efficiency. This is a well-known problem in software engineering. Many systems contain so-called optimizations that are useless, counterproductive, or insignificant because they do not optimize those parts of the system that form the bottleneck. We have become quite conservative about optimizations. Usually we don't bother with them. We create a careful design, and try to ensure that work can be done in large ''chunks.'' A typical example is the old IBM PC BIOS. The routine to print a character on the screen took a single character as an argument. This routine spent almost all of its time on overhead, and only a very small fraction on actually putting the character on the screen. If the interface of the routine had allowed a string as argument, then the entire string could have been printed in only slightly more time than it took to print a single character. The result of this bad design was that all DOS machines had a terribly slow display. This same principle applies to cryptographic designs. Make sure that work can be done in large enough chunks. Then only optimize those parts of your program that you can *measure* as having a significant effect on the performance.

8.4.3 Assertions

Assertions are a good tool to help improve the quality of your code.

When implementing cryptographic code, adopt an attitude of professional paranoia. Each module distrusts the other modules, and always checks parameter validity, enforces calling sequence restrictions, and refuses unsafe operations. Most of the times these are straightforward assertions. If the module specifications state that you have to initialize the object before you use it, then using an object before initialization will result in an assertion error. Assertion failures should always lead to an abort of the program with ample documentation of which assertion failed, and for what reason.

The general rule is: any time you can make a meaningful check on the internal consistency of the system, you should add an assertion. Catch as many errors as you can, both your own and those of other programmers. An error caught by an assertion will not lead to a security breach.

There are some programmers who implement assertion checking in development, but switch it off when they ship the product. This is not the security perspective. What would you think of a nuclear power station where the operators train with all the safety systems in place, but switch them off when they go to work on the real reactor? Or a parachutist who wears his emergency parachute while training on the ground, but leaves it off when he jumps out of the airplane? Why would anyone ever switch off the assertion checking on production code? That is the only place where you really need it! If an assertion fails in production code, then you have just encountered a programming error. Ignoring the error will most likely result in some kind of wrong answer, because at least one assumption the code makes is wrong. Generating wrong answers is probably the worst thing a program can do. It is much better to at least inform the user that a programming error has occurred, so he does not trust the erroneous results of the program. Our recommendation is to leave all your error checking on.

8.4.4 Buffer Overflows

Buffer overflow problems have been known for decades. Perfectly good solutions to avoid them have been available for the same amount of time. Some of the earliest higher-level programming languages, such as Algol 60, completely solved the problem by introducing mandatory array bounds checking. Even so, buffer overflows cause a huge number of security problems on the Internet. There also exist a larger number of software attacks beyond buffer overflows, such as format string attacks and integer overflow attacks.

But these are things we cannot change. We can give you advice on how to write good cryptographic code. Avoid any programming language that allows buffer overflows. Specifically: don't use C or C++. And don't ever switch off the array bounds checking of whichever language you use instead. It is such a simple rule, and will probably solve half of all your security bugs.

8.4.5 Testing

Extensive testing is always part of any good development process. Testing can help find bugs in programs, but it is useless to find security holes. Never confuse testing with security analysis. The two are complementary, but different.

There are two types of tests that should be implemented. The first is a generic set of tests developed from the module's functional specifications. Ideally, one programmer implements the module and a second programmer implements the tests. Both work from the functional specification. Any misunderstanding between the two is a clear indication that the specifications have to be clarified. The generic tests should attempt to cover the entire operational spectrum of the module. For some modules, this is simple; for others, the test program will have to simulate an entire environment. In much of our own code, the test code is about as big as the operational code, and we have not found a way of significantly improving that.

A second set of tests is developed by the programmer of the module itself. These are designed to test any implementation limits. For example, if a module uses a 4 KB buffer internally, then extra tests of the boundary conditions at the start and end of the buffer will help to catch any buffer-management errors. Sometimes it requires knowledge of the internals of a module to devise specific tests.

We frequently write test sequences that are driven by a random generator. We will discuss pseudorandom number generators (PRNGS) extensively in Chapter 9. Using a PRNG makes it very easy to run a very large number of tests. If we save the seed we used for the PRNG, we can repeat the same test sequence, which is very useful for testing and debugging. Details depend on the module in question.

Finally, we have found it useful to have some ''quick test'' code that can run every time the program starts up. In one of Niels's projects, he had to implement AES. The initialization code runs AES on a few test cases and checks the output against the known correct answers. If the AES code is ever destabilized during the further development of the application, this quick test is very likely to detect the problem.

8.5 Side-Channel Attacks

There is a whole class of attacks that we call side-channel attacks [72]. These are possible when an attacker has an additional channel of information about the system. For example, an attacker could make detailed measurements of the time it takes to encrypt a message. Depending on how the system is implemented, this timing information could allow an attacker to infer private information about the message itself or the underlying encryption key. If the cryptography is embedded in a smart card, then the attacker can measure how much current the card draws over time. Magnetic fields, RF emissions, power consumption, timing, and interference on other data channels can all be used for side-channel attacks.

Not surprisingly, side-channel attacks can be remarkably successful against systems that are not designed with these attacks in mind. Power analysis of smart cards is extremely successful [77].

It is very difficult, if not impossible, to protect against all forms of side-channel attacks, but there are some simple precautions you can take. Years ago, when Niels worked on implementing cryptographic systems in smart cards, one of the design rules was that the sequence of instructions that the CPU executed could only depend on information already available to the attacker. This stops timing attacks, and makes power analysis attacks more complicated because the sequence of instructions being executed can no longer leak any information. It is not a full solution, and modern power analysis techniques would have no problem breaking the smart cards that were fielded in those days. Still, that fix was about the best that could be done with the smart cards of the day. Resistance against side-channel attacks will always come from a combination of countermeasures—some of them in the software that implements the cryptographic system, and some of them in the actual hardware.

Preventing side-channel attacks is an arms race. You try to protect yourself against the known side channels, and then a smart person somewhere discovers a new side channel, so then you have to go back and take that one into account as well. In real life, the situation is not that bad, because most side-channel attacks are difficult to perform. Side channels are a real danger to smart cards because the card is under full control of the adversary, but only a few types of side channels are practical against most other computers. In practice, the most important side channels are timing and RF emissions. (Smart cards are particularly vulnerable to measuring the power consumption.)

8.6 Beyond this Chapter

We hope this chapter has made it clear that security does not start or stop with the cryptographic design. All aspects of the system have to do their part to achieve security.

Implementing cryptographic systems is an art in itself. The most important aspect is the quality of the code. Low-quality code is the most common cause of real-world attacks, and it is rather easy to avoid. In our experience, writing high-quality code takes about as long as writing low-quality code, if you count the time from start to finished product, rather than from start to first buggy version. Be fanatical about the quality of your code. It can be done, and it needs to be done, so go do it!

There are a number of great books for further reading. Among these are *Software Security: Building Security In* by McGraw [88], *The Security Development Lifecycle* by Howard and Lipner [62], and *The Art of Software Security Assessment: Identifying and Preventing Software Vulnerabilities* by Dowd, McDonald, and Schuh [37].

8.7 Exercises

Exercise 8.1 Describe how each of the issues in Section 8.3 applies to your personal computer's hardware and software configuration.

Exercise 8.2 Find a new product or system that manipulates transient secrets. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on issues surrounding how the system might store these secrets (Section 8.3).

Exercise 8.3 Find a new product or system that manipulates secret data. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on issues surrounding code quality (Section 8.4).

Exercise 8.4 Monitor the bugtraq mailing list for one week. Create a table listing all the different types of vulnerabilities that are announced or fixed during that week, as well as the number of such vulnerabilities for each type. What sorts of larger inferences can you draw from this table? See `http://www.schneier.com/ce.html` for additional information about the bugtraq mailing list.

Part III

Key Negotiation

In This Part

CHAPTER

9

Generating Randomness

To generate key material, we need a random number generator, or RNG. Generating good randomness is a vital part of many cryptographic operations. Generating good randomness is also very challenging.

We won't go into a detailed discussion of what randomness really is; an informal discussion suffices for our purposes. A good informal definition is that random data is unpredictable to the attacker, even if he is taking active steps to defeat our randomness.

Good random number generators are necessary for many cryptographic functions. Part II discussed the secure channel and its components. We assumed there to be a key known to both Alice and Bob. That key has to be generated somewhere. Key management systems use random number generators to choose keys. If you get the RNG wrong, you end up with a weak key. This is exactly what happened to one of the early versions of the Netscape browser [54].

The measure for randomness is called *entropy* [118]. Here's the high-level idea. If you have a 32-bit word that is completely random, it has 32 bits of entropy. If the 32-bit word takes on only four different values, and each value has a 25% chance of occurring, the word has 2 bits of entropy. Entropy does not measure how many bits are in a value, but how *uncertain* you are about the value. You can think of entropy as the average number of bits you would need to specify the value if you could use an ideal compression algorithm. Note that the entropy of a value depends on how much you know. A random 32-bit word has 32 bits of entropy. Now suppose you happen to know that the value has exactly 18 bits that are 0 and 14 bits that are 1. There are about $2^{28.8}$ values that satisfy these requirements, and the entropy is also limited to 28.8 bits. In other words, the more you know about a value, the smaller its entropy is.

It is a bit more complicated to compute the entropy for values that have a nonuniform probability distribution. The most common definition of entropy for a variable X is

$$H(X) := -\sum_{x} P(X = x) \log_2 P(X = x)$$

where $P(X = x)$ is the probability that the variable X takes on the value x. We won't use this formula, so you don't need to remember it. This definition is what most mathematicians refer to when they talk about entropy. There are a few other definitions of entropy that mathematicians use as well; which one they use depends on what they are working on. And don't confuse our entropy definition with the entropy that physicists talk about. They use the word for a concept from thermodynamics that is only tangentially related to our definition of entropy.

9.1 Real Random

In an ideal world we would use ''real random'' data. The world is not ideal, and real random data is extremely hard to find.

Typical computers have a number of sources of entropy. The exact timing of keystrokes and the exact movements of a mouse are well-known examples. There has even been research into using the random fluctuations in hard-disk access time caused by turbulence inside the enclosure [29]. All of these sources are somewhat suspect because there are situations in which the attacker can influence or perform measurements on the random source.

It is tempting to be optimistic about the amount of entropy that can be extracted from various sources. We've seen software that will generate 1 or 2 bytes of supposedly random data from the timing of a single keystroke. Cryptographers in general are far more pessimistic about the amount of entropy in a single keystroke. A good typist can keep the time between consecutive keystrokes predictable to within a dozen milliseconds. And the keyboard scan frequency limits the resolution with which keystroke timings can be measured. The data being typed is not very random either, even if you ask the user just to hit some keys to generate random data. Furthermore, there is always a risk that the attacker has additional information about the ''random'' events. A microphone can pick up the sounds of the keyboard, which helps to determine the timing of keystrokes. Be very careful in estimating how much entropy you think a particular piece of data contains. We are, after all, dealing with a very clever and active adversary.

There are many physical processes that behave randomly. For example, the laws of quantum physics force certain behavior to be perfectly random. It would be very nice if we could measure such random behavior and use it. Technically, this is certainly possible. However, the attacker has a few lines of attack on this type of solution. First of all, the attacker can try to influence the behavior of the quantum particles in question to make them behave predictably. The attacker can also try to eavesdrop on the measurements we make; if he gets a copy of our measurements, while the data might still be random, it won't have any entropy from the attacker's point of view. (If he knows the value, then it has no entropy for him.) Maybe the attacker can set up a strong RF field in an attempt to bias our detector. There are even some quantum physics–based attacks that can be contemplated. The Einstein-Podolsky-Rosen paradox could be used to subvert the randomness we are trying to measure [11, 19]. Similar comments apply to other sources of entropy, such as thermal noise of a resistor and tunneling and breakdown noise of a Zener diode.

Some modern computers have a built-in real random number generator [63]. This is a significant improvement over a separate real random generator, as it makes some of the attacks more difficult. The random number generator is still only accessible to the operating system, so an application has to trust the operating system to handle the random data in a secure manner.

9.1.1 Problems with Using Real Random Data

Aside from the difficulty of collecting real random data, there are several other problems with its practical use. First of all, it is not always available. If you have to wait for keystroke timings, then you cannot get any random data unless the user is typing. That can be a real problem when your application is a Web server on a machine with no keyboard connected to it. A related problem is that the amount of real random data is always limited. If you need a lot of random data, then you have to wait; something that is unacceptable for many applications.

A second problem is that real random sources, such as a physical random number generator, can break. Maybe the generator will become predictable in some way. Because real random generators are fairly intricate things in the very noisy environment of a computer, they are much more likely to break than the traditional parts of the computer. If you rely on the real random generator directly, then you're out of luck when it breaks. What's worse, you might not know when it breaks.

A third problem is judging how much entropy you can extract from any specific physical event. Unless you have specially designed dedicated hardware for the random generator it is extremely difficult to know how much entropy you are getting. We'll discuss this in greater detail later.

9.1.2 Pseudorandom Data

An alternative to using real random data is to use pseudorandom data. Pseudorandom data is not really random at all. It is generated from a seed by a deterministic algorithm. If you know the seed, you can predict the pseudorandom data. Traditional pseudorandom number generators, or prngs, are not secure against a clever adversary. They are designed to eliminate statistical artifacts, not to withstand an intelligent attacker. The second volume of Knuth's *The Art of Computer Programming* contains an extensive discussion of random number generators, but all generators are analyzed for statistical randomness only [75]. We have to assume that our adversary knows the algorithm that is used to generate the random data. Given some of the pseudorandom outputs, is it possible for him to predict some future (or past) random bits? For many traditional prngs the answer might be yes. For a proper cryptographic prng the answer is no.

In the context of a cryptographic system, we have more stringent requirements. Even if the attacker sees much of the random data generated by the prng, she should not be able to predict anything about the rest of the output of the prng. We call such a prng cryptographically strong. As we have no need for a traditional prng, we will only talk about cryptographically strong prngs.

Forget about the normal random function in your programming library, because it is almost certainly not a cryptographic prng. Unless the cryptographic strength is explicitly documented, you should never use a library prng.

9.1.3 Real Random Data and prngs

We only use real random data for a single thing: to seed a prng. This construction resolves some of the problems of using real random data. Once the prng is seeded, random data is always available. You can keep adding the real random data that you receive to the prng seed, thereby ensuring that it never becomes fully predictable even if the seed becomes known.

There is a theoretical argument that real random data is better than pseudorandom data from a prng. In certain cryptographic protocols you can prove that certain attacks are impossible if you use real random data. The protocol is unconditionally secure. If you use a prng, the protocol is only secure as long as the attacker cannot break the prng; the protocol is computationally secure. This distinction, however, is only of theoretical interest. All cryptographic protocols use computational assumptions for almost everything. Removing the computational assumption for one particular type of attack is an insignificant improvement, and generating real random data, which you need for the unconditional security, is so difficult that you are far more likely to reduce the system security by trying to use real random data. Any weakness in the real random generator immediately leads to a loss of security. However, if you use real random data to seed a PRNG, you can afford to be far more conservative in your assumptions about the entropy sources, which makes it much more likely that you will end up with a secure system in the end.

9.2 Attack Models for a PRNG

The task of generating pseudorandom numbers from a seed is fairly simple. The problem is how to get a random seed, and how to keep it secret in a real-world situation [71]. One of the best designs up to now that we know of is called Yarrow [69], a design we created a few years ago together with John Kelsey. Yarrow tries to prevent all the known attacks.

At any point in time the PRNG has an internal state. Requests for random data are honored by using a cryptographic algorithm to generate pseudorandom data. This algorithm also updates the internal state to ensure that the next request does not return the same random data. This process is easy; any hash function or block cipher can be used for this step.

There are various forms of attack on a PRNG. There is a straightforward attack where the attacker attempts to reconstruct the internal state from the output. This is a classical cryptographic attack, and rather easy to counter using cryptographic techniques.

Things become more difficult if the attacker is at some point able to acquire the internal state. For the purposes of this discussion, it is unimportant how that happens. Maybe there is a flaw in the implementation, or maybe the computer was just booted for the first time and has had no random seed yet, or maybe the attacker managed to read the seed file from disk. Bad things happen, and you have to be able to handle them. In a traditional PRNG, if the attacker acquires the internal state, she can follow all the outputs and all the updates of the internal state. This means that if the PRNG is ever attacked successfully, then it can never recover to a secure state.

Another problem arises if the same PRNG state is used more than once. This can happen when two or more virtual machines (VMs) are booted from the same state and read the same seed file from disk.

Recovering a PRNG whose state has been compromised is difficult, as is avoiding the re-use of the same state across VMs booted from the same instance. We will need some source of entropy from a real random number generator. To keep this discussion simple, we will assume that we have one or more sources that provide some amount of entropy (typically in small chunks that we call events) at unpredictable times.

Even if we mix the small amounts of entropy from an event into the internal state, this still leaves an avenue of attack. The attacker simply makes frequent requests for random data from the PRNG. As long as the total amount of entropy added between two such requests is limited to, say, 30 bits, the attacker can simply try all possibilities for the random inputs and recover the new internal state after the mixing. This would require about 2^{30} tries, which is quite practical to do.[1] The random data generated by the PRNG provides the necessary verification when the attacker hits upon the right solution.

The best defense against this particular attack is to pool the incoming events that contain entropy. You collect entropy until you have enough to mix into the internal state without the attacker being able to guess the pooled data. How much is enough? Well, we want the attacker to spend at least 2^{128} steps on any attack, so you want to have 128 bits of entropy. But here is the real problem: making any kind of estimate of the amount of entropy is extremely difficult, if not impossible. It depends heavily on how much the attacker knows or can know, but that information is not available to the developers during the design phase. This is Yarrow's main problem. It tries to measure the entropy of a source using an entropy estimator, and such an estimator is impossible to get right for all situations.

9.3 Fortuna

In practice you are probably best off using a cryptographic PRNG provided by a well-accepted cryptographic library. For illustrative purposes, we focus now on the design of a PRNG we call Fortuna. Fortuna is an improvement on Yarrow and is named after the Roman goddess of chance.[2] Fortuna solves the problem of having to define entropy estimators by getting rid of them. The rest of this chapter is mostly about the details of Fortuna.

There are three parts to Fortuna. The generator takes a fixed-size seed and generates arbitrary amounts of pseudorandom data. The accumulator collects and pools entropy from various sources and occasionally reseeds the generator. Finally, the seed file control ensures that the PRNG can generate random data even when the computer has just booted.

[1] We are being sloppy with our math here. In this instance we should use guessing entropy, rather than the standard Shannon entropy. For extensive details on entropy measures, see [23].

[2] We thought about calling it Tyche, after the Greek goddess of chance, but nobody would know how to pronounce it.

9.4 The Generator

The generator is the part that converts a fixed-size state to arbitrarily long outputs. We'll use an AES-like block cipher for the generator; feel free to choose AES (Rijndael), Serpent, or Twofish for this function. The internal state of the generator consists of a 256-bit block cipher key and a 128-bit counter.

The generator is basically just a block cipher in counter mode. CTR mode generates a random stream of data, which will be our output. There are a few refinements.

If a user or application asks for random data, the generator runs its algorithm and generates pseudorandom data. Now suppose an attacker manages to compromise the generator's state after the completion of the request. It would be nice if this would not compromise the previous results the generator gave. Therefore, after every request we generate an extra 256 bits of pseudorandom data and use that as the new key for the block cipher. We can then forget the old key, thereby eliminating any possibility of leaking information about old requests.

To ensure that the data we generate will be statistically random, we cannot generate too much data at one time. After all, in purely random data there can be repeated block values, but the output of counter mode never contains repeated block values. (See Section 4.8.2 for details.) There are various solutions; we could use only half of each ciphertext block, which would hide most of the statistical deviation. We could use a different building block called a *pseudorandom function*, rather than a block cipher, but there are no well-analyzed and efficient proposals that we know of. The simplest solution is to limit the number of bytes of random data in a single request, which makes the statistical deviation much harder to detect.

If we were to generate 2^{64} blocks of output from a single key, we would expect close to one collision on the block values. A few repeated requests of this size would quickly show that the output is not perfectly random; it lacks the expected block collisions. We limit the maximum size of any one request to 2^{16} blocks (that is, 2^{20} bytes). For an ideal random generator, the probability of finding a block value collision in 2^{16} output blocks is about 2^{-97}, so the complete absence of collisions would not be detectable until about 2^{97} requests had been made. The total workload for the attacker ends up being 2^{113} steps. Not quite the 2^{128} steps that we're aiming for, but reasonably close.

We know we are being lax here and accepting a (slightly) reduced security level. There seems to be no good alternative. We don't have any suitable cryptographic building blocks that give us a PRNG with a full 128-bit security level. We could use SHA-256, but that would be much slower. We've found that people will argue endlessly not to use a good cryptographic PRNG, and speed has always been one of the arguments. Slowing down the PRNG by a perceptible factor to get a few bits more security is counterproductive. Too many people will simply switch to a really bad PRNG, so the overall system security will drop.

If we had a block cipher with a 256-bit block size, then the collisions would not have been an issue at all. This particular attack is not such a great threat. Not only does the attacker have to perform 2^{113} steps, but the computer that is being attacked has to perform 2^{113} block cipher encryptions. So this attack depends on the speed of the user's computer, rather than on the speed of the attacker's computer. Most users don't add huge amounts of extra computing power just to help an attacker. We don't like these types of security arguments. They are more complicated, and if the PRNG is ever used in an unusual setting, this argument might no longer apply. Still, given the situation, our solution is the best compromise we can find.

When we rekey the block cipher at the end of each request, we do not reset the counter. This is a minor issue, but it avoids problems with short cycles. Suppose we were to reset the counter every time. If the key value ever repeats, and all requests are of a fixed size, then the next key value will also be a repeated key value. We could end up in a short cycle of key values. This is an unlikely situation, but by not resetting the counter we can avoid it entirely. As the counter is 128 bits, we will never repeat a counter value (2^{128} blocks is beyond the computational capabilities of our computers), and this automatically breaks any cycles. Furthermore, we use a counter value of 0 to indicate that the generator has not yet been keyed, and therefore cannot generate any output.

Note that the restriction that limits each request to at most 1 MB of data is not an inflexible restriction. If you need more than 1 MB of random data, just do repeated requests. In fact, the implementation could provide an interface that automatically performs such repeated requests.

The generator by itself is an extremely useful module. Implementations could make it available as part of the interface, not just as a component, of Fortuna. Take a program that performs a Monte Carlo simulation.[3] You really want the simulation to be random, but you also want to be able to repeat the exact same computation, if only for debugging and verification purposes. A good solution is to call the operating system's random generator once at the start of the program to get a random seed. This seed can be logged as part of the simulator output, and from this seed our generator can generate all the random data needed for the simulation. Knowing the original seed of the generator also allows all the computations to be verified by running the program again using the same input data and seed. And for debugging, the

[3] A Monte Carlo simulation is a simulation that is driven by random choices.

same simulation can be run again and again, and it will behave exactly the same every time, as long as the starting seed is kept constant.

We can now specify the operations of the generator in detail.

9.4.1 Initialization

This is rather simple. We set the key and the counter to zero to indicate that the generator has not been seeded yet.

```
function InitializeGenerator
output: $\mathcal{G}$    Generator state.
    Set the key K and counter C to zero.
    $(K, C) \leftarrow (0, 0)$
    Package up the state.
    $\mathcal{G} \leftarrow (K, C)$
    return $\mathcal{G}$
```

9.4.2 Reseed

The reseed operation updates the state with an arbitrary input string. At this level we do not care what this input string contains. To ensure a thorough mixing of the input with the existing key, we use a hash function.

```
function Reseed
input:  $\mathcal{G}$    Generator state; modified by this function.
        $s$    New or additional seed.
    Compute the new key using a hash function.
    $K \leftarrow \text{SHA}_d\text{-256}(K \| s)$
    Increment the counter to make it nonzero and mark the generator as seeded.
            Throughout this generator, C is a 16-byte value treated as an integer
            using the LSByte first convention.
    $C \leftarrow C + 1$
```

The counter C is used here as an integer. Later it will be used as a plaintext block. To convert between the two we use the least-significant-byte-first convention. The plaintext block is a block of 16 bytes $p_0, \ldots, p_{15}$ that corresponds to the integer value

$$\sum_{i=0}^{15} p_i 2^{8i}$$

By using this convention throughout, we can treat C both as a 16-byte string and as an integer.

9.4.3 Generate Blocks

This function generates a number of blocks of random output. This is an internal function used only by the generator. Any entity outside the PRNG should not be able to call this function.

```
function GenerateBlocks
input:  $\mathcal{G}$    Generator state; modified by this function.
        $k$    Number of blocks to generate.
output: $r$    Pseudorandom string of $16k$ bytes.
    assert $C \neq 0$
    Start with the empty string.
    $r \leftarrow \epsilon$
    Append the necessary blocks.
    for $i = 1, \ldots, k$ do
        $r \leftarrow r \parallel E(K, C)$
        $C \leftarrow C + 1$
    od
    return $r$
```

Of course, the $E(K, C)$ function is the block cipher encryption function with key K and plaintext C. The GenerateBlocks function first checks that C is not zero, as that is the indication that this generator has never been seeded. The symbol ϵ denotes the empty string. The loop starts with an empty string in r and appends each newly computed block to r to build the output value.

9.4.4 Generate Random Data

This function generates random data at the request of the user of the generator. It allows for output of up to 2^{20} bytes and ensures that the generator forgets any information about the result it generated.

```
function PseudoRandomData
input:  $\mathcal{G}$    Generator state; modified by this function.
        $n$    Number of bytes of random data to generate.
output: $r$    Pseudorandom string of $n$ bytes.
    Limit the output length to reduce the statistical deviation from perfectly random
            outputs. Also ensure that the length is not negative.
    assert $0 \leq n \leq 2^{20}$
    Compute the output.
    $r \leftarrow$ first-n-bytes(GenerateBlocks($\mathcal{G}$, $\lceil n/16 \rceil$))
    Switch to a new key to avoid later compromises of this output.
    $K \leftarrow$ GenerateBlocks($\mathcal{G}, 2$)
    return $r$
```

The output is generated by a call to GenerateBlocks, and the only change is that the result is truncated to the correct number of bytes. (The $\lceil \cdot \rceil$ operator is the round-upwards operator.) We then generate two more blocks to get a new key. Once the old K has been forgotten, there is no way to recompute the result r. As long as PseudoRandomData does not keep a copy of r, or forget to wipe the memory r was stored in, the generator has no way of leaking any data about r once the function completes. This is exactly why any future compromise of the generator cannot endanger the secrecy of earlier outputs. It does endanger the secrecy of future outputs, a problem that the accumulator will address.

The function PseudoRandomData is limited in the amount of data it can return. One can specify a wrapper around this that can return larger random strings by repeated calls to PseudoRandomData. Note that you should not increase the maximum output size per call, as that increases the statistical deviation from pure random. Doing repeated calls to PseudoRandomData is quite efficient. The only real overhead is that for every 1 MB of random data produced, you have to generate 32 extra random bytes (for the new key) and run the key schedule of the block cipher again. This overhead is insignificant for all of the block ciphers we suggest.

9.4.5 Generator Speed

The generator for Fortuna that we just described is a cryptographically strong prng in the sense that it converts a seed into an arbitrarily long pseudorandom output. It is about as fast as the underlying block cipher; on a PC-type CPU it should run in less than 20 clock cycles per generated byte for large requests. Fortuna can be used as a drop-in replacement for most prng library functions.

9.5 Accumulator

The accumulator collects real random data from various sources and uses it to reseed the generator.

9.5.1 Entropy Sources

We assume there are several sources of entropy in the environment. Each source can produce events containing entropy at any point in time. It does not matter exactly what you use as your sources, as long as there is at least one source that generates data that is unpredictable to the attacker. As you cannot know how the attacker will attack, the best bet is to turn anything that looks like unpredictable data into a random source. Keystrokes and mouse movements make reasonable sources. In addition, you should add as many timing sources as practical. You could use accurate timing of keystrokes, mouse movements and clicks, and responses from the disk drives and printers, preferably all at the same time. Again, it is not a problem if the attacker can predict or copy the data from some of the sources, as long as she cannot do it for all of them.

Implementing sources can be a lot of work. The sources typically have to be built into the various hardware drivers of the operating system. This is almost impossible to do at the user level.

We identify each source by a unique source number in the range $0 \ldots 255$. Implementors can choose whether to allocate the source numbers statically or dynamically. The data in each event is a short sequence of bytes. Sources should only include the unpredictable data in each event. For example, timing information can be represented by the two or four least significant bytes of an accurate timer. There is no point including the day, month, and year. It is safe to assume that the attacker knows those.

We will be concatenating various events from different sources. To ensure that a string constructed from such a concatenation uniquely encodes the events, we have to make sure the string is parsable. Each event is encoded as three or more bytes of data. The first byte contains the random source number. The second byte contains the number of additional bytes of data. The subsequent bytes contain whatever data the source provided.

Of course, the attacker will know the events generated by some of the sources. To model this, we assume that some of the sources are completely under the attacker's control. The attacker chooses which events these sources generate at which times. And like any other user, the attacker can ask for random data from the PRNG at any point in time.

9.5.2 Pools

To reseed the generator, we need to pool events in a pool large enough that the attacker can no longer enumerate the possible values for the events in the pool. A reseed with a ''large enough'' pool of random events destroys the information the attacker might have had about the generator state. Unfortunately, we don't know how many events to collect in a pool before using it to reseed the generator. This is the problem Yarrow tried to solve by using entropy estimators and various heuristic rules. Fortuna solves it in a much better way.

There are 32 pools: $P_0, P_1, \ldots, P_{31}$. Each pool conceptually contains a string of bytes of unbounded length. In practice, the only way that string is used is as the input to a hash function. Implementations do not need to store the unbounded string, but can compute the hash of the string incrementally as it is assembled in the pool.

Each source distributes its random events over the pools in a cyclical fashion. This ensures that the entropy from each source is distributed more or less evenly over the pools. Each random event is appended to the string in the pool in question.

We reseed the generator every time pool P_0 is long enough. Reseeds are numbered $1, 2, 3, \ldots$. Depending on the reseed number r, one or more pools are included in the reseed. Pool P_i is included if 2^i is a divisor of r. Thus, P_0 is used every reseed, P_1 every other reseed, P_2 every fourth reseed, etc. After a pool is used in a reseed, it is reset to the empty string.

This system automatically adapts to the situation. If the attacker knows very little about the random sources, she will not be able to predict P_0 at the next reseed. But the attacker might know a lot more about the random sources, or she might be (falsely) generating a lot of the events. In that case, she probably knows enough of P_0 that she can reconstruct the new generator state from the old generator state and the generator outputs. But when P_1 is used in a reseed, it contains twice as much data that is unpredictable to her; and P_2 will contain four times as much. Irrespective of how many fake random events the attacker generates, or how many of the events she knows, as long as there is at least one source of random events she can't predict, there will always be a pool that collects enough entropy to defeat her.

The speed at which the system recovers from a compromised state depends on the rate at which entropy (with respect to the attacker) flows into the pools. If we assume this is a fixed rate ρ, then after t seconds we have in total ρt bits of entropy. Each pool receives about $\rho t/32$ bits in this time period. The attacker can no longer keep track of the state if the generator is reseeded with a pool with more than 128 bits of entropy in it. There are two cases. If P_0 collects 128 bits of entropy before the next reseed operation, then we have recovered from the compromise. How fast this happens depends on how large we let P_0 grow before we reseed. The second case is when P_0 is reseeding too fast, due to random events known to (or generated by) the attacker. Let t be the time between reseeds. Then pool P_i collects $2^i \rho t/32$ bits of entropy between reseeds and is used in a reseed every $2^i t$ seconds. The recovery from the compromise happens the first time we reseed with pool P_i where $128 \leq 2^i \rho t/32 < 256$. (The upper bound derives from the fact that otherwise pool P_{i-1} would contain 128 bits of entropy between reseeds.) This inequality gives us

$$\frac{2^i \rho t}{32} < 256$$

and thus

$$2^i t < \frac{8192}{\rho}$$

In other words, the time between recovery points ($2^i t$) is bounded by the time it takes to collect 2^{13} bits of entropy ($8192/\rho$). The number 2^{13} seems a bit large, but it can be explained in the following way. We need at least $128 = 2^7$ bits to recover from a compromise. We might be unlucky if the system reseeds just before we have collected 2^7 bits in a particular pool, and then we have to use the next pool, which will collect close to 2^8 bits before the reseed. Finally, we divide our data over 32 pools, which accounts for another factor of 2^5.

This is a very good result. This solution is within a factor of 64 of an ideal solution (it needs at most 64 times as much randomness as an ideal solution would need). This is a constant factor, and it ensures that we can never do terribly badly and will always recover eventually. Furthermore, we do not need to know how much entropy our events have or how much the attacker knows. That is the real advantage Fortuna has over Yarrow. The impossible-to-construct entropy estimators are gone for good. Everything is fully automatic; if there is a good flow of random data, the PRNG will recover quickly. If there is only a trickle of random data, it takes a long time to recover.

So far we've ignored the fact that we only have 32 pools, and that maybe even pool P_{31} does not collect enough randomness between reseeds to recover from a compromise. This could happen if the attacker injected so many random events that 2^{32} reseeds would occur before the random sources that the attacker has no knowledge about have generated 2^{13} bits of entropy. This is unlikely, but to stop the attacker from even trying, we will limit the speed of the reseeds. A reseed will only be performed if the previous reseed was more than 100 ms ago. This limits the reseed rate to 10 reseeds per second, so it will take more than 13 years before P_{32} would ever have been used, had it existed. Given that the economic and technical lifetime of most computer equipment is considerably less than ten years, it seems a reasonable solution to limit ourselves to 32 pools.

9.5.3 Implementation Considerations

There are a couple of implementation considerations in the design of the accumulator.

9.5.3.1 Distribution of Events Over Pools

The incoming events have to be distributed over the pools. The simplest solution would be for the accumulator to take on that role. However, this is dangerous. There will be some kind of function call to pass an event to the accumulator. It is quite possible that the attacker could make arbitrary calls to this function, too. The attacker could make extra calls to this function every time a ''real'' event was generated, thereby influencing the pool that the next ''real'' event would go to. If the attacker manages to get all ''real'' events into pool P_0, the whole multi-pool system is ineffective, and the single-pool attacks apply. If the attacker gets all ''real'' events into P_{31}, they essentially never get used.

Our solution is to let every event generator pass the proper pool number with each event. This requires the attacker to have access to the memory of the program that generates the event if she wants to influence the pool choice. If the attacker has that much access, then the entire source is probably compromised as well.

The accumulator could check that each source routes its events to the pools in the correct order. It is a good idea for a function to check that its inputs are properly formed, so this would be a good idea in principle. But in this situation, it is not always clear what the accumulator should do if the verification fails. If the whole PRNG runs as a user process, the PRNG could throw a fatal error and exit the program. That would deprive the system of the PRNG just because a single source misbehaved. If the PRNG is part of the operating system kernel, it is much harder. Let's assume a particular driver generates random events, but the driver cannot keep track of a simple 5-bit cyclical counter. What should the accumulator do? Return an error code? Chances are that a programmer who makes such simple mistakes doesn't check the return codes. Should the accumulator halt the kernel? A bit drastic, and it crashes the whole machine because of a single faulty driver. The best idea we've come up with is to penalize the driver in CPU time. If the verification fails, the accumulator can delay the driver in question by a second or so.

This idea is not terribly useful, because the reason we let the caller determine the pool number is that we assume the attacker might make false calls to the accumulator with fake events. If this happens and the accumulator checks the pool ordering, the real event generator will be penalized for the misbehavior of the attacker. Our conclusion: the accumulator should not check the pool ordering, because there isn't anything useful the accumulator can do if it detects that something is wrong. Each random source is responsible for distributing its events in cyclical order over the pools. If a random source screws up, we might lose the entropy from that source (which we expect), but no other harm will be done.

9.5.3.2 Running Time of Event Passing

We want to limit the amount of computation necessary when an event is passed to the accumulator. Many of the events are timing events, and they are generated by real-time drivers. These drivers do not want to call an accumulator if once in a while the call takes a long time to complete.

There is a certain minimum number of computations that we will need to do. We have to append the event data to the selected pool. Of course, we are not going to store the entire pool string in memory, because the length of a pool string is potentially unbounded. Recall that popular hash functions are iterative? For each pool we will have a short buffer and compute a partial hash as soon as that buffer is full. This is the minimum amount of computation required per event.

We do not want to do the whole reseeding operation, which uses one or more pools to reseed the generator. This takes an order of magnitude more time than just adding an event to a pool. Instead, this work will be delayed until the next user asks for random data, when it will be performed before the random data is generated. This shifts some of the computational burden from the event generators to the users of random data, which is reasonable since they are also the ones who are benefiting from the PRNG service. After all, most event generators are not benefiting from the random data they help to produce.

To allow the reseed to be done just before the request for random data is processed, we must encapsulate the generator. In other words, the generator will be hidden so that it cannot be called directly. The accumulator will provide a RANDOMDATA function with the same interface as PSEUDORANDOMDATA. This avoids problems with certain users calling the generator directly and bypassing the reseeding process that we worked so hard to perfect. Of course, users can still create their own instance of the generator for their own use.

A typical hash function, like SHA-256, and hence SHA$_d$-256, processes message inputs in fixed-size blocks. If we process each block of the pool string as soon as it is complete, then each event will lead to at most a single hash block computation. However, this also has a disadvantage. Modern computers use a hierarchy of caches to keep the CPU busy. One of the effects of the caches is that it is more efficient to keep the CPU working on the same thing for a while. If you process a single hash code block, then the CPU must read the hash function code into the fastest cache before it can be run. If you process several blocks in sequence, then the first block forces the code into the fastest cache, and the subsequent blocks take advantage of this. In general, performance on modern CPUs can be significantly increased by keeping the CPU working within a small loop and not letting it switch between different pieces of code all the time.

Considering the above, one option is to increase the buffer size per pool and collect more data in each buffer before computing the hash. The advantage is a reduction in the total amount of CPU time needed. The disadvantage is that the maximum time it takes to add a new event to a pool increases. This is an implementation trade-off that we cannot resolve here. It depends too much on the details of the environment.

9.5.4 Initialization

Initialization is, as always, a simple function. So far we've only talked about the generator and the accumulator, but the functions we are about to define are part of the external interface of Fortuna. Their names reflect the fact that they operate on the whole PRNG.

```
function InitializePRNG
output: $\mathcal{R}$      PRNG state.
    Set the 32 pools to the empty string.
    for $i = 0, \ldots, 31$ do
        $P_i \leftarrow \epsilon$
    od
    Set the reseed counter to zero.
    $\textsc{ReseedCnt} \leftarrow 0$
    And initialize the generator.
    $\mathcal{G} \leftarrow \textsc{InitializeGenerator}()$
    Package up the state.
    $\mathcal{R} \leftarrow (\mathcal{G}, \textsc{ReseedCnt}, P_0, \ldots, P_{31})$
    return $\mathcal{R}$
```

9.5.5 Getting Random Data

This is not quite a simple wrapper around the generator component of the PRNG, because we have to handle the reseeds here.

```
function RandomData
input:  $\mathcal{R}$      PRNG state, modified by this function.
        $n$      Number of bytes of random data to generate.
output: $r$      Pseudorandom string of bytes.
    if $length(P_0) \geq \textsc{MinPoolSize} \wedge$ last reseed $>$ 100 ms ago then
        We need to reseed.
        $\textsc{ReseedCnt} \leftarrow \textsc{ReseedCnt} + 1$
        Append the hashes of all the pools we will use.
        $s \leftarrow \epsilon$
        for $i \in 0, \ldots, 31$ do
            if $2^i \mid \textsc{ReseedCnt}$ then
                $s \leftarrow s \parallel \text{SHA}_d\text{-}256(P_i)$
                $P_i \leftarrow \epsilon$
            fi
        od
        Got the data, now do the reseed.
        $\textsc{Reseed}(\mathcal{G}, s)$
    fi
    if ReseedCnt = 0 then
        Generate error, prng not seeded yet
    else
        Reseeds (if needed) are done. Let the generator that is part of $\mathcal{R}$ do the work.
        return PseudoRandomData($\mathcal{G}, n$)
    fi
```

This function starts by checking the size of pool P_0 against the parameter MinPoolSize to see if it should do a reseed. You can use a very optimistic estimate of how large the pool size has to be before it can contain 128 bits of entropy. Assuming that each event contains 8 bits of entropy and takes 4 bytes in the pool (this corresponds to 2 bytes of event data), a suitable value for MinPoolSize would be 64 bytes. It doesn't matter much, although choosing a value smaller than 32 seems inadvisable. Choosing a much larger value is not good, either, because that will delay the reseed even if there are very good random sources available.

The next step is to increment the reseed count. The count was initialized to 0, so the very first reseed uses the value 1. This automatically ensures that the first reseed uses only P_0, which is what we want.

The loop appends the hashes of the pools. We could also have appended the pools themselves, but then every implementation would have to store entire pool strings, not just the running hash-computation of each pool. The notation $2^i \mid$ ReseedCnt is a divisor test. It is true if 2^i is a divisor of the value ReseedCnt. Once an i value fails this test, all tests of the subsequent loop iterations will also fail, which suggests an optimization.

9.5.6 Add an Event

Random sources call this routine when they have another random event. Note that the random sources are each uniquely identified by a source number. We will not specify how to allocate the source numbers because the solution depends on the local situation.

```
function AddRandomEvent
input:  $\mathcal{R}$   prng state, modified by this function.
        $s$   Source number in range $0, \ldots, 255$.
        $i$   Pool number in range $0, \ldots, 31$. Each source must distribute its
              events over all the pools in a round-robin fashion.
        $e$   Event data. String of bytes; length in range $1, \ldots, 32$.
    Check the parameters first.
    assert $1 \leq length(e) \leq 32 \wedge 0 \leq s \leq 255 \wedge 0 \leq i \leq 31$
    Add the data to the pool.
    $P_i \leftarrow P_i \parallel s \parallel length(e) \parallel e$
```

The event is encoded in $2 + length(e)$ bytes, with both s and $length(e)$ being encoded as a single byte. This concatenation is then appended to the pool. Note that our specifications just append data to the pool, but do not mention any hash computation. We only specify the hashing of the pool at the point in time where we use it. A real implementation should compute the hashes on the fly. That is functionally equivalent and easier to implement, but specifying it directly would be far more complicated.

We have limited the length of the event data to 32 bytes. Larger events are fairly useless; random sources should not pass large amounts of data, but rather, only those few bytes that contain unpredictable random data. If a source has a large amount of data that contains some entropy spread throughout it, the source should hash the data first. The AddRandomEvent function should always return quickly. This is especially important because many sources—by their very nature—perform real-time services. These sources cannot spend too much time calling AddRandomEvent. Even if a source produces small events, it should not have to wait on other callers whose events are large. Most implementations will need to serialize the calls to AddRandomEvent by using a mutex of some sort to ensure that only one event is being added at the same time.[4]

Some random sources might not have the time to call AddRandomEvent. In this case, it might be necessary to store the events in a buffer and have a separate process pick the events from the buffer and feed them to the accumulator.

An alternative architecture allows the sources to simply pass the events to the accumulator process, and has a separate thread in the accumulator perform all the hash computations. This is a more complex design, but it does have advantages for the entropy sources. The choice depends very much on the actual situation.

9.6 Seed File Management

Our PRNG so far will collect entropy and generate random data after the first reseed. However, if we reboot a machine we have to wait for the random sources to produce enough events to trigger the first reseed before any random data is available. In addition, there is no guarantee that the state after the first reseed is, in fact, unpredictable to the attacker.

The solution is to use a seed file. The PRNG keeps a separate file full of entropy, called the seed file. This seed is not made available to anyone else. After a reboot, the PRNG reads the seed file and uses it as entropy to get into an

[4] In a multithreaded environment, you should always be very careful to ensure that different threads do not interfere with each other.

unknown state. Of course, once the seed file has been used in this manner, it needs to be rewritten with new data.

We will describe seed file management, first under the assumption that the file system supports atomic operations; later we will discuss the issues involved with implementing seed file management on real systems.

9.6.1 Write Seed File

The first thing to do is generate a seed file. This is done with a simple function.

```
function WriteSeedFile
input: $\mathcal{R}$    PRNG state, modified by this function.
       $f$    File to write to.
   $write(f, \text{RandomData}(\mathcal{R}, 64))$
```

This function simply generates 64 bytes of random data and writes it to the file. This is slightly more data than absolutely needed, but there is little reason to be parsimonious with the bytes here.

9.6.2 Update Seed File

Obviously we need to be able to read a seed file, too. For reasons explained below, we always update the seed file in the same operation.

```
function UpdateSeedFile
input: $\mathcal{R}$    PRNG state, modified by this function.
       $f$    File to be updated.
   $s \leftarrow read(f)$
   assert $length(s) = 64$
   $\text{Reseed}(\mathcal{G}, s)$
   $write(f, \text{RandomData}(\mathcal{R}, 64))$
```

This function reads the seed file, checks its length, and reseeds the generator. It then rewrites the seed file with new random data.

This routine must ensure that no other use is made of the PRNG between the reseed it causes and the writing of the new data to the seed file. Here is the problem: after a reboot, the seed file is read by this function, and the data is used in a reseed. Suppose the attacker asks for random data before the seed file has been updated. As soon as this random data is returned, but before the seed file is updated, the attacker resets the machine. At the next reboot, the same seed file data will be read and used to reseed the generator. This time, an innocent user asks for random data before the seed file has been rewritten. He will get the same random data that the attacker got earlier. This violates the secrecy of the random data. As we often use random data to generate cryptographic keys, this is a rather serious problem.

The implementation should ensure that the seed file is kept secret. Also, all updates to the seed file must be atomic (see Section 9.6.5).

9.6.3 When to Read and Write the Seed File

When the computer is rebooted, the PRNG does not have any entropy to generate random data from. This is why the seed file is there. Thus, the seed file should be read and updated after every reboot.

As the computer runs, it collects entropy from various sources. We eventually want this entropy to affect the seed file as well. One obvious solution is to rewrite the seed file just as the machine is shutting down. As some computers will never be shut down in an orderly fashion, the PRNG should also rewrite the seed file at regular intervals. We won't spell out the details here, as they are quite uninteresting and often depend on the platform. It is important to ensure that the seed file is updated regularly from the PRNG after it has collected a fair amount of entropy. A reasonable solution would be to rewrite the seed file at every shutdown and every 10 minutes or so.

9.6.4 Backups and Virtual Machines

Trying to do the reseeding correctly opens a can of worms. We cannot allow the same state of the PRNG to be repeated twice. We use the file system to store a seed file to prevent this. But most file systems are not designed to avoid repeating the same state twice, and this causes us a lot of trouble.

First of all, there are backups. If you make a backup of the entire file system and then reboot the computer, the PRNG will be reseeded from the seed file. If you later restore the entire file system from the backup and reboot the computer, the PRNG will be reseeded from the very same seed file. In other words, until the accumulator has collected enough entropy, the PRNG will produce the same output after the two reboots. This is a serious problem, as an attacker can do this to retrieve the random data that another user got from the PRNG.

There is no direct defense against this attack. If the backup system is capable of recreating the entire permanent state of the computer, there is nothing we can do to prevent the PRNG state from repeating itself. Ideally, we would fix the backup system to be PRNG-aware, but that is probably too much to ask. Hashing the seed file together with the current time would solve the problem as long as the attacker does not reset the clock to the same time. The same solution could be used if the backup system were guaranteed to keep a counter of how many restore-operations it had done. We could hash the seed file with the restore counter.

Virtual machines pose a similar problem to backups. If a VM's state is saved and then restarted twice, both instances would begin with the same PRNG state. Fortunately, some of the same solutions for backups also apply to multiple VM instances starting from the same state.

The issues with backups and virtual machines deserve further study, but because they are highly platform-dependent, we do not give a general treatment here.

9.6.5 Atomicity of File System Updates

Another important problem associated with the seed file is the atomicity of file system updates. On most operating systems, if you write a seed file, all that happens is that a few memory buffers get updated. The data is not actually written to disk until much later. Some file systems have a ''flush'' command that purports to write all data to disk. However, this can be an extremely slow operation, and we have seen cases where the hardware lied to the software and simply refused to implement the ''flush'' command properly.

Whenever we reseed from our seed file, we must update it before allowing any user to ask for random data. In other words, we must be absolutely sure that the data has been modified on the magnetic media. Things become even more complicated when you consider that many file systems treat file data and file administration information separately. So rewriting the seed file might make the file administration data temporarily inconsistent. If the power fails during that time, we could get a corrupted seed file or even lose the seed file entirely—not a good idea for a security system.

Some file systems use a journal to solve some of these problems. This is a technique originally developed for large database systems. The journal is a sequential list of all the updates that have been done to the file system. When properly used, a journal can ensure that updates are always consistent. Such a file system is always preferable from a reliability point of view. Unfortunately, some of the very common file systems only apply the journal to the administrative information, which isn't quite good enough for our goals.

As long as the hardware and operating system do not support fully atomic and permanent file updates, we cannot create a perfect seed file solution. You will need to investigate the particular platform that you work on and do the best you can to reliably update the seed file.

9.6.6 First Boot

When we start the PRNG for the very first time, there is no seed file to use for a reseed. Take, for example, a new PC that had its OS installed in the factory. The OS is now generating some administrative cryptographic keys for the installation, for which it needs the PRNG. For ease of production, all machines are identical and loaded with identical data. There is no initial seed file, so we cannot use that. We could wait for enough random events to trigger one or more reseeds, but that takes a long time, and we'd never know when we had collected enough entropy to be able to generate good cryptographic keys.

A good idea would be for the installation procedure to generate a random seed file for the PRNG during the configuration. It could, for example, use a PRNG on a separate computer to generate a new seed file for each machine. Or maybe the installation software could ask the tester to wiggle the mouse to collect some initial entropy. The choice of solution depends on the details of the environment, but somehow initial entropy has to be provided. Not providing initial entropy is not an option. The entropy accumulator can take quite a while to seed the PRNG properly, and it is quite likely that some very important cryptographic keys will be generated by the PRNG shortly after the installation of the machine.

Keep in mind that the Fortuna accumulator will seed the generator as soon as it *might* have enough entropy to be really random. Depending on how much entropy the sources actually deliver—something that Fortuna has no knowledge about—it could take quite a while before enough entropy has been gathered to properly reseed the generator. Having an outside source of randomness to create the first seed file is probably the best solution.

9.7 Choosing Random Elements

Our PRNG produces sequences of random bytes. Sometimes this is exactly what you need. In other situations you try to pick a random element from a set. This requires some care to do right.

Whenever we choose a random element, we implicitly assume that the element is chosen uniformly at random from the specified set (unless we specify another distribution). This means that each element should have exactly the same probability of being chosen.[5] This is harder than one might think.

Let n be the number of elements in the set we are choosing from. We will only discuss how to choose a random element from the set $0, 1, \ldots, n-1$. Once you can do this, you can choose elements from any set of size n.

If $n = 0$, there are no elements to choose from, so this is a simple error. If $n = 1$ you have no choice; again a simple case. If $n = 2^k$, then you just get k bits of random data from the PRNG and interpret them as a number in the range $0, \ldots, n-1$. This number is uniformly random. (You might have to get

[5] If we are designing for a 128-bit security level, we could afford a deviation from the uniform probability of 2^{-128}, but it is easier to do it perfectly.

a whole number of bytes from the PRNG and throw away a few bits of the last byte until you're left with k bits, but this is easy.)

What if n is not a power of two? Well, some programs choose a random 32-bit integer and take it modulo n. But that algorithm introduces a bias in the resulting probability distribution. Let's take $n = 5$ as an example and define $m := \lfloor 2^{32}/5 \rfloor$. If we take a uniformly random 32-bit number and reduce it modulo 5, then the results 1, 2, 3, and 4 each occur with a probability of $m/2^{32}$, while the result 0 occurs with a probability of $(m + 1)/2^{32}$. The deviation in probability is small, but could very well be significant. It would certainly be easy to detect the deviation within the 2^{128} steps we allow the attacker.

The proper way to select a random number in an arbitrary range is to use a trial-and-error approach. To generate a random value in the range $0, \ldots, 4$, we first generate a random value in the range $0, \ldots, 7$, which we can do since 8 is a power of 2. If the result is 5 or larger, we throw it away and choose a new random number in the range $0, \ldots, 7$. We keep doing this until the result is in the desired range. In other words, we generate a random number with the right number of bits in it and throw away all the improper ones.

Here is a more formal specification for how to choose a random number in the range $0, \ldots, n - 1$ for $n \geq 2$.

1. Let k be the smallest integer such that $2^k \geq n$.
2. Use the PRNG to generate a k-bit random number K. This number will be in the range $0, \ldots, 2^k - 1$. You might have to generate a whole number of bytes and throw away part of the last byte, but that's easy.
3. If $K \geq n$ go back to step 2.
4. The number K is the result.

This can be a bit of a wasteful process. In the worst case, we throw away half our attempts on average. Here is an improvement. As $2^{32} - 1$ is a multiple of 5, we could choose a random number in the range $0, \ldots, 2^{32} - 2$ and take the result modulo 5 for our answer. To choose a value in the range $0, \ldots, 2^{32} - 2$, we use the ''inefficient'' try-and-throw-away algorithm, but now the probability of having to throw the intermediate result away is very low.

The general formulation is to choose a convenient k such that $2^k \geq n$. Define $q := \lfloor 2^k/n \rfloor$. First choose a random number r in the range $0, \ldots, nq - 1$ using the try-and-throw-away rules. Once a suitable r has been generated, the final result is given by $(r \bmod n)$.

We don't know of any way to generate uniformly random numbers on sizes that are not a power of two without having to throw away some random bits now and again. That is not a problem. Given a decent PRNG, there is no shortage of random bits.

9.8 Exercises

Exercise 9.1 Investigate the random number generators built into three of your favorite programming languages. Would you use these random number generators for cryptographic purposes?

Exercise 9.2 Using an existing cryptography library, write a short program that generates a 256-bit AES key using a cryptographic PRNG.

Exercise 9.3 For your platform, language, and cryptography library of choice, summarize how the cryptographic PRNG works internally. Consider issues including but not limited to the following: how the entropy is collected, how reseeding occurs, and how the PRNG handles reboots.

Exercise 9.4 What are the advantages of using a PRNG over an RNG? What are the advantages of using an RNG over a PRNG?

Exercise 9.5 Using a cryptographic PRNG that outputs a stream of bits, implement a random number generator that outputs random integers in the set $0, 1, \ldots, n-1$ for any n between 1 and 2^{32}.

Exercise 9.6 Implement a naive approach for generating random numbers in the set $0, 1, \ldots, 191$. For this naive approach, generate a random 8-bit value, interpret that value as an integer, and reduce that value modulo 192. Experimentally generate a large number of random numbers in the set $0, 1, \ldots, 191$ and report on the distribution of results.

Exercise 9.7 Find a new product or system that uses (or should use) a cryptographic PRNG. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on the issues surrounding the use of random numbers.

CHAPTER 10

Primes

The following two chapters explain public-key cryptographic systems. This requires some mathematics to get started. It is always tempting to dispense with the understanding and only present the formulas and equations, but we feel very strongly that this is a dangerous thing to do. To use a tool, you should understand the properties of that tool. This is easy with something like a hash function. We have an ''ideal'' model of a hash function, and we desire the actual hash function to behave like the ideal model. This is not so easy to do with public-key systems because there are no ''ideal'' models to work with. In practice, you have to deal with the mathematical properties of the public-key systems, and to do that safely you must understand these properties. There is no shortcut here; you must understand the mathematics. Fortunately, the only background knowledge required is high school math.

This chapter is about prime numbers. Prime numbers play an important role in mathematics, but we are interested in them because some of the most important public-key crypto systems are based on prime numbers.

10.1 Divisibility and Primes

A number a is a divisor of b (notation $a \mid b$, pronounced ''a divides b'') if you can divide b by a without leaving a remainder. For example, 7 is a divisor of 35 so we write $7 \mid 35$. We call a number a *prime* number if it has exactly two positive divisors, namely 1 and itself. For example, 13 is a prime; the two divisors are 1 and 13. The first few primes are easy to find: $2, 3, 5, 7, 11, 13, \ldots$. Any integer greater than 1 that is not prime is called a *composite*. The number 1 is neither prime nor composite.

We will use the proper mathematical notation and terminology in the chapters ahead. This will make it much easier to read other texts on this subject. The notation might look difficult and complicated at first, but this part of mathematics is really easy.

Here is a simple lemma about divisibility:

Lemma 1 *If $a \mid b$ and $b \mid c$ then $a \mid c$.*

Proof. If $a \mid b$, then there is an integer s such that $as = b$. (After all, b is divisible by a so it must be a multiple of a.) And if $b \mid c$ then there is an integer t such that $bt = c$. But this implies that $c = bt = (as)t = a(st)$ and therefore a is a divisor of c. (To follow this argument, just verify that each of the equal signs is correct. The conclusion is that the first item c must be equal to the last item $a(st)$.) □

The lemma is a statement of fact. The proof argues why the lemma is true. The little square box signals the end of the proof. Mathematicians love to use lots of symbols.[1] This is a very simple lemma, and the proof should be easy to follow, as long as you remember what the notation $a \mid b$ means.

Prime numbers have been studied by mathematicians throughout the ages. Even today, if you want to generate all primes below one million, you should use an algorithm developed just over 2000 years ago by Eratosthenes, a friend of Archimedes. (Eratosthenes was also the first person to accurately measure the diameter of the earth. A mere 1700 years later Columbus allegedly used a much smaller—and wrong—estimate for the size of the earth when he planned to sail to India by going due west.) Euclid, another great Greek mathematician, gave a beautiful proof that showed there are an infinite number of primes. This is such a beautiful proof that we'll include it here. Reading through it will help you reacquaint yourself with the math.

Before we start with the real proof we will give a simple lemma.

Lemma 2 *Let n be a positive number greater than 1. Let d be the smallest divisor of n that is greater than 1. Then d is prime.*

Proof. First of all, we have to check that d is well defined. (If there is a number n that has no smallest divisor, then d is not properly defined and the lemma is nonsensical.) We know that n is a divisor of n, and $n > 1$, so there is at least one divisor of n that is greater than 1. Therefore, there must also be a smallest divisor greater than 1.

[1] Using symbols has advantages and disadvantages. We'll use whatever we think is most appropriate for this book.

To prove that d is prime, we use a standard mathematician's trick called *reductio ad absurdum* or *proof by contradiction*. To prove a statement X, we first assume that X is not true and show that this assumption leads to a contradiction. If assuming that X is not true leads to a contradiction, then obviously X must be true.

In our case, we will assume that d is not a prime. If d is not a prime, it has a divisor e such that $1 < e < d$. But we know from Lemma 1 that if $e \mid d$ and $d \mid n$ then $e \mid n$, so e is a divisor of n and is smaller than d. But this is a contradiction, because d was defined as the smallest divisor of n. Because a contradiction cannot be true, our assumption must be false, and therefore d must be prime. □

Don't worry if you find this type of proof a bit confusing; it takes some getting used to.

We can now prove that there are an infinite number of primes.

Theorem 3 (Euclid) *There are an infinite number of primes.*

Proof. We again assume the opposite of what we try to prove. Here we assume that the number of primes is finite, and therefore that the list of primes is finite. Let's call them $p_1, p_2, p_3, \ldots, p_k$, where k is the number of primes. We define the number $n := p_1 p_2 p_3 \cdots p_k + 1$, which is the product of all our primes plus one.

Consider the smallest divisor greater than 1 of n; we'll call it d again. Now d is prime (by Lemma 2) and $d \mid n$. But none of the primes in our finite list of primes is a divisor of n. After all, they are all divisors of $n - 1$, so if you divide n by one of the p_i's in the list, you are always left with a remainder of 1. So d is a prime and it is not in the list. But this is a contradiction, as the list is defined to contain all the primes. Thus, assuming that the number of primes is finite leads to a contradiction. We are left to conclude that the number of primes is infinite. □

This is basically the proof that Euclid gave over 2000 years ago.

There are many more results on the distribution of primes, but interestingly enough, there is no easy formula for the exact number of primes in a specific interval. Primes seem to occur fairly randomly. There are even very simple conjectures that have never been proven. For example, the Goldbach conjecture is that every even number greater than 2 is the sum of two primes. This is easy to verify with a computer for relatively small even numbers, but mathematicians still don't know whether it is true for all even numbers.

The *fundamental theorem of arithmetic* is also useful to know: any integer greater than 1 can be written in exactly one way as the product of primes (if you disregard the order in which you write the primes). For example, $15 = 3 \cdot 5$; $255 = 3 \cdot 5 \cdot 17$; and $60 = 2 \cdot 2 \cdot 3 \cdot 5$. We won't prove this here. Check any textbook on number theory if you want to know the details.

10.2 Generating Small Primes

Sometimes it is useful to have a list of small primes, so here is the Sieve of Eratosthenes, which is still the best algorithm for generating small primes. The 2^{20} in the pseudocode below is a stand-in for any appropriate small constant.

```
function SmallPrimeList
input:  n       Limit on primes to generate. Must satisfy $2 \leq n \leq 2^{20}$.
output: P       List of all primes $\leq n$.
    Limit the size of n. If n is too large we run out of memory.
    assert $2 \leq n \leq 2^{20}$
    Initialize a list of flags all set to one.
    $(b_2, b_3, \ldots, b_n) \leftarrow (1, 1, \ldots, 1)$
    $i \leftarrow 2$
    while $i^2 \leq n$ do
        We have found a prime i. Mark all multiples of i composite.
        for $j \in 2i, 3i, 4i, \ldots, \lfloor n/i \rfloor i$ do
            $b_j \leftarrow 0$
        od
        Look for the next prime in our list. It can be shown that this loop never results
                in the condition $i > n$, which would access a nonexistent $b_i$.
        repeat
            $i \leftarrow i + 1$
        until $b_i = 1$
    od
    All our primes are now marked with a one. Collect them in a list.
    $P \leftarrow [\,]$
    for $k \in 2, 3, 4, \ldots, n$ do
        if $b_k = 1$ then
            $P \leftarrow P \parallel k$
        fi
    od
    return P
```

The algorithm is based on a simple idea. Any composite number c is divisible by a prime that is smaller than c. We keep a list of flags, one for each of the numbers up to n. Each flag indicates whether the corresponding number could be prime. Initially all numbers are marked as potential primes by setting the flag to 1. We start with i being the first prime 2. Of course, none of the multiples of i can be prime so we mark $2i$, $3i$, $4i$, etc. as being composite by setting their flag to 0. We then increment i until we have another candidate prime. Now this candidate is not divisible by any smaller prime, or it would have been marked as a composite already. So the new i must be the next prime. We keep marking the composite numbers and finding the next prime until $i^2 > n$.

It is clear that no prime will ever be marked as a composite, since we only mark a number as a composite when we know a factor of it. (The loop that marks them as composite loops over $2i, 3i, \ldots$. Each of these terms has a factor i and therefore cannot be prime.)

Why can we stop when $i^2 > n$? Well, suppose a number k is composite, and let p be its smallest divisor greater than 1. We already know that p is prime (see Lemma 2). Let $q := k/p$. We now have $p \leq q$; otherwise, q would be a divisor of k smaller than p, which contradicts the definition of p. The crucial observation is that $p \leq \sqrt{k}$, because if p were larger than $\sqrt{k}$ we would have $k = p \cdot q > \sqrt{k} \cdot q \geq \sqrt{k} \cdot p > \sqrt{k} \cdot \sqrt{k} = k$. This last inequality would show that $k > k$, which is an obvious fallacy. So $p \leq \sqrt{k}$.

We have shown that any composite k is divisible by a prime $\leq \sqrt{k}$. So any composite $\leq n$ is divisible by a prime $\leq \sqrt{n}$. When $i^2 > n$ then $i > \sqrt{n}$. But we have already marked the multiples of all the primes less than i as composite in the list, so every composite $< n$ has already been marked as such. The numbers in the list that are still marked as primes are really prime.

The final part of the algorithm simply collects them in a list to be returned.

There are several optimizations you can make to this algorithm, but we have left them out to make things simpler. Properly implemented, this algorithm is very fast.

You might wonder why we need the small primes. It turns out that small primes are useful to generate large primes with, something we will get to soon.

10.3 Computations Modulo a Prime

The main reason why primes are so useful in cryptography is that you can compute modulo a prime.

Let p be a prime. When we compute modulo a prime we only use the numbers $0, 1, \ldots, p-1$. The basic rule for computations modulo a prime is to do the computations using the numbers as integers, just as you normally would, but every time you get a result r you take it modulo p. Taking a modulo is easy: just divide the result r by p, throw away the quotient, and keep the remainder as the answer. For example, if you take 25 modulo 7 you divide 25 by 7, which gives us a quotient of 3 with a remainder of 4. The remainder is the answer, so $(25 \bmod 7) = 4$. The notation $(a \bmod b)$ is used to denote an explicit modulo operation, but modulo computations are used very often, and there are several other notations in general use. Often the entire equation will be written without any modulo operations, and then (mod p) will be added at the end of the equation to remind you that the whole thing is to be taken modulo p. When the situation is clear from the context, even this is left out, and you have to remember the modulo yourself.

You don't need to write parentheses around a modulo computation. We could just as well have written $a \bmod b$, but as the modulo operator looks very much like normal text, this can be a bit confusing for people who are not used to it. To avoid confusion we tend to either put ($a \bmod b$) in parentheses or write $a \pmod b$, depending on which is clearer in the relevant context.

One word of warning: Any integer taken modulo p is always in the range $0, \ldots, p - 1$, even if the original integer is negative. Some programming languages have the (for mathematicians very irritating) property that they allow negative results from a modulo operation. If you want to take -1 modulo p, then the answer is $p - 1$. More generally: to compute ($a \bmod p$), find integers q and r such that $a = qp + r$ and $0 \leq r < p$. The value of ($a \bmod p$) is defined to be r. If you fill in $a = -1$ then you find that $q = -1$ and $r = p - 1$.

10.3.1 Addition and Subtraction

Addition modulo p is easy. Just add the two numbers, and subtract p if the result is greater than or equal to p. As both inputs are in the range $0, \ldots, p - 1$, the sum cannot exceed $2p - 1$, so you have to subtract p at most once to get the result back in the proper range.

Subtraction is similar to addition. Subtract the numbers, and add p if the result is negative.

These rules only work when the two inputs are both modulo p numbers already. If they are outside the range, you have to do a full reduction modulo p.

It takes a while to get used to modulo computations. You get equations like $5 + 3 = 1 \pmod 7$. This looks odd at first. You know that 5 plus 3 is not 1. But while $5 + 3 = 8$ is true in the integer numbers, working modulo 7 we have $8 \bmod 7 = 1$, so $5 + 3 = 1 \pmod 7$.

We use modulo arithmetic in real life quite often without realizing it. When computing the time of day, we take the hours modulo 12 (or modulo 24). A bus schedule might state that the bus leaves at 55 minutes past the hour and takes 15 minutes. To find out when the bus arrives, we compute $55 + 15 = 10 \pmod{60}$, and determine it arrives at 10 minutes past the hour. For now we will restrict ourselves to computing modulo a prime, but you can do computations modulo any number you like.

One important thing to note is that if you have a long equation like $5 + 2 + 5 + 4 + 6 \pmod 7$, you can take the modulo at any point in the computation.

For example, you could sum up $5+2+5+4+6$ to get 22, and the compute $22 \pmod 7$ to get 1. Alternately, you could compute $5+2 \pmod 7$ to get 0, then compute $0+5 \pmod 7$ to get 5, and then $5+4 \pmod 7$ to get 2, and then $2+6 \pmod 7$ to get 1.

10.3.2 Multiplication

Multiplication is, as always, more work than addition. To compute $(ab \bmod p)$ you first compute ab as an integer, and then take the result modulo p. Now ab can be as large as $(p-1)^2 = p^2 - 2p + 1$. Here you have to perform a long division to find (q, r) such that $ab = qp + r$ and $0 \leq r < p$. Throw away the q; the r is the answer.

Let's give you an example: Let $p = 5$. When we compute $3 \cdot 4 \pmod p$ the result is 2. After all, $3 \cdot 4 = 12$, and $(12 \bmod 5) = 2$. So we get $3 \cdot 4 = 2 \pmod p$.

As with addition, you can compute the modulus all at once or iteratively. For example, given a long equation $3 \cdot 4 \cdot 2 \cdot 3 \pmod p$, you can compute $3 \cdot 4 \cdot 2 \cdot 3 = 72$ and then compute $(72 \bmod 5) = 2$. Or you could compute $(3 \cdot 4 \bmod 5) = 2$, then $(2 \cdot 2 \bmod 5) = 4$, and then $(4 \cdot 3 \bmod 5) = 2$.

10.3.3 Groups and Finite Fields

Mathematicians call the set of numbers modulo a prime p a *finite field* and often refer to it as the "mod p" field, or simply "mod p." Here are some useful reminders about computations in a mod p field:

- You can always add or subtract any multiple of p from your numbers without changing the result.
- All results are always in the range $0, 1, \ldots, p-1$.
- You can think of it as doing your entire computation in the integers and only taking the modulo at the very last moment. So all the algebraic rules you learned about the integers (such as $a(b+c) = ab + ac$) still apply.

The finite field of the integers modulo p is referred to using different notations in different books. We will use the notation $\mathbb{Z}_p$ to refer to the finite field modulo p. In other texts you might see GF(p) or even $\mathbb{Z}/p\mathbb{Z}$.

We also have to introduce the concept of a *group*—another mathematical term, but a simple one. A group is simply a set of numbers together with an operation, such as addition or multiplication.[2] The numbers in $\mathbb{Z}_p$ form a group together with addition. You can add any two numbers and get a third number

[2] There are a couple of further requirements, but they are all met by the groups we will be talking about.

in the group. If you want to use multiplication in a group you cannot use the 0. (This has to do with the fact that multiplying by 0 is not very interesting, and that you cannot divide by 0.) However, the numbers $1, \ldots, p-1$ together with multiplication modulo p form a group. This group is called the *multiplicative group modulo p* and is written in various ways; we will use the notation $\mathbb{Z}_p^*$. A finite field consists of two groups: the addition group and the multiplication group. In the case of $\mathbb{Z}_p$ the finite field consists of the addition group, defined by addition modulo p, and the multiplication group $\mathbb{Z}_p^*$.

A group can contain a *subgroup*. A subgroup consists of some of the elements of the full group. If you apply the group operation to two elements of the subgroup, you again get an element of the subgroup. That sounds complicated, so here is an example. The numbers modulo 8 together with addition (modulo 8) form a group. The numbers $\{0, 2, 4, 6\}$ form a subgroup. You can add any two of these numbers modulo 8 and get another element of the subgroup. The same goes for multiplicative groups. The multiplicative subgroup modulo 7 consists of the numbers $1, \ldots, 6$, and the operation is multiplication modulo 7. The set $\{1, 6\}$ forms a subgroup, as does the set $\{1, 2, 4\}$. You can check that if you multiply any two elements from the same subgroup modulo 7, you get another element from that subgroup.

We use subgroups to speed up certain cryptographic operations. They can also be used to attack systems, which is why you need to know about them.

So far we've only talked about addition, subtraction, and multiplication modulo a prime. To fully define a multiplicative group you also need the inverse operation of multiplication: division. It turns out that you can define division on the numbers modulo p. The simple definition is that $a/b \pmod{p}$ is a number c such that $c \cdot b = a \pmod{p}$. You cannot divide by zero, but it turns out that the division $a/b \pmod{p}$ is always well defined as long as $b \neq 0$.

So how do you compute the quotient of two numbers modulo p? This is more complicated, and it will take a few pages to explain. We first have to go back more than 2000 years to Euclid again, and to his algorithm for the GCD.

10.3.4 The GCD Algorithm

Another high-school math refresher course: The *greatest common divisor* (or GCD) of two numbers a and b is the largest k such that $k \mid a$ and $k \mid b$. In other words, $\gcd(a, b)$ is the largest number that divides both a and b.

Euclid gave an algorithm for computing the GCD of two numbers that is still in use today, thousands of years later. For a detailed discussion of this algorithm, see Knuth [75].

```
function GCD
input:  $a$    Positive integer.
        $b$    Positive integer.
```

```
output: k        The greatest common divisor of $a$ and $b$.
    assert $a \geq 0 \wedge b \geq 0$
    while $a \neq 0$ do
        $(a, b) \leftarrow (b \bmod a, a)$
    od
    return $b$
```

Why would this work? The first observation is that the assignment does not change the set of common divisors of a and b. After all, (b mod a) is just $b - sa$ for some integer s. Any number k that divides both a and b will also divide both a and (b mod a). (The converse is also true.) And when $a = 0$, then b is a common divisor of a and b, and b is obviously the largest such common divisor. You can check for yourself that the loop must terminate because a and b keep getting smaller and smaller until they reach zero.

Let's compute the GCD of 21 and 30 as an example. We start with $(a, b) = (21, 30)$. In the first iteration we compute $(30 \bmod 21) = 9$, so we get $(a, b) = (9, 21)$. In the next iteration we compute $(21 \bmod 9) = 3$, so we get $(a, b) = (3, 9)$. In the final iteration we compute $(9 \bmod 3) = 0$ and get $(a, b) = (0, 3)$. The algorithm will return 3, which is indeed the greatest common divisor of 21 and 30.

The GCD has a cousin: the LCM or *least common multiple*. The LCM of a and b is the smallest number that is both a multiple of a and a multiple of b. For example, $\mathrm{lcm}(6, 8) = 24$. The GCD and LCM are tightly related by the equation

$$\mathrm{lcm}(a, b) = \frac{ab}{\gcd(a, b)}$$

which we won't prove here but just state as a fact.

10.3.5 The Extended Euclidean Algorithm

This still does not help us to compute division modulo p. For that, we need what is called the extended Euclidean algorithm. The idea is that while computing $\gcd(a, b)$ we can also find two integers u and v such that $\gcd(a, b) = ua + vb$. This will allow us to compute $a/b \pmod p$.

```
function EXTENDEDGCD
input:  a        Positive integer argument.
        b        Positive integer argument.
output: k        The greatest common divisor of $a$ and $b$.
        (u, v)   Integers such that $ua + vb = k$.
    assert $a \geq 0 \wedge b \geq 0$
    $(c, d) \leftarrow (a, b)$
    $(u_c, v_c, u_d, v_d) \leftarrow (1, 0, 0, 1)$
    while $c \neq 0$ do
```

```
        Invariant: $u_c a + v_c b = c \wedge u_d a + v_d b = d$
        $q \leftarrow \lfloor d/c \rfloor$
        $(c, d) \leftarrow (d - qc, c)$
        $(u_c, v_c, u_d, v_d) \leftarrow (u_d - qu_c, v_d - qv_c, u_c, v_c)$
    od
    return $d, (u_d, v_d)$
```

This algorithm is very much like the GCD algorithm. We introduce new variables c and d instead of using a and b because we need to refer to the original a and b in our invariant. If you only look at c and d, this is exactly the GCD algorithm. (We've rewritten the d mod c formula slightly, but this gives the same result.) We have added four variables that maintain the given invariant; for each value of c or d that we generate, we keep track of how to express that value as a linear combination of a and b. For the initialization this is easy, as c is initialized to a and d to b. When we modify c and d in the loop it is not terribly difficult to update the u and v variables.

Why bother with the extended Euclidean algorithm? Well, suppose we want to compute $1/b \bmod p$ where $1 \leq b < p$. We use the extended Euclidean algorithm to compute EXTENDEDGCD(b, p). Now, we know that the GCD of b and p is 1, because p is prime and it therefore has no other suitable divisors. But the EXTENDEDGCD function also provides two numbers u and v such that $ub + vp = \gcd(b, p) = 1$. In other words, $ub = 1 - vp$ or $ub = 1 \pmod p$. This is the same as saying that $u = 1/b \pmod p$, the inverse of b modulo p. The division a/b can now be computed by multiplying a by u, so we get $a/b = au \pmod p$, and this last formula is something that we know how to compute.

The extended Euclidean algorithm allows us to compute an inverse modulo a prime, which in turn allows us to compute a division modulo p. Together with the addition, subtraction, and multiplication modulo p, this allows us to compute all four elementary operations in the finite field modulo p.

Note that u could be negative, so it is probably a good idea to reduce u modulo p before using it as the inverse of b.

If you look carefully at the EXTENDEDGCD algorithm, you'll see that if you only want u as output, you can leave out the v_c and v_d variables, as they do not affect the computation of u. This slightly reduces the amount of work needed to compute a division modulo p.

10.3.6 Working Modulo 2

An interesting special case is computation modulo 2. After all, 2 is a prime, so we should be able to compute modulo it. If you've done any programming this might look familiar to you. The addition and multiplication tables modulo 2 are shown in Figure 10.1. Addition modulo 2 is exactly the exclusive-or (XOR) function you find in programming languages. Multiplication is just a simple AND operation. In the field modulo 2 there is only one inversion possible ($1/1 = 1$) so division is the same operation as multiplication. It shouldn't surprise you that the field $\mathbb{Z}_2$ is an important tool to analyze certain algorithms used by computers.

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

Figure 10.1: Addition and multiplication modulo 2

10.4 Large Primes

Several cryptographic primitives use very large primes, and we're talking about many hundreds of digits here. Don't worry, you won't have to compute with these primes by hand. That's what the computer is for.

To do any computations at all with numbers this large, you need a multiprecision library. You cannot use floating-point numbers, because they do not have several hundred digits of precision. You cannot use normal integers, because in most programming languages they are limited to a dozen digits or so. Few programming languages provide native support for arbitrary precision integers. Writing routines to perform computations with large integers is fascinating. For a good overview, see Knuth [75, Section 4.3]. However, implementing a multiprecision library is far more work than you might expect. Not only do you have to get the right answer, but you always strive to compute it as quickly as possible. There are quite a number of special situations you have to deal with carefully. Save your time for more important things, and download one of the many free libraries from the Internet, or use a language like Python that has built-in large integer support.

For public-key cryptography, the primes we want to generate are 2000–4000 bits long. The basic method of generating a prime that large is surprisingly simple: take a random number and check whether it is prime. There are very good algorithms to determine whether a large number is prime or not. There are also very many primes. In the neighborhood of a number n, approximately one in every $\ln n$ numbers is prime. (The natural logarithm of n, or $\ln n$ for short, is one of the standard functions on any scientific calculator. To give you an idea of how slowly the logarithm grows when applied to large inputs: the natural logarithm of 2^k is slightly less than $0.7 \cdot k$.) A number that is 2000 bits long falls between 2^{1999} and 2^{2000}. In that range, about one in every 1386 of the numbers is prime. And this includes a lot of numbers that are trivially composite, such as the even numbers.

Generating a large prime looks something like this:

```
function generateLargePrime
input:  $l$      Lower bound of range in which prime should lie.
        $u$      Upper bound of range in which prime should lie.
output: $p$      A random prime in the interval $l, \ldots, u$
    Check for a sensible range.
    assert $2 < l \leq u$
    Compute maximum number of attempts
    $r \leftarrow 100(\lfloor \log_2 u \rfloor + 1)$
    repeat
        $r \leftarrow r - 1$
        assert $r > 0$
        Choose n randomly in the right interval
        $n \in_{\mathcal{R}} l, \ldots, u$
        Continue trying until we find a prime.
    until isPrime($n$)
    return $n$
```

We use the operator $\in_{\mathcal{R}}$ to indicate a random selection from a set. Of course, this requires some output from the PRNG.

The algorithm is relatively straightforward. We first check that we get a sensible interval. The cases $l \leq 2$ and $l \geq u$ are not useful and lead to problems. Note the boundary condition: the case $l = 2$ is not allowed.[3] Next we compute how many attempts we are going to make to find a prime. There are intervals that do not contain a prime. For example, the interval $90, \ldots, 96$ is prime-free. A proper program should never hang, independent of its inputs, so we limit the number of tries and generate a failure if we exceed this number. How many times should we try? As stated before, in the neighborhood of u about one in every $0.7 \log_2 u$ numbers is prime. (The function $\log_2$ is the logarithm to the base 2. The simplest definition is that $\log_2(x) := \ln x / \ln 2$). The number $\log_2 u$ is difficult to compute but $\lfloor \log_2 u \rfloor + 1$ is much easier; it is the number of bits necessary to represent u as a binary number. So if u is an integer that is 2017 bits long, then $\lfloor \log_2 u \rfloor + 1 = 2017$. The factor 100 ensures that it is extremely unlikely that we will not find a prime. For large enough intervals, the probability of a failure due to bad luck is less than 2^{-128}, so we can ignore this risk. At the same time, this limit does ensure that the generateLargePrime function will terminate. We've been a bit sloppy in our use of an assertion to

[3] The Rabin-Miller algorithm we use below does not work well when it gets 2 as an argument. That's okay, we already know that 2 is prime so we don't have to generate it here.

generate the failure; a proper implementation would generate an error with explanations of what went wrong.

The main loop is simple. After the check that limits the number of tries, we choose a random number and check whether it is prime using the isPrime function. We will define this function shortly.

Make sure that the number n you choose is uniformly random in the range $l, \ldots, u$. Also make sure that the range is not too small if you want your prime to be a secret. If the attacker knows the interval you use, and there are fewer than 2^{128} primes in that interval, the attacker could potentially try them all.

If you wish, you can make sure the random number you generate is odd by setting the least significant bit just after you generate a candidate n. As 2 is not in your interval, this will not affect the probability distribution of primes you are choosing, and it will halve the number of attempts you have to make. But this is only safe if u is odd; otherwise, setting the least significant bit might bump n just outside the allowed range. Also, this will generate some small bias away from l if l is odd.

The isPrime function is a two-step filter. The first phase is a simple test where we try to divide n by all the small primes. This will quickly weed out the great majority of numbers that are composite and divisible by a small prime. If we find no divisors, we employ a heavyweight test called the Rabin-Miller test.

```
function isPrime
input:  $n$    Integer $\geq 3$.
output: $b$    Boolean whether $n$ is prime.
    assert $n \geq 3$
    for $p \in \{$ all primes $\leq 1000 \}$ do
        if $p$ is a divisor of $n$ then
            return $p = n$
        fi
    od
    return Rabin-Miller($n$)
```

If you are lazy and don't want to generate the small primes, you can cheat a bit. Instead of trying all the primes, you can try 2 and all odd numbers $3, 5, 7, \ldots, 999$, in that order. This sequence contains all the primes below 1000, but it also contains a lot of useless composite numbers. The order is important to ensure that a small composite number like 9 is properly detected as being composite. The bound of 1000 is arbitrary and can be chosen for optimal performance.

All that remains to explain is the mysterious Rabin-Miller test that does the hard work.

10.4.1 Primality Testing

It turns out to be remarkably easy to test whether a number is prime. At least, it is remarkably easy compared to factoring a number and finding its prime divisors. These easy tests are not perfect. They are all probabilistic. There is a certain chance they give the wrong answer. By repeatedly running the same test we can reduce the probability of error to an acceptable level.

The primality test of choice is the Rabin-Miller test. The mathematical basis for this test is well beyond the scope of this book, although the outline is fairly simple. The purpose of this test is to determine whether an odd integer n is prime. We choose a random value a less than n, called the *basis*, and check a certain property of a modulo n that always holds when n is prime. However, you can prove that when n is not a prime, this property holds for at most 25% of all possible basis values. By repeating this test for different random values of a, you build your confidence in the final result. If n is a prime, it will always test as a prime. If n is not a prime, then at least 75% of the possible values for a will show so, and the chance that n will pass multiple tests can be made as small as you want. We limit the probability of a false result to 2^{-128} to achieve our required security level.

Here is how it goes:

```
function Rabin-Miller
input:  $n$    An odd number $\geq 3$.
output: $b$    Boolean indicating whether $n$ is prime or not.
    assert $n \geq 3 \wedge n \bmod 2 = 1$
    First we compute $(s, t)$ such that $s$ is odd and $2^t s = n - 1$.
    $(s, t) \leftarrow (n - 1, 0)$
    while $s \bmod 2 = 0$ do
        $(s, t) \leftarrow (s/2, t + 1)$
    od
    We keep track of the probability of a false result in $k$. The probability is at most
            $2^{-k}$. We loop until the probability of a false result is small enough.
    $k \leftarrow 0$
    while $k < 128$ do
        Choose a random $a$ such that $2 \leq a \leq n - 1$.
        $a \in_{\mathcal{R}} 2, \ldots, n - 1$
        The expensive operation: a modular exponentiation.
        $v \leftarrow a^s \bmod n$
        When $v = 1$, the number $n$ passes the test for basis $a$.
        if $v \neq 1$ then
            The sequence $v, v^2, \ldots, v^{2^t}$ must finish on the value 1, and the last value
                    not equal to 1 must be $n - 1$ if $n$ is a prime.
            $i \leftarrow 0$
            **while** $v \neq n - 1$ **do**
                **if** $i = t - 1$ **then**
                    **return false**
                **else**
                    $(v, i) \leftarrow (v^2 \bmod n, i + 1)$
                **fi**
            **od**
        **fi**
        *When we get to this point, n has passed the primality test for the basis a. We
                have therefore reduced the probability of a false result by a factor of
                $2^2$, so we can add 2 to k.*
        $k \leftarrow k + 2$
    **od**
    **return true**
```

This algorithm only works for an odd n greater or equal to 3, so we test that first. The isPrime function should only call this function with a suitable argument, but each function is responsible for checking its own inputs and outputs. You never know how the software will be changed in future.

The basic idea behind the test is known as Fermat's little theorem.[4] For any prime n and for all $1 \leq a < n$, the relation $a^{n-1} \bmod n = 1$ holds. To fully understand the reasons for this requires more math than we will explain here. A simple test (also called the Fermat primality test) verifies this relation for a number of randomly chosen a values. Unfortunately, there are some obnoxious numbers called the Carmichael numbers. These are composite but they pass the Fermat test for (almost) all basis a.

The Rabin-Miller test is a variation of the Fermat test. First we write $n - 1$ as $2^t s$, where s is an odd number. If you want to compute a^{n-1} you can first compute a^s and then square the result t times to get $a^{s \cdot 2^t} = a^{n-1}$. Now if $a^s = 1 \pmod{n}$ then repeated squaring will not change the result so we have $a^{n-1} = 1 \pmod{n}$. If $a^s \neq 1 \pmod{n}$, then we look at the numbers $a^s, a^{s \cdot 2}, a^{s \cdot 2^2}, a^{s \cdot 2^3}, \ldots, a^{s \cdot 2^t}$ (all modulo n, of course). If n is a prime, then we know that the last number must be 1. If n is a prime, then the only numbers that satisfy $x^2 = 1 \pmod{n}$ are 1 and $n - 1$.[5] So if n is prime, then one of the numbers in the sequence must be $n - 1$, or we could never have the last number be equal to 1. This is really all the Rabin-Miller test checks. If any choice of a demonstrates that n is composite, we return immediately. If n continues to test as a prime, we repeat the test for different a values until the probability that we have generated a

[4] There are several theorems named after Fermat. Fermat's last Theorem is the most famous one, involving the equation $a^n + b^n = c^n$ and a proof too large to fit in the margin of the page.

[5] It is easy to check that $(n - 1)^2 = 1 \pmod{n}$.

wrong answer and claimed that a composite number is actually prime is less than 2^{-128}.

If you apply this test to a random number, the probability of failure of this test is much, much smaller than the bound we use. For almost all composite numbers n, almost all basis values will show that n is composite. You will find a lot of libraries that depend on this and perform the test for only 5 or 10 bases or so. This idea is fine, though we would have to investigate how many attempts are needed to reach an error level of 2^{-128} or less. But it only holds as long as you apply the isPrime test to *randomly* chosen numbers. Later on we will encounter situations where we apply the primality test to numbers that we received from someone else. These might be maliciously chosen, so the isPrime function must achieve a 2^{-128} error bound all by itself.

Doing the full 64 Rabin-Miller tests is necessary when we receive the number to be tested from someone else. It is overkill when we try to generate a prime randomly. But when generating a prime, you spend most of your time rejecting composite numbers. (Almost all composite numbers are rejected by the very first Rabin-Miller test that you do.) As you might have to try hundreds of numbers before you find a prime, doing 64 tests on the final prime is only marginally slower than doing 10 of them.

In an earlier version of this chapter, the Rabin-Miller routine had a second argument that could be used to select the maximum error probability. But it was a perfect example of a needless option, so we removed it. Always doing a good test to a 2^{-128} bound is simpler, and much less likely to be improperly used.

There is still a chance of 2^{-128} that our isPrime function will give you the wrong answer. To give you an idea of how small this chance actually is, the chance that you will be killed by a meteorite while you read this sentence is far larger. Still alive? Okay, so don't worry about it.

10.4.2 Evaluating Powers

The Rabin-Miller test spends most of its time computing $a^s \bmod n$. You cannot compute a^s first and then take it modulo n. No computer in the world has enough memory to even store a^s, much less the computing power to compute it; both a and s can be thousands of bits long. But we only need $a^s \bmod n$; we can apply the mod n to all the intermediate results, which stops the numbers from growing too large.

There are several ways of computing $a^s \bmod n$, but here is a simple description. To compute $a^s \bmod n$, use the following rules:

- If $s = 0$ the answer is 1.
- If $s > 0$ and s is even, then first compute $y := a^{s/2} \bmod n$ using these very same rules. The answer is given by $a^s \bmod n = y^2 \bmod n$.

- If $s > 0$ and s is odd, then first compute $y := a^{(s-1)/2} \bmod n$ using these very same rules. The answer is given by $a^s \bmod n = a \cdot y^2 \bmod n$.

This is a recursive formulation of the so-called binary algorithm. If you look at the operations performed, it builds up the desired exponent bit by bit from the most significant part of the exponent down to the least significant part. It is also possible to convert this from a recursive algorithm to a loop.

How many multiplications are required to compute $a^s \bmod n$? Let k be the number of bits of s; i.e., $2^{k-1} \leq s < 2^k$. Then this algorithm requires at most $2k$ multiplications modulo n. This is not too bad. If we are testing a 2000-bit number for primality, then s will also be about 2000 bits long, and we only need 4000 multiplications. That is still a lot of work, but certainly within the capabilities of most desktop computers.

Many public-key cryptographic systems make use of modular exponentiations like this. Any good multiprecision library will have an optimized routine for evaluating modular exponentiations. A special type of multiplication called Montgomery multiplication is well suited for this task. There are also ways of computing a^s using fewer multiplications [18]. Each of these tricks can save 10%–30% of the time it takes to compute a modular exponentiation, so used in combination they can be important.

Straightforward implementations of modular exponentiation are often vulnerable to timing attacks. See Section 15.3 for details and possible remedies.

10.5 Exercises

Exercise 10.1 Implement SmallPrimeList. What is the worst-case performance of SmallPrimeList? Generate a graph of the timings for your implementation and $n = 2, 4, 8, 16, \ldots, 2^{20}$.

Exercise 10.2 Compute $13635 + 16060 + 8190 + 21363 \pmod{29101}$ in two ways and verify the equivalence: by reducing modulo 29101 after each addition and by computing the entire sum first and then reducing modulo 29101.

Exercise 10.3 Compute the result of $12358 \cdot 1854 \cdot 14303 \pmod{29101}$ in two ways and verify the equivalence: by reducing modulo 29101 after each multiplication and by computing the entire product first and then reducing modulo 29101.

Exercise 10.4 Is $\{1, 3, 4\}$ a subgroup of the multiplicative group of integers modulo 7?

Exercise 10.5 Use the GCD algorithm to compute the GCD of 91261 and 117035.

Exercise 10.6 Use the ExtendedGCD algorithm to compute the inverse of 74 modulo the prime 167.

Exercise 10.7 Implement GenerateLargePrime using a language or library that supports big integers. Generate a prime within the range $l = 2^{255}$ and $u = 2^{256} - 1$.

Exercise 10.8 Give pseudocode for the exponentiation routine described in Section 10.4.2. Your pseudocode should not be recursive but should instead use a loop.

Exercise 10.9 Compute $27^{35} \pmod{569}$ using the exponentiation routine described in Section 10.4.2. How many multiplications did you have to perform?

CHAPTER

11

Diffie-Hellman

For our discussion of public-key cryptography, we're going to follow the historical path. Public-key cryptography was really started by Whitfield Diffie and Martin Hellman when they published their article ''New Directions in Cryptography'' in 1976 [33].

So far in this book, we've only talked about encryption and authentication with shared secret keys. But where do we get those shared secret keys from? If you have 10 friends you want to communicate with, you can meet them all and exchange a secret key with each of these friends for future use. But like all keys, these keys should be refreshed regularly, so at some point you will have to meet and exchange keys all over again. A total of 45 keys are needed for a group of 10 friends. But as the group gets larger, the number of keys grows quadratically. For 100 people all communicating with each other, you need 4950 keys. Specifically, in a group of N people, we would need $N(N-1)/2$ keys. This quickly becomes unmanageable.

Diffie and Hellman posed the question of whether it would be possible to do this more efficiently. Suppose you have an encryption algorithm where the encryption and decryption keys are different. You could publish your encryption key and keep your decryption key secret. Anyone could then send you an encrypted message, and only you could decrypt it. This would solve the problem of having to distribute so many different keys.

Diffie and Hellman posed the question, but they could only provide a partial answer. Their partial solution is today known as the Diffie-Hellman key exchange protocol, often shortened to DH protocol [33].

The DH protocol is a really nifty idea. It turns out that two people communicating over an insecure line can agree on a secret key in such a way that both of them end up with the same key, without divulging it to someone who is listening in on their conversation.

11.1 Groups

If you've read the last chapter, it won't surprise you that primes are involved. For the rest of this chapter, p is a large prime. Think of p as being 2000 to 4000 bits long. Most of our computations in this chapter will be modulo p—in many places we will not specify this again explicitly. The DH protocol uses $\mathbb{Z}_p^*$, the multiplicative group modulo p that we discussed in Section 10.3.3.

Choose any g in the group and consider the numbers $1, g, g^2, g^3, \ldots,$ all modulo p, of course. This is an infinite sequence of numbers, but there is only a finite set of numbers in $\mathbb{Z}_p^*$. (Remember, $\mathbb{Z}_p^*$ is the numbers $1, \ldots, p-1$ together with the operation of multiplication modulo p.) At some point, the numbers must start to repeat. Let us assume this happens at $g^i = g^j$ with $i < j$. As we can do division modulo p, we can divide each side by g^i and get $1 = g^{j-i}$. In other words, there is a number $q := j - i$ such that $g^q = 1 \pmod{p}$. We call the smallest positive value q for which $g^q = 1 \pmod{p}$ the *order* of g. (Unfortunately, there is quite a bit of terminology associated with this stuff. We feel it is better to use the standard terminology than invent our own words; this will avoid confusion when reading other books.)

If we keep on multiplying gs, we can reach the numbers $1, g, g^2, \ldots, g^{q-1}$. After that, the sequence repeats as $g^q = 1$. We say that g is a generator and that it generates the set $1, g, g^2, \ldots, g^{q-1}$. The number of elements that can be written as a power of g is exactly q, the order of g.

One property of multiplication modulo p is that there is at least one g that generates the entire group. That is, there is at least one g value for which $q = p - 1$. So instead of thinking of $\mathbb{Z}_p^*$ as the numbers $1, \ldots, p-1$, we can also think of them as $1, g, g^2, \ldots, g^{p-2}$. A g that generates the entire group is called a *primitive element* of the group.

Other values of g can generate smaller sets. Observe that if we multiply two numbers from the set generated by g, we get another power of g, and therefore another element from the set. If you go through all the math, it turns out that the set generated by g is another group. That is, you can multiply and divide in this group just as you can in the large group modulo p. These smaller groups are called subgroups (see Section 10.3.3). They will be important in various attacks.

There is one last thing to explain. For any element g, the order of g is a divisor of $p - 1$. This isn't too hard to see. Choose g to be a primitive element. Let h be any other element. As g generates the whole group, there is an x such that $h = g^x$. Now consider the elements generated by h. These are $1, h, h^2, h^3, \ldots$ which are equal to $1, g^x, g^{2x}, g^{3x}, \ldots$. (All our computations are still modulo p, of course.) The order of h is the smallest q at which $h^q = 1$, which is the same as saying it is the smallest q such that $g^{xq} = 1$. For any t, $g^t = 1$ is the same as saying $t = 0 \pmod{p - 1}$. So q is the smallest q such that $xq = 0 \pmod{p - 1}$. This happens when $q = (p - 1)/\gcd(x, p - 1)$. So q is obviously a factor of $p - 1$.

Here's a simple example. Let's choose $p = 7$. If we choose $g = 3$ then g is a primitive element because $1, g, g^2, \ldots, g^5 = 1, 3, 2, 6, 4, 5$. (Again, all computations are modulo p.) The element $h = 2$ generates the subgroup $1, h, h^2 = 1, 2, 4$ because $h^3 = 2^3 \bmod 7 = 1$. The element $h = 6$ generates the subgroup $1, 6$. These subgroups have sizes 3 and 2 respectively, which are both divisors of $p - 1$.

This also explains parts of the Fermat test we talked about in Section 10.4.1. Fermat's test is based on the fact that for any a we have $a^{p-1} = 1$. This is easy to check. Let g be a generator of $\mathbb{Z}_p^*$, and let x be such that $g^x = a$. As g is a generator of the entire group, there is always such an x. But now $a^{p-1} = g^{x(p-1)} = (g^{p-1})^x = 1^x = 1$.

11.2 Basic DH

For the original DH protocol, we first choose a large prime p, and a primitive element g that generates the whole group $\mathbb{Z}_p^*$. Both p and g are public constants in this protocol, and we assume that all parties, including the attackers, know them. The protocol is shown in Figure 11.1. This is one of the usual ways in which we write cryptographic protocols. There are two parties involved: Alice and Bob. Time progresses from the top to the bottom. First Alice chooses a random x in $\mathbb{Z}_p^*$, which is the same as choosing a random number in $1, \ldots, p - 1$. She computes $g^x \bmod p$ and sends the result to Bob. Bob in turn chooses a random y in $\mathbb{Z}_p^*$. He computes $g^y \bmod p$ and sends the result to Alice. The final result k is defined as g^{xy}. Alice can compute this by raising the g^y she got from Bob to the power x that she knows. (High-school math: $(g^y)^x = g^{xy}$.) Similarly, Bob can compute k as $(g^x)^y$. They both end up with the same value k, which they can use as a secret key.

But what about an attacker? The attacker gets to see g^x and g^y, but not x or y. The problem of computing g^{xy} given g^x and g^y is known as the Diffie-Hellman problem, or DH problem for short. As long as p and g are chosen correctly, there is no known efficient algorithm to compute this. The best method known is to first compute x from g^x, after which the attacker can compute k as $(g^y)^x$ just like Alice did. In the real numbers, computing x from g^x is called the logarithm function, which you find on any scientific calculator. In the finite field $\mathbb{Z}_p^*$, it is called a *discrete logarithm*, and in general the problem of computing x from g^x in a finite group is known as the discrete logarithm problem, or DL problem.

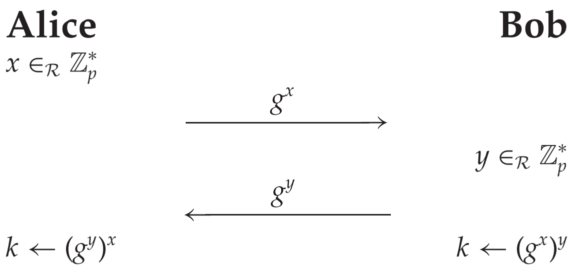
Alice
Bob
$x \in_{\mathcal{R}} \mathbb{Z}_p^*$
g^x
$y \in_{\mathcal{R}} \mathbb{Z}_p^*$
g^y
$k \leftarrow (g^y)^x$
$k \leftarrow (g^x)^y$

Figure 11.1: The original Diffie-Hellman protocol.

The original DH protocol can be used in many ways. We've written it as an exchange of messages between two parties. Another way of using it is to let everybody choose a random x, and publish $g^x \pmod p$ in the digital equivalent of a phone book. If Alice wants to communicate with Bob securely, she gets g^y from the phone book, and using her x, computes g^{xy}. Bob can similarly compute g^{xy} without any interaction with Alice. This makes the system usable in settings such as e-mail where there is no direct interaction.

11.3 Man in the Middle

The one thing that DH does not protect against is the so-called man-in-the-middle attack.[1] Look back at the protocol. Alice knows she is communicating with somebody, but she does not know with whom she is communicating. Eve can sit in the middle of the protocol and pretend to be Bob when speaking to Alice, and pretend to be Alice when speaking to Bob. This is shown in Figure 11.2. To Alice, this protocol looks just like the original DH protocol. There is no way in which Alice can detect that she is talking to Eve, not Bob. The same holds for Bob. Eve can keep up these pretenses for as long as she likes. Suppose Alice and Bob start to communicate using the secret key they think they have set up. All Eve needs to do is forward all the communications between Alice and Bob. Of course, Eve has to decrypt all the data she gets from Alice that was encrypted with key k, and then encrypt it again with key k' to send to Bob. She has to do the same with the traffic in the other direction as well, but that is not a lot of work.

With a digital phone book, this attack is harder. As long as the publisher of the book verifies the identity of everybody when they send in their g^x, Alice knows she is using Bob's g^x. We'll discuss other solutions when we talk about digital signatures and PKIs later on in this book.

[1] The terminology may look similar, but a man-in-the-middle attack is different than a meet-in-the-middle attack from Section 2.7.2.

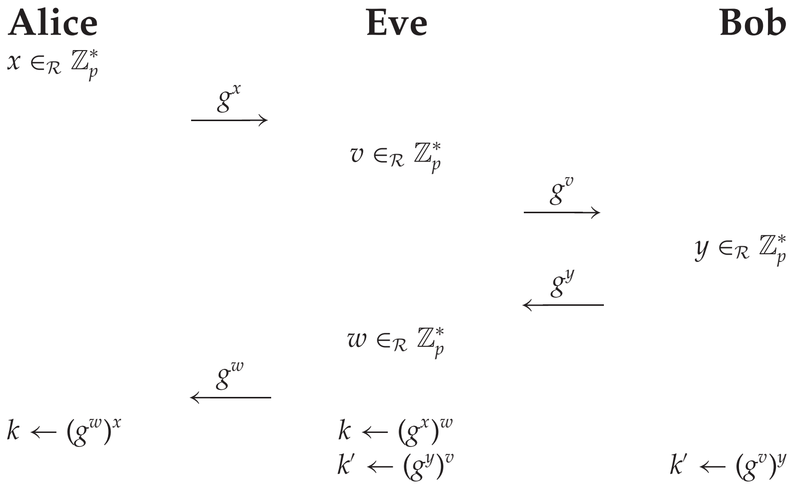
Alice
Eve
Bob
$x \in_{\mathcal{R}} \mathbb{Z}_p^*$
g^x
$v \in_{\mathcal{R}} \mathbb{Z}_p^*$
g^v
$y \in_{\mathcal{R}} \mathbb{Z}_p^*$
g^y
$w \in_{\mathcal{R}} \mathbb{Z}_p^*$
g^w
$k \leftarrow (g^w)^x$
$k \leftarrow (g^x)^w$
$k' \leftarrow (g^y)^v$
$k' \leftarrow (g^v)^y$

Figure 11.2: Diffie-Hellman protocol with Eve in the middle.

There is at least one setting where the man-in-the-middle attack can be addressed without further infrastructure. If the key k is used to encrypt a phone conversation (or a video link), Alice can talk to Bob and recognize him by his voice. Let h be a hash function of some sort. If Bob reads the first few digits of $h(k)$ to Alice, then Alice can verify that Bob is using the same key she is. Alice can read the next few digits of $h(k)$ to Bob, to allow Bob to do the same verification. This works, but only in situations where you can tie knowledge of the key k to the actual person on the other side. In most computer communications, this solution is not possible. And if Eve ever succeeds in building a speech synthesizer that can emulate Bob, it all falls apart. Finally, the biggest problem with this solution is that it requires discipline from the users, which is risky since users regularly ignore security procedures. In general, it is much better to have technical mechanisms for thwarting man-in-the-middle attacks.

11.4 Pitfalls

Implementing the DH protocol can be a bit tricky. For example, if Eve intercepts the communications and replaces both g^x and g^y with the number 1, then both Alice and Bob will end up with $k = 1$. The result is a key negotiation protocol that looks as if it completed successfully, except that Eve knows the resulting key. That is bad, and we will have to prevent this attack in some way.

A second problem is if the generator g is not a primitive element of $\mathbb{Z}_p^*$ but rather generates only a small subgroup. Maybe g has an order of one million.

In that case, the set $\{1, g, g^2, \ldots, g^{q-1}\}$ only contains a million elements. As k is in this set, Eve can easily search for the correct key. Obviously, one of the requirements is that g must have a high order. But who chooses p and g? All users are using the same values, so most of them get these values from someone else. To be safe, they have to verify that p and g are chosen properly. Alice and Bob should each check that p is prime, and that g is a primitive element modulo p.

The subgroups modulo p form a separate problem. Eve's attack of replacing g^x with the number 1 is easy to counter by having Bob check for this. But Eve could also replace g^x with the number h, where h has a small order. The key that Bob derives now comes from the small set generated by h, and Eve can try all possible values to find k. (Of course, Eve can play the same attack against Alice.) What both Alice and Bob have to do is verify that the numbers they receive do not generate small subgroups.

Let's have a look at the subgroups. Working modulo a prime, all (multiplicative) subgroups can be generated from a single element. The entire group $\mathbb{Z}_p^*$ consists of the elements $1, \ldots, p-1$ for a total of $p-1$ elements. Each subgroup is of the form $1, h, h^2, h^3, \ldots, h^{q-1}$ for some h and where q is the order of h. As we discussed earlier, it turns out that q must be a divisor of $p-1$. In other words: the size of any subgroup is a divisor of $p-1$. The converse also holds: for any divisor d of $p-1$ there is a single subgroup of size d. If we don't want any small subgroups, then we must avoid small divisors of $p-1$.

There is another reason for wanting large subgroups. It turns out that if the prime factorization of $p-1$ is known, then computing the discrete log of g^x can be broken down into a set of discrete log computations over subgroups.

This is a problem. If p is a large prime, then $p-1$ is always even, and therefore divisible by 2. Thus there is a subgroup with two elements; it consists of the elements 1 and $p-1$. But apart from this subgroup that is always present, we can avoid other small subgroups by insisting that $p-1$ have no other small factors.

11.5 Safe Primes

One solution is to use a *safe prime* for p. A safe prime is a (large enough) prime p of the form $2q+1$ where q is also prime. The multiplicative group $\mathbb{Z}_p^*$ now has the following subgroups:

- The trivial subgroup consisting only of the number 1.
- The subgroup of size 2, consisting of 1 and $p-1$.
- The subgroup of size q.
- The full group of size $2q$.

The first two are trivial to avoid. The third is the group we want to use. The full group has one remaining problem. Consider the set of all numbers modulo p that can be written as a square of some other number (modulo p, of course). It turns out that exactly half the numbers in $1, \ldots, p-1$ are squares, and the other half are non-squares. Any generator of the entire group is a non-square. (If it were a square, then raising it to some power could never generate a non-square, so it does not generate the whole group.)

There is a mathematical function called the Legendre symbol that determines whether a number modulo p is a square or not, without ever needing to find the root. There are efficient algorithms for computing the Legendre symbol. So if g is a non-square and you send out g^x, then any observer, such as Eve, can immediately determine whether x is even or odd. If x is even, then g^x is a square. If x is odd, then g^x is a non-square. As Eve can determine the squareness of a number using the Legendre symbol function, she can determine whether x is odd or even; Eve cannot learn the value x, except for the least significant bit. The solution for avoiding this problem is to use only squares modulo p. This is exactly the subgroup of order q. Another nice property is that q is prime, so there are no further subgroups we have to worry about.

Here is how to use a safe prime. Choose (p, q) such that $p = 2q + 1$ and both p and q are prime. (You can use the isPrime function to do this on a trial-and-error basis.) Choose a random number α in the range $2, \ldots, p-2$ and set $g = \alpha^2 \pmod{p}$. Check that $g \neq 1$ and $g \neq p - 1$. (If g is one of these forbidden values, choose another α and try again.) The resulting parameter set (p, q, g) is suitable for use in the Diffie-Hellman protocol.

Every time Alice (or Bob) receives a value that is supposed to be a power of g, she (or he) must check that the value received is indeed in the subgroup generated by g. When you use a safe prime as described above, you can use the Legendre symbol function to check for proper subgroup membership. There is also a simpler but slower method. A number r is a square if and only if $r^q = 1 \pmod{p}$. You also want to forbid the value 1, as its use always leads to problems. So the full test is: $r \neq 1 \wedge r^q \bmod p = 1$.

11.6 Using a Smaller Subgroup

The disadvantage of using the safe prime approach is that it is inefficient. If the prime p is n bits long, then q is $n - 1$ bits long and so all exponents are $n - 1$ bits long. The average exponentiation will take about $3n/2$ multiplications of numbers modulo p. For large primes p, this is quite a lot of work.

The standard solution is to use a smaller subgroup. Here is how that is done. We start by choosing q as a 256-bit prime. (In other words: $2^{255} < q < 2^{256}$). Next we find a (much) larger prime p such that $p = Nq + 1$ for some arbitrary value N. To do this, we choose N randomly in the suitable range, compute p as $Nq + 1$, and check whether p is prime. As p must be odd, it is easy to see that N must be even. The prime p will be thousands of bits long.

Next, we have to find an element of order q. We do that in a similar fashion to the safe prime case. Choose a random α in $\mathbb{Z}_p^*$ and set $g := \alpha^N$. Now verify that $g \neq 1$ and $g^q = 1$. (The case $g = p - 1$ is covered by the second test, as q is odd.) If g is not satisfactory, choose a different α and try again. The resulting parameter set (p, q, g) is suitable for use in the Diffie-Hellman protocol.

When we use this smaller subgroup, the values that Alice and Bob will exchange are all in the subgroup generated by g. But Eve could interfere and substitute a completely different value. Therefore, every time Alice or Bob receives a value that is supposed to be in the subgroup generated by g, he or she should check that it actually is. This check is the same as in the safe prime case. A number r is in the proper subgroup if $r \neq 1 \wedge r^q \bmod p = 1$. Of course, they should also check that r is not outside the set of modulo-p numbers, so the full check becomes $1 < r < p \wedge r^q = 1$.

For all numbers r in the subgroup generated by g we have that $r^q = 1$. So if you ever need to raise number r to a power e, you only have to compute $r^{e \bmod q}$, which can be considerably less work than computing r^e directly if e is much larger than q.

How much more efficient is the subgroup case? The large prime p is at least 2000 bits long. In the safe-prime situation, computing a general g^x takes about 3000 multiplications. In our subgroup case, g^x takes about 384 multiplies because x can be reduced modulo q and is therefore only 256 bits long. This is a savings of a factor of nearly eight. When p grows larger, the savings increase further. This is the reason that subgroups are widely used.

11.7 The Size of *p*

Choosing the right sizes for the parameters of a DH system is difficult. Up to now, we have been using the requirement that an attacker must spend 2^{128} steps to attack the system. That was an easy target for all the symmetric key primitives. Public-key operations like the DH system are far more expensive to start with, and the computational cost grows much more quickly with the desired security level.

If we keep to our requirement of forcing the attacker to use 2^{128} steps to attack the system, the prime p should be about 6800 bits long. In practical systems today, that would be a real problem from a performance point of view.

There is a big difference between key sizes for symmetric primitives and key sizes for public-key primitives like DH. Never, ever fall into the trap of comparing a symmetric key size (such as 128 or 256 bits) to the size of a public key that can be thousands of bits. Public-key sizes are always much larger than symmetric-key sizes.[2]

Public-key operations are far slower than the encryption and authentication functions we presented earlier. In most systems, the symmetric-key operations are insignificant, whereas the public-key operations can have a real effect on performance. We must therefore look much more closely at the performance aspects of public-key operations.

Symmetric-key sizes are typically fixed in a system. Once you design your system to use a particular block cipher and hash function, you also fix the key size. That means that the symmetric key size is fixed for the life of the system. Public-key sizes, on the other hand, are almost always variable. This makes it much easier to change the key size. Our intent in this book is to design a system that will be used for 30 years, and the data must be kept secure for 20 years after it has first been processed. The symmetric key size must be chosen large enough to protect the data up to 50 years from now. But the variable-sized public keys only have to protect the data for the next 20 years. After all, all keys have a limited lifetime. A public key might be valid for one year, and should protect data for 20 more years. This means that the public key only needs to protect data 21 years, rather than the 50 years needed for symmetric keys. Each year, you generate a new public key, and you can choose larger public keys as computing technology progresses.

The best estimates of how large your prime p needs to be can be found in [85]. A prime of 2048 bits can be expected to secure data until around 2022; 3072 bits is secure until 2038; and 4096 bits until 2050. The 6800 bits we mentioned above is derived from the formulas in [85]. That is the size of p if you want to force the attacker to perform 2^{128} steps in an attack.

Be very careful with these types of predictions. There is some reasonable basis for these numbers, but predicting the future is always dangerous. We might be able to make some sensible predictions about key sizes for the next 10 years, but making predictions about what things will be like 50 years from now is really rather silly. Just compare the current state of the art in computers and cryptography with the situation 50 years ago. The predictions in [85] are by far the best estimates we have; nevertheless, treat them with caution.

So what are we to do? As cryptographic designers, we have to choose a key size that will be secure for at least the next 20 years. Obviously 2048 bits is a lower bound. Larger is better, but larger keys have a significant extra cost. In the face of so much uncertainty, we would like to be conservative. So here is our advice: as of today, use 2048 bits as an absolute minimum. (And don't forget that as time passes this minimum will grow.) If at all possible from a performance point of view, use 4096 bits, or as close to 4096 bits as you can

[2] This holds for the public-key schemes we discuss in this book. Other public-key schemes, such as those based on elliptic curves, can have completely different key size parameters.

afford. Furthermore, make absolutely sure that your system can handle sizes up to 8192 bits. This will save the day if there are unexpected developments in attacking public-key systems. Improvements in cryptanalysis will most likely lead to attacks on smaller key sizes. Switching to a very much larger key size can be done while the system is in the field. It will cost some performance, but the basic operation of the system will be preserved. This is far better than losing all security and having to reengineer the system, which is what you would have to do if the system could not use larger keys.

Some applications require data to be kept secret for much longer than 20 years. In these cases, you need to use the larger keys now.

11.8 Practical Rules

Here are our practical rules for setting up a subgroup you can use for the DH protocol.

Choose q as a 256-bit prime. (There are collision-style attacks on the exponent in DH, so all our exponents should be 256 bits long to force the attacker to use at least 2^{128} operations.) Choose p as a large prime of the form $Nq + 1$ for some integer N. (See Section 11.7 for a discussion of how large p should be. Computing the corresponding range for N is trivial.) Choose a random g such that $g \neq 1$ and $g^q = 1$. (The easy way to do this is to choose a random α, set $g = \alpha^N$, and check g for suitability. Try another α if g fails the criteria.)

Any party receiving the subgroup description (p, q, g) should verify that:

- Both p and q are prime, q is 256 bits long, and p is sufficiently large. (Don't trust keys that are too small.)
- q is a divisor of $(p - 1)$.
- $g \neq 1$ and $g^q = 1$.

This should be done even if the description is provided by a trusted source. You would be amazed at how often systems fail in some interesting way, especially when they are under attack. Checking a set (p, q, g) takes a little time, but in most systems the same subgroup is used for a long time, so these checks need only be performed once.

Any time a party receives a number r that is supposed to be in the subgroup, it should be verified that $1 < r < p$ and $r^q = 1$. Note that $r = 1$ is *not* allowed.

Using these rules, we get the version of the Diffie-Hellman protocol shown in Figure 11.3. Both parties start by checking the group parameters. Each of them only has to do this once at start-up, not every time they run a DH protocol. (They should do it after every reboot or reinitialization, however, because the parameters could have changed.)

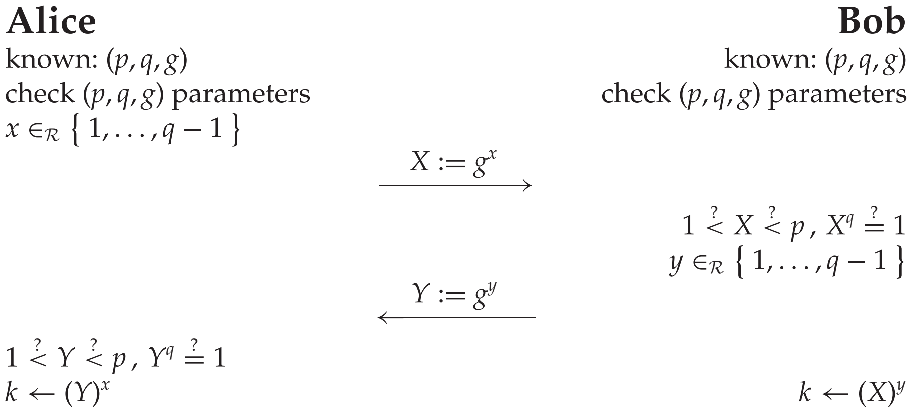
Alice
known: (p, q, g)
check (p, q, g) parameters
$x \in_{\mathcal{R}} \{1, \ldots, q-1\}$
Bob
known: (p, q, g)
check (p, q, g) parameters
$X := g^x$
$1 \overset{?}{<} X \overset{?}{<} p\,,\ X^q \overset{?}{=} 1$
$y \in_{\mathcal{R}} \{1, \ldots, q-1\}$
$Y := g^y$
$1 \overset{?}{<} Y \overset{?}{<} p\,,\ Y^q \overset{?}{=} 1$
$k \leftarrow (Y)^x$
$k \leftarrow (X)^y$

Figure 11.3: Diffie-Hellman in a subgroup.

The rest of the protocol is very much the same as the original DH protocol in Figure 11.1. Alice and Bob now use the subgroup, so the two exponents x and y are in the range $1, \ldots, q-1$. Both Alice and Bob check that the number they receive is in the proper subgroup to avoid any small-subgroup attacks by Eve.

The notation we use for the checks is a relational operator (such as $=$ or $<$) with a question mark above it. This means that Alice (or Bob) should check that the relation holds. If it does, then everything is all right. If the relation is not correct, then Alice has to assume that she is under attack. The standard behavior is to stop the execution of the protocol, not send any other messages, and destroy all protocol-specific data. For example, in this protocol Alice should destroy x and Y if the last set of checks fails. See Section 13.5.5 for a detailed discussion of how to handle these failures.

This protocol describes a secure variant of DH, but it should not be used in exactly this form. The result k has to be hashed before it is used by the rest of the system. See Section 14.6 for a more detailed discussion.

11.9 What Can Go Wrong?

Very few books or articles talk about the importance of checking that the numbers you receive are in the correct subgroup. Niels first found this problem in the Internet Key Exchange (IKE) protocol of IPsec [60]. Some of the IKE protocols include a DH exchange. As IKE has to operate in the real world, it has to deal with lost messages. So IKE specifies that if Bob receives no answer, he should resend his last message. IKE does not specify how Alice should process the message that Bob sent again. And it is easy for Alice to make a serious mistake.

For simplicity, let us suppose Alice and Bob use the DH protocol in the subgroup illustrated in Figure 11.3 without checking that X and Y are proper values. Furthermore, after this exchange Alice starts using the new key k to send an encrypted and authenticated message to Bob that contains some further protocol data. (This is a very common situation, and similar situations can occur in IKE.)

Here is the dangerous behavior by Alice: when she receives a resend of the second message containing Y, she simply recomputes the key k and sends the appropriate reply to Bob. Sounds entirely harmless, right? But the attacker Eve can now start to play games. Let d be a small divisor of $(p-1)$. Eve can replace Y by an element of order d. Alice's key k is now limited to d possible values, and is completely determined by Y and $(x \bmod d)$. Eve tries all possible values for $(x \bmod d)$, computes the key k that Alice would have gotten, and tries to decrypt the next message that Alice sends. If Eve guesses $(x \bmod d)$ correctly, this message will decrypt properly, and Eve has learned $(x \bmod d)$.

But what if $p-1$ contains a number of small factors $(d_1, d_2, \ldots, d_k)$? Then Eve can run this attack repeatedly for each of these factors and learn $(x \bmod d_1), \ldots, (x \bmod d_k)$. Using the general form of the Chinese Remainder Theorem (see Section 12.2) she can combine this knowledge to obtain $(x \bmod d_1 d_2 d_3 \cdots d_k)$. So if the product of all small divisors of $p-1$ is large, Eve can get a significant amount of information about x. As x is supposed to be secret, this is always a bad development. In this particular case, Eve can finish by forwarding the original Y to Alice and letting Alice and Bob complete the protocol. But Eve has collected enough information about x that she can now find the key k that Alice and Bob use.

To be quite clear: this is not an attack on IKE. It is an attack on an implementation of IKE that is allowed by the standard [60]. Still, in our opinion the protocol should include enough information for a competent programmer to create a secure implementation. Leaving this type of information out is dangerous, as somebody somewhere will implement it the wrong way. (We have not verified whether this attack applies to newer versions of IKE.)

For this attack to work, Eve has to be lucky enough to have a $p-1$ with sufficient small divisors. We are designing against an adversary that can perform 2^{128} steps of computing. This allows Eve to take advantage of all divisors of $p-1$ up to 2^{128} or so. We've never seen a good analysis of the probabilities of how much information Eve could get, but a quick estimate indicates that on average Eve will be able to get approximately 128 bits of information about x from the factors smaller than 2^{128}. She can then attack the unknown part of x using a collision-style attack, and as x is only 256 bits long, this leads to a real attack. At least, it would if we didn't check that X and Y were in the proper subgroup.

The attack becomes even easier if Eve was the person selecting the subgroup (p, q, g). She may have put the small divisors into $p - 1$ herself when she selected p in the first place. Or maybe she sat on the committee that recommended certain parameters for a standard. This isn't as crazy as it seems. The U.S. government, in the form of NIST, helpfully provides primes that can be used with DSA, a signature scheme that uses subgroups like this. Other parts of that same U.S. government (e.g., NSA, CIA, FBI) have a vested interest in being able to break into private communications. We certainly don't want to imply that these primes are bad, but it is something that you would want to check before you use them. This is easy to do; in fact, NIST published an algorithm for choosing parameters that does not insert additional small factors, and you can check whether the algorithm was indeed followed. But few people ever do.

In the end, the simplest solution is to check that every value you receive is in the proper subgroup. All other ways of stopping small subgroup attacks are much more complicated. You could try to detect the small factors of $p - 1$ directly, but that is way too complicated. You could require the person who generated the parameter set to provide the factorization of $p - 1$, but that adds a great deal of complexity to the whole system. Verifying that the received values are in the right subgroup is a bit of work, but it is by far the simplest and most robust solution.

11.10 Exercises

Exercise 11.1 Assume 200 people wish to communicate securely using symmetric keys, one symmetric key for each pair of people. How many symmetric keys would this system use in total?

Exercise 11.2 What are the subgroups generated by 3, 7, and 10 in the multiplicative group of integers modulo $p = 11$?

Exercise 11.3 Why is a number r a square modulo p, $p = 2q + 1$ and p and q both prime, if and only if $r^q = 1 \pmod{p}$.

Exercise 11.4 What problems, if any, could arise if Alice uses the same x and g^x for all her communications with Bob, and Bob uses the same y and g^y for all his communications with Alice?

Exercise 11.5 Alice and Bob wish to agree on a 256-bit AES key. They are trying to decide between using 256-bit, 512-bit, or some other length DH public keys g^x and g^y. What would be your recommendation to them?

CHAPTER 12

RSA

The RSA system is probably the most widely used public-key cryptosystem in the world. It is certainly the best known. It provides both digital signatures and public-key encryption, which makes it a very versatile tool, and it is based on the difficulty of factoring large numbers, a problem that has fascinated many people over the last few millennia and has been studied extensively.

12.1 Introduction

RSA is similar to, yet very different from, Diffie-Hellman (see Chapter 11). Diffie-Hellman (DH for short) is based on a one-way function: assuming p and g are publicly known, you can compute ($g^x \bmod p$) from x, but you cannot compute x given $g^x \bmod p$. RSA is based on a trapdoor one-way function. Given the publicly known information n and e, it is easy to compute $m^e \bmod n$ from m, but not the other way around. However, if you know the factorization of n, then it is easy to do the inverse computation. The factorization of n is the trapdoor information. If you know it, you can invert the function; if you do not know it, you cannot invert the function. This trapdoor functionality allows RSA to be used both for encryption and digital signatures. RSA was invented by Ronald Rivest, Adi Shamir, and Leonard Adleman, and first published in 1978 [105].

Throughout this chapter we will use the values p, q, and n. The values p and q are different large primes, each on the order of a thousand bits long or more. The value n is defined by $n := pq$. (An ordinary product, that is, not modulo something.)

12.2 The Chinese Remainder Theorem

Instead of doing computations modulo a prime p as in the DH system, we will be doing computations modulo the composite number n. To explain what is going on, we will need a little more number theory about computations modulo n. A very useful tool is the *Chinese Remainder Theorem*, or CRT. It is named so because the basic version was first stated by the first-century Chinese mathematician Sun Tzu. (Most of the math you need for DH and RSA dates back thousands of years, so it can't be too difficult, right?)

The numbers modulo n are $0, 1, \ldots, n-1$. These numbers do not form a finite field as they would if n were a prime. Mathematicians still write $\mathbb{Z}_n$ for these numbers and call this a ring, but that is a term we won't need. For each x in $\mathbb{Z}_n$, we can compute the pair $(x \bmod p, x \bmod q)$. The Chinese Remainder Theorem states that you can compute the inverse function: if you know $(x \bmod p, x \bmod q)$, you can reconstruct x.

For ease of notation, we will define $(a, b) := (x \bmod p, x \bmod q)$.

First, we show that reconstruction is possible, then we'll give an algorithm to compute the original x. To be able to compute x given (a, b), we must be sure there is not a second number x' in $\mathbb{Z}_n$ such that $x' \bmod p = a$ and $x' \bmod q = b$. If this were the case, both x' and x would result in the same (a, b) pair, and no algorithm could figure out which of these two numbers was the original input.

Let $d := x - x'$, the difference between the numbers that lead to the same (a, b) pair. We have $(d \bmod p) = (x - x') \bmod p = (x \bmod p) - (x' \bmod p) = a - a = 0$; thus, d is a multiple of p. For much the same reason, d is a multiple of q. This implies that d is a multiple of $\mathrm{lcm}(p, q)$, because lcm is, after all, the *least* common multiple. As p and q are different primes, $\mathrm{lcm}(p, q) = pq = n$, and thus $x - x'$ is a multiple of n. But both x and x' are in the range $0, \ldots, n-1$, so $x - x'$ must be a multiple of n in the range $-n+1, \ldots, n-1$. The only valid solution is $x - x' = 0$, or $x = x'$. This proves that for any given pair (a, b), there is at most one solution for x. All we have to do now is find that solution.

12.2.1 Garner's Formula

The most practical way of computing the solution is *Garner's formula*.

$$x = (((a - b)(q^{-1} \bmod p)) \bmod p) \cdot q + b$$

Here the $(q^{-1} \bmod p)$ term is a constant that depends only on p and q. Remember that we can divide modulo p, and therefore we can compute $(1/q \bmod p)$, which is just a different way of writing $(q^{-1} \bmod p)$.

We don't need to understand Garner's formula. All we need to do is prove that the result x is correct.

First of all, we show that x is in the right range $0, \ldots, n-1$. Obviously $x \geq 0$. The part $t := (((a - b)(q^{-1} \bmod p)) \bmod p)$ must be in the range

$0, \ldots, p-1$ because it is a modulo p result. If $t \leq p-1$, then $tq \leq (p-1)q$ and $x = tq + b \leq (p-1)q + (q-1) = pq - 1 = n - 1$. This shows that x is in the range $0, \ldots, n-1$.

The result should also be correct modulo both p and q.

$$\begin{aligned} x \bmod q &= ((((a-b)(q^{-1} \bmod p)) \bmod p) \cdot q + b) \bmod q \\ &= (K \cdot q + b) \bmod q && \text{for some } K \\ &= b \bmod q \\ &= b \end{aligned}$$

The whole thing in front of the multiplication by q is some integer K, but any multiple of q is irrelevant when computing modulo q. Modulo p is a bit more complicated:

$$\begin{aligned} x \bmod p &= ((((a-b)(q^{-1} \bmod p)) \bmod p) \cdot q + b) \bmod p \\ &= (((a-b)q^{-1}) \cdot q + b) \bmod p \\ &= ((a-b)(q^{-1}q) + b) \bmod p \\ &= ((a-b) + b) \bmod p \\ &= a \bmod p \\ &= a \end{aligned}$$

In the first line, we simply expand ($x \bmod p$). In the next line, we eliminate a couple of redundant $\bmod p$ operators. We then change the order of the multiplications, which does not change the result. (You might remember from school that multiplication is associative, so $(ab)c = a(bc)$.) The next step is to observe that $q^{-1}q = 1 \pmod p$, so we can remove this term altogether. The rest is trivial.

This derivation is a bit more complicated than the ones we have seen so far, especially as we use more of the algebraic properties. Don't worry if you can't follow it.

We can conclude that Garner's formula gives a result x that is in the right range and for which $(a, b) = (x \bmod p, x \bmod q)$. As we already know that there can only be one such solution, Garner's formula solves the CRT problem completely.

In real systems, you typically precompute the value $q^{-1} \bmod p$, so Garner's formula requires one subtraction modulo p, one multiplication modulo p, one full multiplication, and an addition.

12.2.2 Generalizations

The CRT also works when n is the product of multiple primes that are all different.[1] Garner's formula can be generalized to these situations, but we won't need that in this book.

[1] There are versions that work when n is divisible by the square or higher power of some primes, but those are even more complicated.

12.2.3 Uses

So what is the CRT good for? If you ever have to do a lot of computations modulo n, then using the CRT saves a lot of time. For a number $0 \leq x < n$, we call the pair $(x \bmod p, x \bmod q)$ the CRT representation of x. If we have x and y in CRT representation, then the CRT representation of $x + y$ is $((x + y) \bmod p, (x + y) \bmod q)$, which is easy to compute from the CRT representations of x and y. The first component $(x + y) \bmod p$ can be computed as $((x \bmod p) + (y \bmod p) \bmod p)$. This is just the sum (modulo p) of the first half of each of the CRT representations. The second component of the result can be computed in a similar manner.

You can compute a multiplication in much the same way. The CRT representation of xy is $(xy \bmod p, xy \bmod q)$, which is easy to compute from the CRT representations. The first part $(xy \bmod p)$ is computed by multiplying $(x \bmod p)$ and $(y \bmod p)$ and taking the result modulo p again. The second part is computed in the same manner modulo q.

Let k be the number of bits of n. Each of the primes p and q is about $k/2$ bits long. One addition modulo n would require one k-bit addition, perhaps followed by a k-bit subtraction if the result exceeded n. In the CRT representation, you have to do two modulo additions on numbers half the size. This is approximately the same amount of work.

For multiplication, the CRT saves a lot of time. Multiplying two k-bit numbers requires far more work than twice multiplying two $k/2$-bit numbers. For most implementations, CRT multiplication is twice as fast as a full multiplication. That is a significant savings.

For exponentiations, the CRT saves even more. Suppose you have to compute $x^s \bmod n$. The exponent s can be up to k bits long. This requires about $3k/2$ multiplications modulo n. Using the CRT representation, each multiplication is less work, but there is also a second savings. We want to compute $(x^s \bmod p, x^s \bmod q)$. When computing modulo p, we can reduce the exponent s modulo $(p - 1)$, and similarly modulo q. So we only have to compute $(x^{s \bmod (p-1)} \bmod p, x^{s \bmod (q-1)} \bmod q)$. Each of the exponents is only $k/2$ bits long and requires only $3k/4$ multiplications. Instead of $3k/2$ multiplications modulo n, we now do $2 \cdot 3k/4 = 3k/2$ multiplications modulo one of the primes. This saves a factor of 3–4 in computing time in a typical implementation.

The only costs of using the CRT are the additional software complexity and the necessary conversions. If you do more than a few multiplications in one computation, the overhead of these conversions is worthwhile. Most textbooks only talk about the CRT as an implementation technique for RSA. We find that the CRT representation makes it much easier to understand the RSA system. This is why we explained the CRT first. We'll soon use it to explain the behavior of the RSA system.

12.2.4 Conclusion

In conclusion: a number x modulo n can be represented as a pair (x mod p, x mod q) when $n = pq$. Conversion between the two representations is fairly straightforward. The CRT representation is useful if you have to do many multiplications modulo a composite number that you know the factorization of. (You cannot use it to speed up your computations if you don't know the factorization of n.)

12.3 Multiplication Modulo *n*

Before we delve into the details of RSA, we must look at how numbers modulo n behave under multiplication. This is somewhat different from the modulo p case we discussed before.

For any prime p, we know that for all $0 < x < p$ the equation $x^{p-1} = 1 \pmod{p}$ holds. This is not true modulo a composite number n. For RSA to work, we need to find an exponent t such that $x^t = 1 \bmod n$ for (almost) all x. Most textbooks just give the answer, which does not help you understand why the answer is true. It is actually relatively easy to find the correct answer by using the CRT.

We want a t such that, for almost all x, $x^t = 1 \pmod{n}$. This last equation implies that $x^t = 1 \pmod{p}$ and $x^t = 1 \pmod{q}$. As both p and q are prime, this only holds if $p - 1$ is a divisor of t, and $q - 1$ is a divisor of t. The smallest t that has this property is therefore $\mathrm{lcm}(p-1, q-1) = (p-1)(q-1)/\gcd(p-1, q-1)$. For the rest of this chapter, we will use the convention that $t = \mathrm{lcm}(p-1, q-1)$.

The letters p, q, and n are used by everybody, although some use capital letters. Most books don't use our t, but instead use the Euler totient function $\phi(n)$. For an n of the form $n = pq$, the Euler totient function can be computed as $\phi(n) = (p-1)(q-1)$, which is a multiple of our t. It is certainly true that $x^{\phi(n)} = 1$, and that using $\phi(n)$ instead of t gives correct answers, but using t is more precise.

We've skipped over one small issue in our discussion: x^t mod p cannot be equal to 1 if $x \bmod p = 0$. So the equation $x^t \bmod n = 1$ cannot hold for *all* values x. There are not many numbers that suffer from this deficiency; there are q numbers with $x \bmod p = 0$ and p numbers with $x \bmod q = 0$, so the total number of values that have this problem is $p + q$. Or $p + q - 1$, to be more precise, because we counted the value 0 twice. This is an insignificant fraction of the total number of values $n = pq$. Even better, the actual property used by RSA is that $x^{t+1} = x \pmod{n}$, and this still holds even for these special numbers. Again, this is easy to see when using the CRT representation. If $x = 0$

(mod p), then $x^{t+1} = 0 = x \pmod{p}$, and similarly modulo q. The fundamental property $x^{t+1} = x \pmod{n}$ is preserved, and holds for all numbers in $\mathbb{Z}_n$.

12.4 RSA Defined

We can now define the RSA system. Start by randomly choosing two different large primes p and q, and compute $n = pq$. The primes p and q should be of (almost) equal size, and the modulus n ends up being twice as long as p and q are.

We use two different exponents, traditionally called e and d. The requirement for e and d is that $ed = 1 \pmod{t}$ where $t := \mathrm{lcm}(p-1, q-1)$ as before. Recall that many texts write $ed = 1 \pmod{\phi(n)}$. We choose the public exponent e to be some small odd value and use the EXTENDEDGCD function from Section 10.3.5 to compute d as the inverse of e modulo t. This ensures that $ed = 1 \pmod{t}$.

To encrypt a message m, the sender computes the ciphertext $c := m^e \pmod{n}$. To decrypt a ciphertext c, the receiver computes $c^d \pmod{n}$. This is equal to $(m^e)^d = m^{ed} = m^{kt+1} = (m^t)^k \cdot m = (1)^k \cdot m = m \pmod{n}$, where k is some value that exists. So the receiver can decrypt the ciphertext m^e to get the plaintext m.

The pair (n, e) forms the public key. These are typically distributed to many different parties. The values (p, q, t, d) are the private key and are kept secret by the person who generated the RSA key.

For convenience, we often write $c^{1/e} \bmod n$ instead of $c^d \bmod n$. The exponents of a modulo n computation are all taken modulo t, because $x^t = 1 \pmod{n}$, so multiples of t in the exponent do not affect the result. And we computed d as the inverse of e modulo t, so writing d as $1/e$ is natural. The notation $c^{1/e}$ is often easier to follow, especially when multiple RSA keys are in use. That is why we also talk about taking the e'th root of a number. Just remember that computations of any roots modulo n require knowledge of the private key.

12.4.1 Digital Signatures with RSA

So far, we've only talked about encrypting messages with RSA. One of the great advantages of RSA is that it can be used for both encrypting messages and signing messages. These two operations use the same computations. To sign a message m, the owner of the private key computes $s := m^{1/e} \bmod n$. The pair (m, s) is now a signed message. To verify the signature, anyone who knows the public key can verify that $s^e = m \pmod{n}$.

As with encryption, the security of the signature is based on the fact that the e'th root of m can only be computed by someone who knows the private key.

12.4.2 Public Exponents

The procedure described so far has one problem. If e has a common factor with $t = \text{lcm}(p-1, q-1)$, there is no solution for d. So we have to choose p, q, and e such that this situation does not occur. This is more of a nuisance than a problem, but it has to be dealt with.

Choosing a short public exponent makes RSA more efficient, as fewer computations are needed to raise a number to the power e. We therefore try to choose a small value for e. In this book, we will choose a fixed value for e, and choose p and q to satisfy the conditions above.

You have to be careful that the encryption functions and digital signature functions don't interact in undesirable ways. You don't want it to be possible for an attacker to decrypt a message c by convincing the owner of the private key to sign c. After all, signing the ''message'' c is the same operation as decrypting the ciphertext c. The encoding functions presented later in this book will prevent this. These encodings are remotely akin to block cipher modes of operation; you should not use the basic RSA operation directly. But even then, we still don't want to use the same RSA operation for both functions. We could use different RSA keys for encryption and authentication, but that would increase complexity and double the amount of key material.

Another approach, which we use here, is to use two different public exponents on the same n. We will use $e = 3$ for signatures and $e = 5$ for encryption. This decouples the systems because cube roots and fifth roots modulo n are independent of each other. Knowing one does not help the attacker to compute the other [46].

Choosing fixed values for e simplifies the system and also gives predictable performance. It does impose a restriction on the primes that you can use, as both $p - 1$ and $q - 1$ cannot be multiples of 3 or 5. It is easy to check for this when you generate the primes in the first place.

The rationale for using 3 and 5 is simple. These are the smallest suitable values.[2] We choose the smaller public exponent for signatures, because signatures are often verified multiple times, whereas any piece of data is only encrypted once. It therefore makes more sense to let the signature verification be the more efficient operation.

Other common values used for e are 17 and 65537. We prefer the smaller values, as they are more efficient. There are some minor potential problems with the small public exponents, but we will eliminate them with our encoding functions further on.

It would also be nice to have a small value for d, but we have to disappoint you here. Although it is possible to find a pair (e, d) with a small d, using a

[2] You could in principle use $e = 2$, but that would introduce a lot of extra complexities.

small d is insecure [127]. So don't play any games by choosing a convenient value for d.

12.4.3 The Private Key

It is extremely difficult for the attacker to find any of the values of the private key p, q, t, or d if she knows only the public key (n, e). As long as n is large enough, there is no known algorithm that will do this in an acceptable time. The best solution we know of is to factor n into p and q, and then compute t and d from that. This is why you often hear about factoring being so important for cryptography.

We've been talking about the private key consisting of the values p, q, t, and d. It turns out that knowledge of any one of these values is sufficient to compute all the other three. This is quite instructive to see.

We assume that the attacker knows the public key (n, e), as that is typically public information. If she knows p or q, things are easy. Given p she can compute $q = n/p$, and then she can compute t and d just as we did above.

What if the attacker knows (n, e, t)? First of all, $t = (p-1)(q-1)/\gcd(p-1, q-1)$, but as $(p-1)(q-1)$ is very close to n, it is easy to find $\gcd(p-1, q-1)$ as it is the closest integer to n/t. (The value $\gcd(p-1, q-1)$ is never very large because it is very unlikely that two random numbers share a large factor.) This allows the attacker to compute $(p-1)(q-1)$. She can also compute $n - (p-1)(q-1) + 1 = pq - (pq - p - q + 1) + 1 = p + q$. So now she has both $n = pq$ and $s := p + q$. She can now derive the following equations:

$$\begin{aligned} s &= p + q \\ s &= p + n/p \\ ps &= p^2 + n \\ 0 &= p^2 - ps + n \end{aligned}$$

The last is just a quadratic equation in p that she can solve with high-school math. Of course, once the attacker has p, she can compute all the other private key values as well.

Something similar happens if the attacker knows d. In all our systems, e will be very small. As $d < t$, the number $ed - 1$ is only a small factor times t. The attacker can just guess this factor, compute t, and then try to find p and q as above. If she fails, she just tries the other possibilities. (There are faster techniques, but this one is easy to understand.)

In short, knowing any one of the values p, q, t, or d lets the attacker compute all the others. It is therefore reasonable to assume that the owner of the private key has all four values. Implementations only need to store one of these values, but often store several of the values they need to perform the RSA decryption operation. This is implementation dependent, and is not relevant from a cryptographic point of view.

If Alice wants to decrypt or sign a message, she obviously must know d. As knowing d is equivalent to knowing p and q, we can safely assume that she knows the factors of n and can therefore use the CRT representation for her computations. This is nice, because raising a number to the power d is the most expensive operation in RSA, and using the CRT representation saves a factor of 3–4 work.

12.4.4 The Size of *n*

The modulus n should be the same size as the modulus p that you would use in the DH case. See Section 11.7 for the detailed discussion. To reiterate: the absolute minimum size for n is 2048 bits or so if you want to protect your data for 20 years. This minimum will slowly increase as computers get faster. If you can afford it in your application, let n be 4096 bits long, or as close to this size as you can manage. Furthermore, make sure that your software supports values of n up to 8192 bits long. You never know what the future will bring, and it could be a lifesaver if you can switch to using larger keys without replacing software or hardware.

The two primes p and q should be of equal size. For a k-bit modulus n, you can just generate two random $k/2$-bit primes and multiply them. You might end up with a $k-1$-bit modulus n, but that doesn't matter much.

12.4.5 Generating RSA Keys

To pull everything together, we present two routines that generate RSA keys with the desired properties. The first one is a modification of the generateLargePrime function of Section 10.4. The only functional change is that we require that the prime satisfies $p \bmod 3 \neq 1$ and $p \bmod 5 \neq 1$ to ensure that we can use the public exponents 3 and 5. Of course, if you want to use a different fixed value for e, you have to modify this routine accordingly.

```
function generateRSAPrime
input:  k    Size of the desired prime, in number of bits.
output: p    A random prime in the interval $2^{k-1}, \ldots, 2^k - 1$ subject to $p \bmod$
             $3 \neq 1 \wedge p \bmod 5 \neq 1$.

  Check for a sensible range.
  assert $1024 \leq k \leq 4096$

  Compute maximum number of attempts.
  $r \leftarrow 100k$
  repeat
      $r \leftarrow r - 1$
      assert $r > 0$
```

```
        Choose n as a random k-bit number.
        $n \in_{\mathcal{R}} 2^{k-1}, \ldots, 2^k - 1$
        Keep on trying until we find a prime.
    until $n \bmod 3 \neq 1 \wedge n \bmod 5 \neq 1 \wedge$ isPrime($n$)
    return $n$
```

Instead of specifying a full range in which the prime should fall, we only specify the size of the prime. This is a less-flexible definition, but somewhat simpler, and it is sufficient for RSA. The extra requirements are in the loop condition. A clever implementation will not even call isPrime(n) if n is not suitable modulo 3 or 5, as isPrime can take a significant amount of computations.

So why do we still include the loop counter with the error condition? Surely, now that the range is large enough, we will always find a suitable prime? We'd hope so, but stranger things have happened. We are not worried about getting a range with no primes in it—we're worried about a broken prng that always returns the same composite result. This is, unfortunately, a common failure mode of random number generators, and this simple check makes generateRSAPrime safe from misbehaving prngs. Another possible failure mode is a broken isPrime function that always claims that the number is composite. Of course, we have more serious problems to worry about if any of these functions is misbehaving.

The next function generates all the key parameters.

```
function generateRSAKey
input:  $k$      Size of the modulus, in number of bits.
output: $p, q$   Factors of the modulus.
        $n$      Modulus of about $k$ bits.
        $d_3$    Signing exponent.
        $d_5$    Decryption exponent.
    Check for a sensible range.
    assert $2048 \leq k \leq 8192$
    Generate the primes.
    $p \leftarrow$ generateRSAPrime($\lfloor k/2 \rfloor$)
    $q \leftarrow$ generateRSAPrime($\lfloor k/2 \rfloor$)
    A little test just in case our prng is bad . . . .
    assert $p \neq q$
    Compute t as lcm($p - 1, q - 1$).
    $t \leftarrow (p-1)(q-1)/\mathrm{GCD}(p-1, q-1)$
    Compute the secret exponents using the modular inversion feature of the extended
            GCD algorithm.
    $g, (u, v) \leftarrow$ extendedGCD($3, t$)
```

```
*Check that the GCD is correct, or we don't get an inverse at all.*
**assert** $g = 1$
*Reduce u modulo t, as u could be negative and $d_3$ shouldn't be.*
$d_3 \leftarrow u \bmod t$
*And now for $d_5$.*
$g, (u, v) \leftarrow \textsc{extended}\mathrm{GCD}(5, t)$
**assert** $g = 1$
$d_5 \leftarrow u \bmod t$
**return** $p, q, pq, d_3, d_5$
```

Note that we've used the fixed choices for the public exponents, and that we generate a key that can be used both for signing ($e = 3$) and for encryption ($e = 5$).

12.5 Pitfalls Using RSA

Using RSA as presented so far is very dangerous. The problem is the mathematical structure. For example, if Alice digitally signs two messages m_1 and m_2, then Bob can compute Alice's signature on $m_3 := m_1 m_2 \bmod n$. After all, Alice has computed $m_1^{1/e}$ and $m_2^{1/e}$ and Bob can multiply the two results to get $(m_1 m_2)^{1/e}$.

Another problem arises if Bob encrypts a very small message m with Alice's public key. If $e = 5$ and $m < \sqrt[5]{n}$, then $m^e = m^5 < n$, so no modular reduction ever takes place. The attacker Eve can recover m by simply taking the fifth root of m^5, which is easy to do because there are no modulo reductions involved. A typical situation in which this could go wrong is if Bob tries to send an AES key to Alice. If she just takes the 256-bit value as an integer, then the encrypted key is less than $2^{256 \cdot 5} = 2^{1280}$, which is much smaller than our n. There is never a modulo reduction, and Eve can compute the key by simply computing the fifth root of the encrypted key value.

One of the reasons we have explained the theory behind RSA in such detail is to teach you some of the mathematical structure that we encounter. This very same structure invites many types of attack. We've mentioned some simple ones in the previous paragraph. There are far more advanced attacks, based on techniques for solving polynomial equations modulo n. All of them come down to a single thing: it is very bad to have any kind of structure in the numbers that RSA operates on.

The solution is to use a function that destroys any available structure. Sometimes this is called a padding function, but this is a misnomer. The word padding is normally used for adding additional bytes to get a result of the right length. People have used various forms of padding for RSA encryption and signatures, and quite a few times this has resulted in attacks on their designs. What you need is a function that removes as much structure as possible. We'll call this the encoding function.

There are standards for this, most notably PKCS #1 v2.1 [110]. As usual, this is not a single standard. There are two RSA encryption schemes and two RSA signature schemes, each of which can take a variety of hash functions. This is not necessarily bad, but from a pedagogical perspective we don't like the extra complexity. We'll therefore present some simpler methods, even though they might not have all the features of some of the PKCS methods. And, as we mentioned before in the case of AES, there are many advantages to using a standardized algorithm in practice. For example, for encryption you might use RSA-OAEP [9], and for signatures you might use RSA-PSS [8].

The PKCS #1 v2.1 standard also demonstrates a common problem in technical documentation: it mixes specification with implementation. The RSA decryption function is specified twice; once using the equation $m = c^d \bmod n$ and once using the CRT equations. These two computations have the same result: one is merely an optimized implementation of the other. Such implementation descriptions should not be part of the standard, as they do not produce different behavior. They should be discussed separately. We don't want to criticize this PKCS standard in particular; it is a very widespread problem that you find throughout the computer industry.

12.6 Encryption

Encrypting a message is the canonical application of RSA, yet it is almost never used in practice. The reason is simple: the size of the message that can be encrypted using RSA is limited by the size of n. In real systems, you cannot even use all the bits, because the encoding function has an overhead. This limited message size is too impractical for most applications, and because the RSA operation is quite expensive in computational terms, you don't want to split a message into smaller blocks and encrypt each of them with a separate RSA operation.

The solution used almost everywhere is to choose a random secret key K, and encrypt K with the RSA keys. The actual message m is then encrypted with key K using a block cipher or stream cipher. So instead of sending something like $E_{\mathrm{RSA}}(m)$, you send $E_{\mathrm{RSA}}(K), E_K(m)$. The size of the message is no longer limited, and only a single RSA operation is required, even for large messages. You have to transmit a small amount of extra data, but this is usually a minor price to pay for the advantages you get.

We will use an even simpler method of encryption. Instead of choosing a K and encrypting K, we choose a random $r \in \mathbb{Z}_n$ and define the bulk encryption key as $K := h(r)$ for some hash function h. Encrypting r is done by simply raising it to the fifth power modulo n. (Remember, we use $e = 5$ for encryption.) This solution is simple and secure. As r is chosen randomly, there is no structure in r that can be used to attack the RSA portion of the encryption. The hash function in turn ensures that no structure between different r's propagates to structure in the K's, except for the obvious requirement that equal inputs must yield equal outputs.

For simplicity of implementation, we choose our r's in the range $0, \ldots, 2^k - 1$, where k is the largest number such that $2^k < n$. It is easier to generate a random k-bit number than to generate a random number in $\mathbb{Z}_n$, and this small deviation from the uniform distribution is harmless in this situation.

Here is a more formal definition:

```
function encryptRandomKeyWithRSA
input:  $(n, e)$  RSA public key, in our case $e = 5$.
output: $K$       Symmetric key that was encrypted.
        $c$       RSA ciphertext.
    Compute k.
    $k \leftarrow \lfloor \log_2 n \rfloor$
    Choose a random r such that $0 \leq r < 2^k - 1$.
    $r \in_{\mathcal{R}} \{ 0, \ldots, 2^k - 1 \}$
    $K \leftarrow \text{SHA}_d\text{-256}(r)$
    $c \leftarrow r^e \bmod n$
    return $(K, c)$
```

The receiver computes $K = h(c^{1/e} \bmod n)$ and gets the same key K.

```
function decryptRandomKeyWithRSA
input:  $(n, d)$  RSA private key with $e = 5$.
        $c$       Ciphertext.
output: $K$       Symmetric key that was encrypted.
    assert $0 \leq c < n$
    This is trivial.
    $K \leftarrow \text{SHA}_d\text{-256}(c^{1/e} \bmod n)$
    return $K$
```

We previously dealt extensively with how to compute $c^{1/e}$ given the private key, so we won't discuss that here again. Just don't forget to use the CRT for a factor of 3–4 speed-up.

Here is a good way to look at the security. Let's assume that Bob encrypts a key K for Alice, and Eve wants to know more about this key. Bob's message depends only on some random data and on Alice's public key. So at worst this message could leak data to Eve about K, but it cannot leak any data about any other secret, such as Alice's private key. The key K is computed using a hash function, and we can pretend that the hash function is a random mapping. (If we cannot treat the hash function as a random mapping, it doesn't satisfy our security requirement for hash functions.) The only way to get information about the output of a hash function is to know most of the input. That means having information about r. But if RSA is secure—and we have to assume that since we have chosen to use it—then it is impossible to get any significant amount of information about a randomly chosen r given just (r^e mod n). This leaves the attacker with a lot of uncertainty about r, and consequently, no knowledge about K.

Suppose the key K is later revealed to Eve, maybe due to a failure of another component of the system. Does this reveal anything about Alice's private key? No. K is the output of a hash function, and it is impossible for Eve to derive any information about the inputs to the hash function. So even if Eve chose c in some special way, the K she acquires does not reveal anything about r. Alice's private key was only used to compute r, so Eve cannot learn anything about Alice's private key either.

This is one of the advantages of having a hash function in the DECRYPTRANDOMKEYWITHRSA function. Suppose it just returned $c^{1/e}$ mod n. This routine could then be used to play all kinds of games. Suppose some other part of the system had a weakness and Eve learned the least significant bit of the output. Eve could then send specially chosen values $c_1, c_2, c_3, \ldots$ to Alice and get the least significant bits of $c_1^{1/e}, c_2^{1/e}, c_3^{1/e}, \ldots$. These answers have all kinds of algebraic properties, and it is quite conceivable that Eve could learn something useful from a situation like this. The hash function h in DECRYPTRANDOMKEYWITHRSA destroys all mathematical structure. Learning one bit from the output K gives Eve almost no information about $c^{1/e}$. Even the full result K divulges very little useful information; the hash function is not invertible. Adding the hash function here makes the RSA routines more secure against failures in the rest of the system.

This is also the reason why DECRYPTRANDOMKEYWITHRSA does *not* check that the r we compute from c falls in the range $0, \ldots, 2^k - 1$. If we checked this condition, we would have to handle the error that could result. As error handling always leads to different behavior, it is quite probable that Eve could detect whether this error occurred. This would provide Eve with a function that reveals information: Eve could choose any value c and learn whether $c^{1/e}$ mod $n < 2^k$. Eve cannot compute this property without Alice's help, and we don't want to help Eve if we can avoid it. By not checking the condition, we at most generate a nonsense output, and that is something that can happen in any case, as c might have been corrupted without resulting in an invalid r value.[3]

An aside: there is a big difference between revealing a random pair $(c, c^{1/e})$, and computing $c^{1/e}$ for a c chosen by someone else. Anybody can produce pairs

[3] Placing more restrictions on r does not stop the problem of nonsensical outputs. Eve can always use Alice's public key and a modified ENCRYPTRANDOMKEYWITHRSA function to send Alice encryptions of nonsensical keys.

of the form $(c, c^{1/e})$. All you do is choose a random r, compute the pair (r^e, r), and then set $c := r^e$. There is nothing secret about pairs like that. But if Alice is kind enough to compute $c^{1/e}$ for a c she receives from Eve, Eve can choose c values with some special properties—something she can't do for the $(c, c^{1/e})$ pairs she generates herself. Don't provide this extra service for your attacker.

12.7 Signatures

For signatures, we have to do a bit more work. The problem is that the message m we want to sign can have a lot of structure to it, and we do not want any structure in the number we compute the RSA root on. We have to destroy the structure.

The first step is to hash the message. So instead of a variable-length message m, we deal with a fixed-size value $h(m)$ where h is a hash function. If we use SHA$_d$-256, we get a 256-bit result. But n is much bigger than that, so we cannot use $h(m)$ directly.

The simple solution is to use a pseudorandom mapping to expand $h(m)$ to a random number s in the range $0, \ldots, n-1$. The signature on m is then computed as $s^{1/e} \pmod{n}$. Mapping $h(m)$ to a modulo n value is a bit of work (see the discussion in Section 9.7). In this particular situation, we can safely simplify our problem by mapping $h(m)$ to a random element in the range $0, \ldots, 2^k - 1$, where k is the largest number such that $2^k < n$. Numbers in the range $0, \ldots, 2^k - 1$ are easy to generate because we only need to generate k random bits. In this particular situation, this is a safe solution, but don't use it just anywhere. There are many situations in cryptography where this will break your entire system.

We will use the generator from our Fortuna PRNG from Chapter 9. Many systems use the hash function h to build a special little random number generator for this purpose, but we've already defined a good one. Besides, you need the PRNG to choose the primes to generate the RSA keys, so you have the PRNG in the software already.

This results in three functions—one to map the message to s, one to sign the message, and one to verify the signature.

```
function MsgToRSANumber
input:  $n$     RSA public key, modulus.
        $m$     Message to be converted to a value modulo $n$.
output: $s$     A number modulo $n$.
    Create a new PRNG generator.
    $\mathcal{G} \leftarrow$ InitializeGenerator()
    Seed it with the hash of the message.
    ReSeed($\mathcal{G}$, SHA$_d$-256($m$))
    Compute k.
    $k \leftarrow \lfloor \log_2 n \rfloor$
    $x \leftarrow$ PseudoRandomData($\mathcal{G}$, $\lceil k/8 \rceil$)
    As usual, we treat the byte-string x as an integer using the LSByte first convention.
            The modulo reduction can be implemented with a simple AND on the
            last byte of x.
    $s \leftarrow x \bmod 2^k$
    return $s$

function SignWithRSA
input:  $(n, d)$  RSA private key with $e = 3$.
        $m$       Message to be signed.
output: $\sigma$  Signature on $m$.
    $s \leftarrow$ MsgToRSANumber($n, m$)
    $\sigma \leftarrow s^{1/e} \bmod n$
    return $\sigma$
```

The letter σ, or sigma, is often used for signatures because it is the Greek equivalent of our letter s. By now you should know how to compute $s^{1/e} \bmod n$, given the private key.

```
function VerifyRSASignature
input:  $(n, e)$  RSA public key with $e = 3$.
        $m$       Message that is supposed to be signed.
        $\sigma$  Signature on the message.
    $s \leftarrow$ MsgToRSANumber($n, m$)
    assert $s = \sigma^e \bmod n$
```

Of course, in a real application there will be some action to take if the signature verification fails. We've just written an assertion here to indicate that normal operations should not proceed. A signature failure should be taken like any other failure in a cryptographic protocol: as a clear signal that you are under active attack. Don't send any replies unless you absolutely have to, and destroy all the material you are working on. The more information you send out, the more information you give the attacker.

The security argument for our RSA signatures is similar to that of the RSA encryptions. If you ask Alice to sign a bunch of messages $m_1, m_2, \ldots, m_i$, then you are getting pairs of the form $(s, s^{1/e})$, but the s values are effectively random. As long as the hash function is secure, you can only affect $h(m)$ by trial and error. The random generator is again a random mapping. Anyone can create pairs of the form $(s, s^{1/e})$ for random s values, so this provides no new information that helps the attacker forge a signature. However, for any particular message m, only someone who knows the private key can compute the corresponding $(s, s^{1/e})$ pair, because s must be computed from $h(m)$, then $s^{1/e}$ must be computed from s. This requires the private key. Therefore, anyone who verifies the signature knows that Alice must have signed it.

This brings us to the end of our treatment of RSA, and to the end of the math-heavy part of this book. We will be using DH and RSA for our key negotiation protocol and the PKI, but that only uses the math we have already explained. No new mathematics will be introduced.

12.8 Exercises

Exercise 12.1 Let $p = 89$, $q = 107$, $n = pq$, $a = 3$, and $b = 5$. Find x in $\mathbb{Z}_n$ such that $a = x \pmod{p}$ and $b = x \pmod{q}$.

Exercise 12.2 Let $p = 89$, $q = 107$, $n = pq$, $x = 1796$, and $y = 8931$. Compute $x + y \pmod{n}$ directly. Compute $x + y \pmod{n}$ using CRT representations.

Exercise 12.3 Let $p = 89$, $q = 107$, $n = pq$, $x = 1796$, and $y = 8931$. Compute $xy \pmod{n}$ directly. Compute $xy \pmod{n}$ using CRT representations.

Exercise 12.4 Let $p = 83$, $q = 101$, $n = pq$, and $e = 3$. Is (n, e) a valid RSA public key? If so, compute the corresponding private RSA key d. If not, why not?

Exercise 12.5 Let $p = 79$, $q = 89$, $n = pq$, and $e = 3$. Is (n, e) a valid RSA public key? If so, compute the corresponding private RSA key d. If not, why not?

Exercise 12.6 To speed up decryption, Alice has chosen to set her private key $d = 3$ and computes e as the inverse of d modulo t. Is this a good design decision?

Exercise 12.7 Does a 256-bit RSA key (a key with a 256-bit modulus) provide strength similar to that of a 256-bit AES key?

Exercise 12.8 Let $p = 71$, $q = 89$, $n = pq$, and $e = 3$. First find d. Then compute the signature on $m_1 = 5416$, $m_2 = 2397$, and $m_3 = m_1 m_2 \pmod{n}$ using the basic RSA operation. Show that the third signature is equivalent to the product of the first two signatures.

CHAPTER

13

Introduction to Cryptographic Protocols

Cryptographic protocols consist of an exchange of messages between participants. We've already seen a simple cryptographic protocol in Chapter 11.

Creating secure protocols is challenging. The main problem is that as a designer or implementer, you are not in control. Up to now we have been designing a system, and have had control over the behavior of various parts. Once you start communicating with other parties, you have no control over their behavior. The other party has a different set of interests than you do, and he could deviate from the rules to try to get an advantage. When working on protocols, you must assume that you are dealing with the enemy.

13.1 Roles

Protocols are typically described as being executed by Alice and Bob, or between a customer and a merchant. Names like ''Alice,'' ''Bob,'' ''customer,'' and ''merchant'' are not really meant to identify a particular individual or organization. They identify a role within the protocol. If Mr. Smith wants to communicate with Mr. Jones, he might run a key agreement protocol. Mr. Smith could take the role of Alice, and Mr. Jones the role of Bob. The next day the roles might be reversed. It is important to keep in mind that a single entity can take on any of the roles.[1] This is especially important to remember when you analyze the protocol for security. We've already seen the

[1] In protocols with three or more participants, it is even possible for a single person to take on more than one role at the same time.

man-in-the-middle attack on the DH protocol. In that attack, Eve takes on the roles of both Alice and of Bob. (Of course, Eve is just another role, too.)

13.2 Trust

Trust is the ultimate basis for all dealings that we have with other people. If you don't trust anybody with anything at all, why bother interacting with them? For example, buying a candy bar requires a basic level of trust. The customer has to trust the merchant to provide the candy and give proper change. The merchant has to trust the customer to pay. Both have recourse if the other party misbehaves. Shoplifters are prosecuted. Cheating merchants risk bad publicity, lawsuits, and getting punched in the nose.

There are several sources of trust:

Ethics Ethics has a large influence in our society. Although very few, if any, people behave ethically all the time, most people behave ethically most of the time. Attackers are few. Most people pay for their purchases, even when it would be laughably easy to steal them.

Reputation Having a ''good name'' is very important in our society. People and companies want to protect their reputation. Often the threat of bad publicity gives them an incentive to behave properly.

Law In civilized societies, there is a legal infrastructure that supports lawsuits and prosecution of people who misbehave. This gives people an incentive to behave properly.

Physical Threat Another incentive to behave properly is the fear of harm if you cheat and are caught. This is one of the sources of trust for drug deals and other illegal trades.

MAD A Cold War term: Mutually Assured Destruction. In milder forms, it is the threat to do harm to both yourself and the other party. If you cheat your friend, she might break off the friendship, doing you both harm. Sometimes you see two companies in a MAD situation, especially when they file patent infringement lawsuits against each other.

All of these sources are mechanisms whereby a party has an incentive not to cheat. The other party knows this incentive, and therefore feels he can trust his opponent to some extent. This is why these incentives all fail when you deal with completely irrational people: you can't trust them to act in their own best interest, which undermines all these mechanisms.

It is hard to develop trust over the Internet. Suppose Alice lives abroad and connects to the ACME website. ACME has almost no reason to trust Alice; of the mechanisms of trust we mentioned, only ethics remains. Legal recourse against private individuals abroad is almost impossible, and prohibitively expensive in most cases. You can't effectively harm their reputation or threaten them, even with MAD.

There is nevertheless a basis of trust between Alice and ACME, because ACME has a reputation to protect. This is important to remember when you design a protocol for e-commerce. If there are any failure modes (and there always are), the failure should be to ACME's advantage, because ACME has an incentive to settle the matter properly by manual intervention.[2] If the failure is to Alice's advantage, the issue is less likely to be settled properly. Furthermore, ACME will be vulnerable to attackers who try to induce the failure mode and then profit by it.

Trust is not a black-and-white issue. It is not that you either trust someone or you don't trust him. You trust different people to different degrees. You might trust an acquaintance with $100 but not with your lottery ticket that just won a $5,000,000 prize. We trust the bank to keep our money safe, but we get receipts and copies of canceled checks because we don't fully trust their administration. The question ''Do you trust him?'' is incomplete. It should be ''Do you trust him with *X*?''

13.2.1 Risk

Trust is fundamental to business, but it is usually expressed as risk rather than trust. Risk can be seen as the converse of trust. Risks are evaluated, compared, and traded in many forms.

When working on cryptographic protocols, it is easier to talk in terms of trust than in terms of risks. But a lack of trust is simply a risk, and that can sometimes be handled by standard risk-management techniques such as insurance. We talk about trust when we design protocols. Always keep in mind that business people think and talk in terms of risks. You'll have to convert between the two perspectives if you want to be able to talk to them.

13.3 Incentive

The incentive structure is another fundamental component of any analysis of a protocol. What are the goals of the different participants? What would they like to achieve? Even in real life, analyzing the incentive structure gives insightful conclusions.

[2] Almost all telephone, mail, and electronic commerce to individuals follows this rule by having the customer pay for the order before it is shipped.

Several times every week we get press reports that announce things like, ''New research has shown that . . . .'' Our first reaction is always to ask: who paid for the research? Research whose results are advantageous to the party who paid for it is always suspect. Several factors are at play here. First, the researchers know what their customer wants to hear, and know they can get repeat contracts if they produce ''good'' results. This introduces a bias. Second, the sponsor of the research is not going to publish any negative reports. Publishing only the positive reports introduces another bias. Tobacco companies published ''scientific'' reports that nicotine was not addictive. Microsoft pays for research that ''proves'' that open source software is bad in some way. Don't ever trust research that supports the company that paid for it.

The authors are personally quite familiar with these pressures. During our years as consultants, we performed many security evaluations for paying customers. We were often harsh—the average product we evaluated was quite bad—and our evaluations often had significantly negative components. That didn't always make us popular with our customers. One of them even called Bruce and said: ''Stop your work and send me your bill. I've found someone who is cheaper and who writes better reports.'' Guess which meaning of ''better'' was intended here?

We see exactly the same problem in other areas. As we wrote the first edition of this book, the press was filled with stories about the accounting and banking industries. Analysts and accountants were writing reports favorable for their clients rather than unbiased evaluations. We blame the incentive structure that gave these people a reason to bias their reports. Looking at the incentives is quite instructive, and something we've done for years. With a bit of practice, it is surprisingly easy, and it yields valuable insights.

If you pay your management in stock options, you give them the following incentive structure: increase the share price over the next three years and make a fortune; decrease the share price, and get a golden handshake. It is a ''Heads I win a lot, tails I win a little'' incentive, so guess what some managers do? They go for a high-risk, short-term strategy. If they get the opportunity to double the amount they gamble they will always take it, because they will only collect the winnings and never pay the loss. If they can inflate the share price for a few years with bookkeeping tricks, they will, because they can cash out before they are found out. Some of the gambles fail, but others pay the bills.

A similar thing happened with the savings and loan industry in the United States in the 1980s. The federal government liberalized the rules, allowing S&Ls to invest their money more freely. At the same time, the government guaranteed the deposits. Now look at the incentive structure. If the investments pay off, the S&L makes a profit, and no doubt management gets a nice bonus. If the investments lose money, the federal government pays off the depositors. Not surprisingly, a bunch of S&Ls lost a lot of money on high-risk investments—and the federal government picked up the bill.

Fixing the incentive structure is often relatively easy. For example, instead of the company itself paying for the audit, the stock exchange can arrange and pay for the audit of the books. Give the auditors a significant bonus for every error they find and you'll get a much more accurate report.

Examples of undesirable incentive structures abound. Divorce lawyers have an incentive to make the divorce very acrimonious, as they are paid for every hour spent fighting over the estate. It is a safe bet that they will advise you to settle as soon as the legal fees exceed the value of the estate.

In American society, lawsuits are common. If an accident happens, every participant has a great incentive to hide, deny, or otherwise avoid the blame. Strict liability laws and huge damage awards might seem good for society at first, but it greatly hinders our ability to figure out why the accident happened, and how we can avoid it in future. Liability laws that are supposed to protect consumers make it all but impossible (as an example) for a company like Firestone to admit there is a problem with their product so we can all learn how to build better tires.

Cryptographic protocols interact in two ways with incentive structures. First, they rely on incentive structures. Some electronic payment protocols do not stop the merchant from cheating the customer, but provide the customer with proof of the cheating. This works because it creates a cryptographic forensic trail. The merchant now has an incentive not to have people out there with proof they were cheated. The proof could be used either in a court case or just to damage the reputation of the merchant.

Second, cryptographic protocols change the incentive structure. They make certain things impossible, removing them from the incentive structure. They can also open up new possibilities and new incentives. Once you have online banking, you create an incentive for a thief to break into your computer and steal your money by that method.

At first, incentives look as if they are mostly materialistic, but that is only part of it. Many people have nonmaterialistic motives. In personal relationships, the most fundamental incentives have little to do with money. Keep an open mind, and try to understand what drives people. Then create your protocols accordingly.

13.4 Trust in Cryptographic Protocols

The function of cryptographic protocols is to minimize the amount of trust required. This is important enough to repeat. The function of cryptographic protocols is to minimize the amount of trust required. This means minimizing both the number of people who need to trust each other and the amount of trust they need to have.

One powerful tool for designing cryptographic protocols is the paranoia model. When Alice takes part in a protocol, she assumes that all other participants are conspiring together to cheat her. This is really the ultimate conspiracy theory. Of course, each of the other participants is making the same assumption. This is the default model in which all cryptographic protocols are designed.

Any deviations from this default model must be explicitly documented. It is surprising how often this step is overlooked. We sometimes see protocols used in situations where the required trust is not present. For example, most secure websites use the SSL protocol. The SSL protocol requires trusted certificates. But a certificate is easy to get. The result is that the user is communicating securely with a website, but he doesn't know which website he is communicating with. Numerous phishing scams against PayPal users have exploited this vulnerability, for example.

It is very tempting not to document the trust that is required for a particular protocol, as it is often ''obvious.'' That might be true to the designer of the protocol, but like any module in the system, the protocol should have a clearly specified interface, for all the usual reasons.

From a business point of view, the documented trust requirements also list the risks. Each point of required trust implies a risk that has to be dealt with.

13.5 Messages and Steps

A typical protocol description consists of a number of messages that are sent between the participants of the protocol and a description of the computations that each participant has to do.

Almost all protocol descriptions are done at a very high level. Most of the details are not described. This allows you to focus on the core functionality of the protocol, but it creates a great danger. Without careful specifications of all the actions that each participant should take, it is extremely difficult to create a safe implementation of the protocol.

Sometimes you see protocols specified with all the minor details and checks. Such specifications are often so complicated that nobody fully understands them. This might help an implementer, but anything that is too complicated cannot be secure.

The solution is, as always, modularization. With cryptographic protocols, as with communication protocols, we can split the required functionality into several protocol layers. Each layer works on top of the previous layer. All the layers are important, but most of the layers are the same for all protocols. Only the topmost layer is highly variable, and that is the one you always find documented.

13.5.1 The Transport Layer

Network specialists must forgive us for reusing one of their terms here. For cryptographers, the transport layer is the underlying communication system that allows parties to communicate. This consists of sending strings of bytes from one participant to another. How this is done is irrelevant for our purposes. What we as cryptographers care about is that we can send a string of bytes from one participant to the other. You can use UDP packets, a TCP data stream, e-mail, or any other method. In many cases, the transport layer needs some additional encoding. For example, if a program executes multiple protocols simultaneously, the transport layer must deliver the message to the right protocol execution. This might require an extra destination field of some sort. When using TCP, the length of the message needs to be included to provide message-oriented services over the stream-oriented TCP protocol.

To be quite clear, we expect that transport layer to transmit arbitrary strings of bytes. Any byte value can occur in the message. The length of the string is variable. The string received should, of course, be identical to the string that was sent; deleting trailing zero bytes, or any other modification, is not allowed.

Some transport layers include things like magic constants to provide an early detection of errors or to check the synchronization of the TCP stream. If the magic constant is not correct on a received message, the rest of the message should be discarded.

There is one important special case. Sometimes we run a cryptographic protocol over a cryptographically secured channel like the one we designed in Chapter 7. In cases like that, the transport layer also provides confidentiality, authentication, and replay protection. That makes the protocol much easier to design, because there are far fewer types of attack to worry about.

13.5.2 Protocol and Message Identity

The next layer up provides protocol and message identifiers. When you receive a message, you want to know which protocol it belongs to and which message within that protocol it is.

The protocol identifier typically contains two parts. The first part is the version information, which provides room for future upgrades. The second part identifies which particular cryptographic protocol the message belongs to. In an electronic payment system, there might be protocols for withdrawal, payment, deposit, refund, etc. The protocol identifier avoids confusion among messages of different protocols.

The message identifier indicates which of the messages of the protocol in question this is. If there are four messages in a protocol, you don't want there to be any confusion about which message is which.

Why do we include so much identifying information? Can't an attacker forge all of this? Of course he can. This layer doesn't provide any protection against active forgery; rather, it detects accidental errors. It is important to have good detection of accidental errors. Suppose you are responsible for maintaining a system, and you suddenly get a large number of error messages. Differentiating between active attacks and accidental errors such as configuration and version problems is a valuable service.

Protocol and message identifiers also make the message more self-contained, which makes much of the maintenance and debugging easier. Cars and airplanes are designed to be easy to maintain. Software is even more complex—all the more reason why it should be designed for ease of maintenance.

Probably the most important reason to include message identifying information has to do with the Horton Principle. When we use authentication (or a digital signature) in a protocol, we typically authenticate several messages and data fields. By including the message identification information, we avoid the risk that a message will be interpreted in the wrong context.

13.5.3 Message Encoding and Parsing

The next layer is the encoding layer. Each data element of the message has to be converted to a sequence of bytes. This is a standard programming problem and we won't go into too much detail about that here.

One very important point is the parsing. The receiver must be able to parse the message, which looks like a sequence of bytes, back into its constituent fields. This parsing must not depend on contextual information.

A fixed-length field that is the same in all versions of the protocol is easy to parse. You know exactly how long it is. The problems begin when the size or meaning of a field depends on some context information, such as earlier messages in the protocol. This is an invitation to trouble.

Many messages in cryptographic protocols end up being signed or otherwise authenticated. The authentication function authenticates a string of bytes, and usually it is simplest to authenticate the message at the level of the transport layer. If the interpretation of a message depends on some contextual information, the signature or authentication is ambiguous. We've broken several protocols based on this type of failure.

A good way to encode fields is to use Tag-Length-Value or TLV encoding. Each field is encoded as three data elements. The tag identifies the field in question, the length is the length of the value encoding, and the value is the actual data to be encoded. The best-known TLV encoding is ASN.1 [64], but it is incredibly complex and we shy away from it. A subset of ASN.1 could be very useful.

Another alternative is XML. Forget the XML hype; we're only using XML as a data encoding system. As long as you use a fixed Document Template Definition (DTD), the parsing is not context-dependent, and you won't have any problems.

13.5.4 Protocol Execution States

In many implementations, it is possible for a single computer to take part in several protocol executions at the same time. To keep track of all the protocols requires some form of protocol execution state. The state contains all the information necessary to complete the protocol.

Implementing protocols requires some kind of event-driven programming, as the execution has to wait for external messages to arrive before it can proceed. This can be implemented in various ways, such as using one thread or process per protocol execution, or using some kind of event dispatch system.

Given an infrastructure for event-driven programming, implementing a protocol is relatively straightforward. The protocol state contains a state machine that indicates the type of message expected next. As a general rule, no other type of message is acceptable. If the expected type of message arrives, it is parsed and processed according to the rules.

13.5.5 Errors

Protocols always contain a multitude of checks. These include verifying the protocol type and message type, checking that it is the expected type of message for the protocol execution state, parsing the message, and performing the cryptographic verifications specified. If any of these checks fail, we have encountered an error.

Errors need very careful handling, as they are a potential avenue of attack. The safest procedure is not to send any reply to an error and immediately delete the protocol state. This minimizes the amount of information the attacker can get about the protocol. Unfortunately, it makes for an unfriendly system, as there is no indication of the error.

To make systems usable, you often need to add error messages of some sort. If you can get away with it, don't send an error message to the other parties in the protocol. Log an error message on a secure log so the system administrator can diagnose the problem. If you *must* send an error message, make it as uninformative as possible. A simple ''There was an error'' message is often sufficient.

One dangerous interaction is between errors and timing attacks. Eve can send a bogus message to Alice and wait for her error reply. The time it takes

Alice to detect the error and send the reply often provides detailed information about what was wrong and exactly where it went wrong.

Here is a good illustration of the dangers of these interactions. Years ago, Niels worked with a commercially available smart card system. One of the features was a PIN code that was needed to enable the card. The four-digit PIN code was sent to the card, and the card responded with a message indicating whether the card was now enabled or not. Had this been implemented well, it would have taken 10,000 tries to exhaust all the possible PIN codes. The smart card allowed five failed PIN attempts before it locked up, after which it would require special unlocking by other means. The idea was that an attacker who didn't know the PIN code could make five attempts to guess the four-digit PIN code, which gave her a 1 in 2000 probability of guessing the PIN code before the card locked up.

The design was good, and similar designs are widely used today. A 1 in 2000 chance is good enough for many applications. But unfortunately, the programmer of that particular smart card system made a problematic design decision. To verify the four-digit PIN code, the program first checked the first digit, then the second, etc. The card reported the PIN code failure as soon as it detected that one of the digits was wrong. The weakness was that the time it took the smart card to send the ''wrong PIN'' error depended on how many of the digits of the PIN were correct. A smart attacker could measure this time and learn a lot of information. In particular, the attacker could find out at which position the first wrong digit was. Armed with that knowledge, it would take the attacker only 40 attempts to exhaustively search the PIN space. (After 10 attempts the first digit would have to be right, after another 10 attempts the second, etc.) After five tries, her chances of finding the correct PIN code rose to 1 in 143. That is much better for the attacker than the 1 in 2000 chance she should have had. If she got 20 tries, her chances rose to 60%, which is a lot more than the 0.2% she should have had.

Even worse, there are certain situations where having 20 or 40 tries is not infeasible. Smart cards that lock up after a number of failed PIN tries always reset the counter once the correct PIN has been used, so the user gets another five tries to type the correct PIN the next time. Suppose your roommate has a smart card like the one described above. If you can get at your roommate's smart card, you can run one or two tries before putting the smart card back. Wait for him to use the card for real somewhere, using the correct PIN and resetting the failed-PIN attempt counter in the smart card. Now you can do one or two more tries. Soon you'll have the whole PIN code because it takes at most 40 tries to find it.

Error handling is too complex to give you a simple set of rules. This is something we as a community do not know enough about yet. At the moment, the best advice we can give is to be very careful and reveal as little information as possible.

13.5.6 Replay and Retries

A replay attack occurs when the attacker records a message and then later resends that same message. Message replays have to be protected against. They can be a bit tricky to detect, as the message looks exactly like a proper one. After all, it *is* a proper one.

Closely related to the replay attack is the retry. Suppose Alice is performing a protocol with Bob, and she doesn't get a response. There could be many reasons for this, but one common one is that Bob didn't receive Alice's last message and is still waiting for it. This happens in real life all the time, and we solve this by sending another letter or e-mail, or repeating our last remark. In automated systems this is called a retry. Alice retries her last message to Bob and again waits for a reply.

So Bob can receive replays of messages sent by the attacker and retries sent by Alice. Somehow, Bob has to deal properly with them and ensure correct behavior without introducing a security weakness.

Sending retries is relatively simple. Each participant has a protocol execution state of some form. All you need to do is keep a timer and send the last message again if you do not receive an answer within a reasonable time. The exact time limit depends on the underlying communication infrastructure. If you use UDP packets (a protocol that uses IP packets directly), there is a reasonable probability that the message will get lost, and you want a short retry time, on the order of a few seconds. If you send your messages over TCP, then TCP retries any data that was not received properly using its own timeouts. There is little reason to do a retry at the cryptographic protocol level, and most systems that use TCP do not do this. Nevertheless, for the rest of this discussion we are going to assume that retries are being used, as the general techniques of handling received retries also work even if you never send them.

When you receive a message, you have to figure out what to do with it. We assume that each message is recognizable, so that you know which message in the protocol it is supposed to be. If it is the message you expect, there is nothing out of the ordinary and you just follow the protocol rules. Suppose it is a message from the ''future'' of the protocol; i.e., one that you only expect at a later point in time. This is easy; ignore it. Don't change your state, don't send a reply, just drop it and do nothing. It is probably part of an attack. Even in weird protocols where it could be part of a sequence of errors induced by lost messages, ignoring a message has the same effect as the message being lost in transit. As the protocol is supposed to recover from lost messages, ignoring a message is always a safe solution.

That leaves the case of ''old'' messages: messages you already processed in the protocol you are running. There are three situations in which this could occur. In the first one, the message you receive has the same message identification as the previous one you responded to, and it is identical in content to the message you responded to, too. In this case, the message is probably a retry, so you send exactly the same reply you sent the first time. Note that the reply should be the same. Don't recompute the reply with a different random value, and don't just assume that the message you get is identical to the first one you replied to. You have to check.

The second case is when you receive a message that has the same message identification as the message you last responded to, but the message contents are different. For example, suppose in the DH protocol Bob receives the first message from Alice, and then later receives another message that claims to be the first message in the protocol, but which contains different data while still passing the relevant integrity checks. This situation is indicative of an attack. No retry would ever create this situation, as the resent message is never different from the first try. Either the message you just received is bogus, or the earlier one you responded to is bogus. The safe choice is to treat this as a protocol error, with all the consequences we discussed. (Ignoring the message you just received is safe, but it means that fewer forms of active attacks are detected as such. This has a detrimental effect on the detection and response parts of the security system.)

The third case is when you receive a message that is even older than the previous message you responded to. There is not much you can do with this. If you still have a copy of the original message you received at that phase in the protocol, you can check if it is identical to that one. If it is, ignore it. If it is different, you have detected an attack and should treat it as a protocol error. Many implementations do not store all the messages that were received in a protocol execution, which makes it impossible to know whether the message you receive now is or is not identical to the one originally processed. The safe option is to ignore these messages. You'd be surprised how often this actually happens. Sometimes messages get delayed for a long time. Suppose Alice sends a message that is delayed. After a few seconds, she sends a retry that does arrive, and both Alice and Bob continue with the protocol. Half a minute later, Bob receives the original message. This is a situation in which Bob receives a copy of—in protocol terms—a very old message.

Things get more complicated if you have a protocol in which there are more than two participants. These exist, but are beyond the scope of this book. If you ever work on a multiparty protocol, think carefully about replay and retries.

One final comment: it is impossible to know whether the last message of a protocol arrived or not. If Alice sends the last message to Bob, then she will never get a confirmation that it arrived. If the communication link is broken and Bob never receives the last message, then Bob will retry the previous message but that will not reach Alice either. This is indistinguishable to Alice from the normal end of the protocol. You could add an acknowledgment from

Bob to Alice to the end of the protocol, but then this acknowledgment becomes the new last message and the same problem arises. Cryptographic protocols have to be designed in a way that this ambiguity does not lead to insecure behavior.

13.6 Exercises

Exercise 13.1 Describe a protocol you engage in on a regular basis. This might be ordering a drink at a local coffee shop or boarding an airplane. Who are the explicit actors directly involved in this protocol? Are there other actors involved peripherally in this protocol, such as during the setup phase? For simplicity, list at most 5 actors. Create a matrix, where each row is labeled by an actor and each column is labeled by an actor. For each cell, describe how the actor in the row trusts the actor in the column.

Exercise 13.2 Consider the security of your personal computer. List the attackers who might break into your computer, their incentives, and the associated costs and risks to the attacker.

Exercise 13.3 Repeat exercise 13.2, except for a bank instead of your personal computer.

Exercise 13.4 Repeat exercise 13.2, except for a computer at the Pentagon instead of your personal computer.

Exercise 13.5 Repeat exercise 13.2, except for a computer belonging to a criminal organization instead of your personal computer.

CHAPTER

14

Key Negotiation

Finally, we are ready to tackle the key negotiation protocol. The purpose of this protocol is to derive a shared key that can then be used for the secure channel we defined in Chapter 7.

Complete protocols get quite complicated, and it can be confusing to present the final protocol all at once. Instead, we will present a sequence of protocols, each of which adds a bit more functionality. Keep in mind that the intermediate protocols are not fully functional, and will have various weaknesses.

There are different methods for designing key negotiation protocol, some with supporting proofs of security and some without. We designed our protocol from the ground up—not only because it leads to a cleaner explanation, but also because it allows us to highlight nuances and challenges at each stage of the protocol's design.

14.1 The Setting

There are two parties in the protocol: Alice and Bob. Alice and Bob want to communicate securely. They will first conduct the key negotiation protocol to set up a secret session key k, and then use k for a secure channel to exchange the actual data.

For a secure key negotiation, Alice and Bob must be able to identify each other. This basic authentication capability is the subject of the third part of this book. For now, we will just assume that Alice and Bob can authenticate messages to each other. This basic authentication can be done using RSA signatures (if Alice and Bob know each other's keys or are using a PKI), or using a shared secret key and a MAC function.

But wait! Why do a key negotiation if you already have a shared secret key? There are many reasons why you might want to do this. First of all, the key negotiation can decouple the session key from the existing (long-term) shared key. If the session key is compromised (e.g., because of a flawed secure channel implementation), the shared secret still remains safe. And if the shared secret key is compromised *after* the key negotiation protocol has been run, the attacker who learns the shared secret key still does not learn the session key negotiated by the protocol. So yesterday's data is still protected if you lose your key today. These are important properties: they make the entire system more robust.

There are also situations in which the shared secret key is a relatively weak one, like a password. Users don't like to memorize 30-letter passwords, and tend to choose much simpler ones. A standard attack is the *dictionary attack*, where a computer searches through a large number of simple passwords. Although we do not consider them here, some key negotiation protocols can turn a weak password into a strong key.

14.2 A First Try

There are standard protocols you might use to do key negotiation. A well-known one based on the DH protocol is the Station-to-Station protocol [34]. Here we will walk you through the design of a different protocol for illustrative purposes. We'll start with the simplest design we can think of, shown in Figure 14.1. This is just the DH protocol in a subgroup with some added authentication. Alice and Bob perform the DH protocol using the first two messages. (We've left out some of the necessary checks, for simplicity.) Alice then computes an authentication on the session key k and sends it to Bob, who checks the authentication. Similarly, Bob sends an authentication of k to Alice.

We don't know the exact form of the authentication at the moment. Remember, we said we assume that Alice and Bob can authenticate messages to each other. So Bob is able to check $\textsc{Auth}_A(k)$ and Alice is able to check $\textsc{Auth}_B(k)$. Whether this is done using digital signatures or using a MAC function is not our concern here. This protocol merely turns an authentication capability into a session key.

There are some problems with this protocol:

- The protocol is based on the assumption that (p, q, g) are known to both Alice and Bob. Choosing constants for these values is a bad idea.
- It uses four messages, whereas it is possible to achieve the goal using only three.

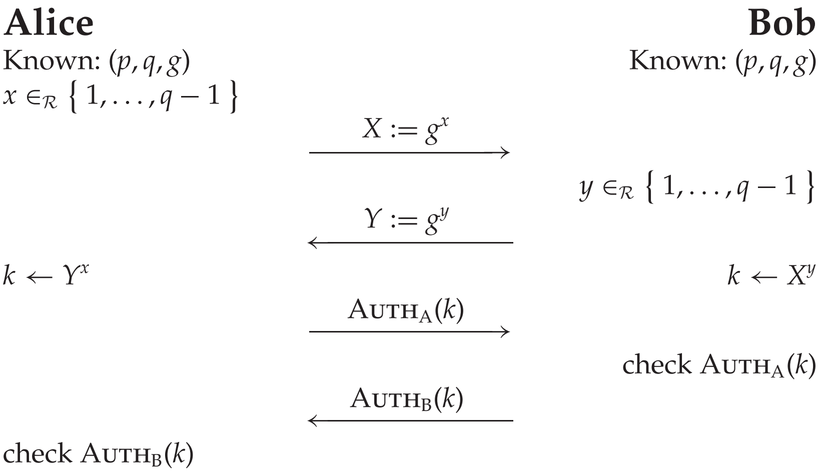
Alice
Bob
Known: (p, q, g)
Known: (p, q, g)
$x \in_{\mathcal{R}} \{ 1, \ldots, q-1 \}$
$X := g^x$
$y \in_{\mathcal{R}} \{ 1, \ldots, q-1 \}$
$Y := g^y$
$k \leftarrow Y^x$
$k \leftarrow X^y$
$\textsc{Auth}_{\mathrm{A}}(k)$
check $\textsc{Auth}_{\mathrm{A}}(k)$
$\textsc{Auth}_{\mathrm{B}}(k)$
check $\textsc{Auth}_{\mathrm{B}}(k)$

Figure 14.1: A first attempt at key negotiation.

- The session key is used as an input to the authentication function. This is not a problem if the authentication function is strong, but suppose the authentication function leaks a few bits about the session key. That would be bad. It certainly would require a new analysis of the entire protocol. A good rule of thumb is to use a secret only for a single thing. Here k will be used as a session key, so we don't want to use it as an argument to the authentication function.
- The two authentication messages are too similar. If, for example, the authentication function is a simple MAC using a secret key known to both Alice and Bob, then Bob could just send the authentication value he received from Alice, and he would not need the secret key to complete the protocol. Thus Alice would not be convinced by the last authentication message.
- Implementations have to be careful not to use k until the authentication messages have been exchanged. This is not a major issue and is a rather simple requirement, but you wouldn't believe what sometimes happens when people try to optimize a program.

We will fix all of these problems over the course of this chapter.

14.3 Protocols Live Forever

We've emphasized the importance of designing systems to withstand the future. This is even more important for protocols. If you limit the size of database fields to 2000 bytes, it might be a problem for some users, but you can remove the limit in the next version. Not so for protocols. Protocols are run between different participants, and every new version needs to be interoperable with the old version. Modifying a protocol and still keeping it compatible with older versions is rather complicated. Before you know it, you have to implement several versions of the protocol, with a system to decide which version to use.

The protocol version switch becomes a point of attack, of course. If an older protocol is less secure, an attacker has an incentive to force you to use that older protocol. You'd be surprised at how many systems we've seen that suffer from what's known as a version-rollback attack.

It is of course impossible to know all the future requirements, so it might be necessary to define a second version of a protocol at some point. However, the cost of having several protocol versions is high, especially in overall complexity.

Successful protocols live almost forever (we don't care about unsuccessful ones). It is extremely difficult to completely remove a protocol from the world. So it is even more important to design protocols to be future-proof. This is why we can't specify a fixed set of DH parameters for our key negotiation protocol. Even if we chose them to be very large, there is always a danger that future cryptanalytical improvements might force us to change them.

14.4 An Authentication Convention

Before we go on, we will introduce an authentication convention. Protocols often have many different data elements, and it can be hard to figure out exactly which data elements need to be authenticated. Some protocols break because they neglect to authenticate certain data fields. We use a simple convention to solve these problems.

In our protocols, every time a party sends an authentication, the authentication data consists of all the data exchanged so far: all the previous messages, and all the data fields that precede the authentication in the authenticator's message. The authenticator will also cover (be computed over) the identities of the communicants. In the protocol shown in Figure 14.1, Alice's authenticator would not be on k, but on Alice's identifier, Bob's identifier, X, and Y. Bob's authenticator would cover Alice's identifier, Bob's identifier, X, Y, and $\textsc{Auth}_A$.

This convention removes many avenues of attack. It also costs very little. Cryptographic protocols don't exchange that much data, and authentication computations almost always start by hashing the input string. Hash functions are so fast that the extra cost is insignificant.

This convention also allows us to shorten the notation. Instead of writing something like $\textsc{Auth}_A(X, Y)$ we simply write $\textsc{Auth}_A$. As the data to be authenticated is specified by the convention, we no longer need to write it down explicitly. All further protocols in this book will use this convention.

Just as a reminder: authentication functions only authenticate a string of bytes. Each string of bytes to be authenticated must start with a unique identifier that identifies the exact point in the protocol where this authenticator is used. Also, the encoding of the previous messages and the data fields into this string of bytes must be such that the messages and fields can be recovered from the string without further context information. We've already talked about this in detail, but it is an important point that is easily overlooked.

14.5 A Second Attempt

How do we fix the problems of the previous protocol? We don't want to use a constant DH parameter set, so we'll let Alice choose it and send it to Bob. We'll also collapse the four messages into two, as shown in Figure 14.2. Alice starts by choosing DH parameters and her DH contribution, and sends it all to Bob with an authentication. Bob has to check that the DH parameters are properly chosen and that X is valid. (See Chapter 11 for details of these checks.) The rest of the protocol is similar to the previous version. Alice receives Y and $\text{A}\textsc{uth}_\text{B}$, checks them, and computes the DH result.

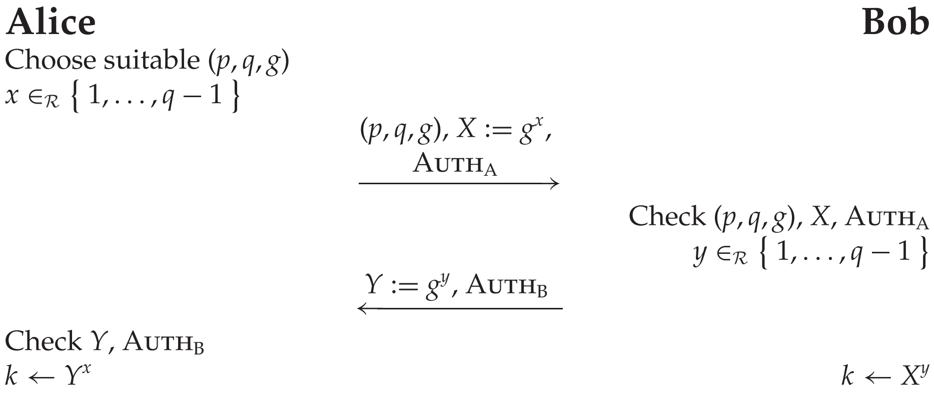
Alice
Bob
Choose suitable (p, q, g)
$x \in_\mathcal{R} \{ 1, \ldots, q-1 \}$
(p, q, g), $X := g^x$,
$\text{A}\textsc{uth}_\text{A}$
Check (p, q, g), X, $\text{A}\textsc{uth}_\text{A}$
$y \in_\mathcal{R} \{ 1, \ldots, q-1 \}$
$Y := g^y$, $\text{A}\textsc{uth}_\text{B}$
Check Y, $\text{A}\textsc{uth}_\text{B}$
$k \leftarrow Y^x$
$k \leftarrow X^y$

Figure 14.2: A second attempt at key negotiation.

We no longer have fixed DH parameters. We use only two messages, we don't use the authentication key directly in any way, and our authentication convention ensures that the strings being authenticated are not similar.

But now we have some new problems:

- What do we do if Bob wants a larger DH prime than Alice? Perhaps Bob has stricter security policies and thinks the DH prime chosen by Alice isn't secure enough. Bob will have to abort the protocol. Maybe he could send an error message along the lines of ''Require DH prime to be at least k bits long,'' but that gets messy and complicated. Alice would have to restart the protocol with new parameters.
- There is a problem with the authentication. Bob isn't sure he is talking to Alice at all. Anybody can record the first message that Alice sends and then later send it to Bob. Bob thinks the message comes from Alice (after all, the authentication checked), and finishes the protocol, thinking he shares a key k with Alice. The attacker doesn't learn k, as he doesn't know x, and without k the attacker cannot break into the rest of the system that uses k. But Bob's logs will show a completed authenticated protocol with Alice, and that is a problem by itself, as it provides erroneous information to investigating administrators.

Bob's problem is called a lack of ''liveness.'' He isn't sure that Alice is ''alive,'' and that he's not talking to a replaying ghost. The traditional way to solve this is to make sure that Alice's authenticator covers a random element chosen by Bob.

14.6 A Third Attempt

We will fix these problems with a few more changes. Instead of Alice choosing the DH parameters, she will simply send her minimal requirements to Bob, and Bob will choose the parameters. This does increase the number of messages to three. (It turns out that most interesting cryptographic protocols require at least three messages. We don't know why, they just do.) Bob only sends a single message: the second one. This message will contain his authenticator, so Alice should send a randomly chosen element in the first message. We use a random nonce for this.

This leads to the protocol shown in Figure 14.3. Alice starts by choosing s, the minimal size of the prime p she wants to use. She also chooses a random 256-bit string as nonce N_a and sends them both to Bob. Bob chooses a suitable DH parameter set and his random exponent, and sends the parameters, his DH contribution, and his authenticator to Alice. Alice completes the DH protocol as usual with the added authenticator.

There is one more problem to be solved. The final result k is a variable-sized number. Other parts of the system might find this difficult to work with. Furthermore, k is computed using algebraic relations, and leaving algebraic structure in a cryptographic system always scares us. There are a few places where you absolutely need such structure, but we avoid it wherever possible. The danger of algebraic structure is that an attacker might find some way of exploiting it. Mathematics can be an extremely powerful tool. Over the past few decades, we have seen many new proposals for public-key systems, almost all of which have been broken—mostly due to the algebraic structure they contained. Always remove any algebraic structure that you can.

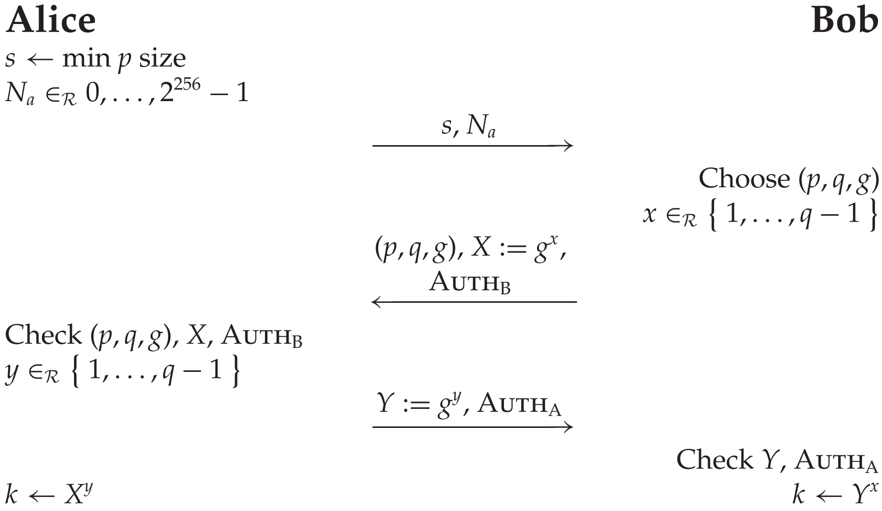
Alice
Bob
$s \leftarrow \min p$ size
$N_a \in_{\mathcal{R}} 0, \ldots, 2^{256} - 1$
s, N_a
Choose (p, q, g)
$x \in_{\mathcal{R}} \{ 1, \ldots, q - 1 \}$
(p, q, g), $X := g^x$, Auth_{B}
Check (p, q, g), X, Auth_{B}
$y \in_{\mathcal{R}} \{ 1, \ldots, q - 1 \}$
$Y := g^y$, Auth_{A}
Check Y, Auth_{A}
$k \leftarrow X^y$
$k \leftarrow Y^x$

Figure 14.3: A third attempt at key negotiation.

The obvious solution is to hash the final key. This reduces it to a fixed size, and also destroys any remaining algebraic structure.

14.7 The Final Protocol

The final protocol is shown in short form in Figure 14.4. This is the form that is easiest to read and understand. However, we've left a lot of verification steps out of the protocol to make it easy to read and to focus on the key properties. We simply write ''Check (p, q, g),'' which stands for several verifications. To show you all the required cryptographic checks, the long form of the protocol is given in Figure 14.5.

Bob needs to choose a suitable size for p. This depends on the minimum size required by Alice and his own required minimum size. Of course, Bob should ensure that the value of s is reasonable. We don't want Bob to be required to start generating 100,000-bit primes just because he received an unauthenticated message with a large value for s in it. Similarly, Alice should not have to start checking very large primes just because Bob sent them. Therefore, both Alice and Bob limit the size of p. Using a fixed maximum limits flexibility; if cryptanalytical progress suddenly forces you to use larger primes, then a fixed maximum is going to be a real problem. A configurable maximum brings with it all the problems of a configuration parameter that almost nobody understands. We've chosen to use a dynamic maximum. Both Alice and Bob refuse to use a prime that is more than twice as long as the prime they would prefer to use. A dynamic maximum provides a nice upgrade path and avoids excessively large primes. You can argue about whether the choice of the factor two is best. Maybe you should use three; it doesn't matter much.

Alice		**Bob**
$s \leftarrow \min p$ size		
$N_a \in_{\mathcal{R}} 0, \ldots, 2^{256} - 1$		
	$\xrightarrow{s, N_a}$	
		Choose (p, q, g)
		$x \in_{\mathcal{R}} \{ 1, \ldots, q - 1 \}$
	$\xleftarrow[\text{Auth}_{\text{B}}]{(p, q, g),\, X := g^x,}$	
Check (p, q, g), X, Auth_{B}		
$y \in_{\mathcal{R}} \{ 1, \ldots, q - 1 \}$		
	$\xrightarrow{Y := g^y,\, \text{Auth}_{\text{A}}}$	
		Check Y, Auth_{A}
$k \leftarrow \text{SHA}_d\text{-256}(X^y)$		$k \leftarrow \text{SHA}_d\text{-256}(Y^x)$

Figure 14.4: The final protocol in short form.

The rest of the protocol is just an expansion of the earlier short form. If Bob and Alice are smart, they'll both use caches of suitable DH parameters. This saves Bob from having to generate new DH parameters every time, and it saves Alice having to check them every time. Applications can even use a fixed set of DH parameters, or encode them as defaults in a configuration file, in which case you don't have to send them explicitly. A single DH parameter set identifier would be enough. But be careful when optimizing. Optimizations can end up modifying the protocol enough to break it. There are no simple rules we can give you to check if an optimization breaks a protocol or not. Protocol design is still more an art than a science, and there are no hard rules to live by.

Alice		**Bob**
$s_a \leftarrow \min p$ size		
$N_a \in_{\mathcal{R}} 0, \ldots, 2^{256} - 1$		
	$\xrightarrow{s_a,\, N_a}$	
		$s_b \leftarrow \min p$ size
		$s \leftarrow \max(s_a, s_b)$
		$s \overset{?}{\leq} 2 \cdot s_b$
		Choose (p, q, g) with $\log_2 p \geq s - 1$
		$x \in_{\mathcal{R}} \{\, 1, \ldots, q-1 \,\}$
	$\xleftarrow[\textsc{Auth}_{\mathrm{B}}]{(p, q, g),\, X := g^x,}$	
Check $\textsc{Auth}_{\mathrm{B}}$		
$s_a - 1 \overset{?}{\leq} \log_2 p \overset{?}{\leq} 2 \cdot s_a$		
$255 \overset{?}{\leq} \log_2 q \overset{?}{\leq} 256$		
Check p, q both prime		
$q \overset{?}{\mid} (p-1) \wedge g \overset{?}{\neq} 1 \wedge g^q \overset{?}{=} 1$		
$X \overset{?}{\neq} 1 \wedge X^q \overset{?}{=} 1$		
$y \in_{\mathcal{R}} \{\, 1, \ldots, q-1 \,\}$		
	$\xrightarrow{Y := g^y,\, \textsc{Auth}_{\mathrm{A}}}$	
		Check $\textsc{Auth}_{\mathrm{A}}$
		$Y \overset{?}{\neq} 1 \wedge Y^q \overset{?}{=} 1$
$k \leftarrow \mathrm{SHA}_d\text{-}256(X^y)$		$k \leftarrow \mathrm{SHA}_d\text{-}256(Y^x)$

Figure 14.5: The final protocol in long form.

14.8 Different Views of the Protocol

There are a number of instructive ways to look at a protocol like this. There are a few properties that the protocol should have, and we can look at why the protocol provides them all.

14.8.1 Alice's View

Let's look at the protocol from Alice's point of view. She receives a single message from Bob. She's sure this message is from Bob because it is authenticated, and the authentication includes her random nonce N_a. There is no way anyone could forge this message or replay an old message.

Alice checks that the DH parameters are properly chosen, showing that the DH protocol has all its expected properties. So when she keeps y secret and sends out Y, she knows that only persons who know an x such that $g^x = X$ can compute the resulting key k. This is the basic DH protocol property. Bob authenticated X, and Alice trusts Bob to only do this when he is following the protocol. Thus, Bob knows the appropriate x, and is keeping it secret. Therefore, Alice is sure that only Bob knows the final key k that she derives.

So Alice is convinced she is really talking to Bob, and that the key she derives can be known only to her and Bob.

14.8.2 Bob's View

Now let's look at Bob's side. The first message he receives gives him almost no useful information; it basically states that someone out there has chosen a value s_a and some random bits N_a.

The third message (the second one Bob receives) is different. This is a message that definitely came from Alice, because Alice authenticated it, and we assumed at the outset that Bob can verify an authentication by Alice. The authentication includes X, a random value chosen by Bob, so the third message is not a replay but has been authenticated by Alice specifically for this protocol run. Also, Alice's authentication covers the first message that Bob received, so now he knows that the first message was proper, too.

Bob knows the DH parameters are safe; after all, he chose them. So just like Alice, he knows that only someone who knows a y such that $g^y = Y$ can compute the final key k. But Alice authenticated the Y she sent, and Bob trusts Alice, so she is the only person who knows the corresponding y. This convinces Bob that Alice is the only other person who can compute k.

14.8.3 Attacker's View

Finally, we look at the protocol from the viewpoint of an attacker. If we just listen in on the communications, we see all the messages that Alice and Bob exchange. But the key k is computed using the DH protocol, so as long as the DH parameters are safe, a passive attack like this is not going to reveal anything about k. In other words: we'll have to try an active attack.

One instructive exercise is to look at each data element and try to change it. Here we are quickly stopped by the two authentications. Alice's final authentication covers all the data that was exchanged between Alice and Bob. That means we can't change any data elements, other than to try a replay attack of a prerecorded protocol run. But the nonce and the random X value stop any replay attempts.

That doesn't mean we can't try to play around. We could, for example, change s_a to a larger value. As long as this larger value is acceptable to Bob, most of the protocol would complete normally. There are just three problems. First of all, increasing s_a isn't an attack because it only makes the DH prime larger, and therefore the DH parameters stronger. The second and third problems are the two authentications, which will both fail.

There are some other things that might look like attacks at first. For example, suppose Alice sends Bob s_a and N_a. Bob sends s_a and N_a to Charlie. Charlie replies to Bob with (p, q, g), X, and Auth_C. Bob now turns this around and forwards (p, q, g) and X to Alice, along with a new authenticator Auth_B that he computes. Alice replies to Bob with Y and Auth_A. Bob then sends Y and a new authenticator Auth_B that he computes to Charlie. What's the result of all this? Alice thinks she's sharing a key k with Bob when in fact she's sharing it with Charlie. And Charlie thinks he's sharing a key with Bob when he's in fact sharing it with Alice. Is this an attack? Not really. Notice that Bob could just do the normal key negotiation with both Alice and Charlie, and then forward all the messages on the secure channel (decrypting each message he receives from Alice and re-encrypting it to Charlie, and vice-versa). This has the same effect; Alice thinks she is communicating with Bob, and Charlie thinks he is communicating with Bob, but they are sending messages to each other instead. And in this scenario Bob knows more (and can do more) than if he ran the ''attack.'' It is true that Alice might send a message to Charlie that makes Charlie believe that Bob agreed to something, but that can only be to Bob's detriment. And an attack that harms the attacker is not one we worry about.

In the real world, you will find many protocols where there are unauthenticated data elements. Most designers wouldn't bother authenticating s_a in our protocol, because changing it would not lead to an attack. (Both Alice and Bob independently verify that the size of p is large enough for them.) Allowing attackers to play around is always a bad idea. We don't want to give them any more tools than necessary. And we can certainly imagine a situation where not authenticating s_a could be dangerous. For example, assume that Bob prefers to use DH parameters from a list built into the program, and only generates new parameters when necessary. As long as Alice and Bob choose to use DH prime sizes that are still in the list, Bob never generates a new parameter set. But this also means that Bob's parameter generation code and Alice's parameter verification code are never used and therefore unlikely to be properly tested. A bug in the parameter generation and testing code could remain hidden until an attacker increases s_a. Yes, this is an unlikely scenario, but there are thousands of unlikely scenarios that are all bad for security. And thousands of low-probability risks add up to a high-probability risk. This is why we are so paranoid about stopping any type of attack anytime we can. It gives us defense in depth.

14.8.4 Key Compromise

So what happens if some other part of the system is compromised? Let's have a look.

If Alice merely loses her authentication key without it becoming known to an attacker, she simply loses the ability to run this protocol. She can still use session keys that were already established. This is very much how you'd expect the protocol to behave. The same holds for Bob if he loses his key.

If Alice loses the session key, without it becoming known to an attacker, she will have to run the key negotiation protocol again with Bob to establish a new session key.

Things get worse if an attacker manages to learn a key. If Alice's authentication key is compromised, the attacker can impersonate Alice from that moment on until the time that Bob is informed and stops accepting Alice's authentications. This is an unavoidable consequence. If you lose your car keys, anyone who finds them can use the car. That is one of the main functions of keys: they allow access to certain functions. This protocol does have the desirable property that past communications between Alice and Bob still remain secret. Even knowing Alice's authentication key doesn't let the attacker find the session key k for a protocol that has already finished, even if the attacker recorded all the messages. This is called *forward secrecy*.[1] The same properties hold with regard to Bob's authentication key.

Finally, we consider the situation where the session key is compromised. The key k is the hash of g^{xy}, where both x and y are randomly chosen. This provides no information about any other key. It certainly provides no information about Alice's or Bob's authentication keys. The value of k in one protocol run is completely independent of the k in another protocol run (at least, it is if we assume that Alice and Bob use a good PRNG).

Our protocol offers the best possible protection against key compromises.

14.9 Computational Complexity of the Protocol

Let's have a look at the computational complexity of our solution. We'll assume that the DH parameter selection and verification are all cached, so we don't count them in the workload of a single protocol run. That leaves the following computations, which Alice and Bob must each perform:

- Three exponentiations in the DH subgroup.
- One authentication generation.

[1] You sometimes see the term *perfect forward secrecy*, or PFS, but we don't use words like ''perfect'' because it never is.

- One authentication verification.
- Various relatively efficient operations, such as random number generation, comparisons, and hash functions.

If symmetric-key authentication is used, the run time of the protocol is dominated by the DH exponentiations. Let's look at how much work that is. Bob and Alice each have to do three modular exponentiations with a 256-bit exponent. This requires about 1150 modular multiplications.[2] To get an idea of how much work this really is, we'll compare this to the computational cost of an RSA signature where the RSA modulus and the DH prime are the same size. For an s-bit modulus, the signature algorithm requires $3s/2$ multiplications if you do not use the CRT (Chinese Remainder Theorem). Using the CRT representation saves a factor of four, so the cost of an RSA signature on s-bit numbers is similar to the cost of doing $3s/8$ multiplications. This leads us to an interesting conclusion: RSA signatures are relatively slower than DH computations when the moduli are large, and relatively faster when the moduli are small. The break-even point is around 3000 bits. This is because DH always uses 256-bit exponents, and for RSA the exponent grows with the modulus size.

We conclude that for the public-key sizes we use, the DH computations cost roughly the same as an RSA signature computation. The DH operations are still the dominant factors in the computations for the protocol, but the cost is quite reasonable.

If RSA signatures are used for the authentication, the computational load more or less doubles. (We can ignore RSA verifications as they are very fast.) This still isn't excessive. CPU speeds are rapidly increasing, and in most practical implementations you'll see that communications delays and overhead take up more time than the computations.

14.9.1 Optimization Tricks

There are a few optimizations that can be applied to the DH operations. Using addition chain heuristics, each exponentiation can be done using fewer multiplications. Furthermore, Alice computes both X^q and X^y. You can use addition sequence heuristics to compute these two results simultaneously and save about 250 multiplications. See Bos [18] for a detailed discussion.

There are also various tricks that make it faster to generate a random y and compute g^y, but these tricks require so much extra system complexity that we'd rather not use them.

[2] This is for the simple binary exponentiation algorithm. A better-optimized algorithm reduces this to less than 1000 multiplications.

14.10 Protocol Complexity

This protocol is also an excellent example of why protocol design is so hideously difficult. Even a simple protocol like this quickly expands to a full page, and we didn't even include all the rules for DH parameter generation or the checks for the authentication scheme that are unknown at our abstraction level. Yet it is already difficult to keep track of everything that goes on. More complicated protocols get much larger. One particular smart card payment system that Niels worked on had a dozen or so protocols specified in 50 pages of symbols and protocol specifications, and that was using a proprietary, highly compact notation! There were 50 more densely written pages needed to cover the security-critical implementation issues.

Full documentation of a set of cryptographic protocols can run into hundreds of pages. Protocols quickly get too complicated to keep in your head, and that is dangerous. Once you don't understand it all, it is almost inevitable that a weakness slips in. The above-mentioned project was probably too complex to be fully understood, even by the designers.

A few years later Niels worked with another, commercially available smart card system. This was a well-known and established system that was widely used for many different smart card applications. One day Marius Schilder, a colleague, showed up with a question—or rather, with a large hole in the system. It turns out that two of the protocols had a destructive interference with each other. One protocol computed a session key from a long-term card key, a bit like the key negotiation protocol of this chapter. A second protocol computed an authentication value from the long-term card key. With a bit of tweaking, you could use the second protocol to let the smart card compute the session key, and then send half of the bits to you. With half of the key bits known, breaking the rest of the system was trivial. Oops! This bug was fixed in the next version, but it is a good illustration of the problems of large protocol specifications.

Real-world systems always have very large protocol specifications. Communicating is very complex, and adding cryptographic functions and distrust makes things even harder. Our advice: be very careful with protocol complexity.

One of the fundamental problems in this area is that there are no good modularization notations for protocols, so everything ends up being mixed together. We've already seen that here in this chapter: the DH parameter size negotiation, DH key exchange, and authentication are all merged together. This is not just a combination of loose parts; the specification and implementation mash them all together. It is rather like a really bad and complex computer program without any modularization. We all know what that leads to, but we've developed modularization techniques to deal with program complexity. Unfortunately, we lack modularization techniques for protocols, and developing such modularization techniques may not be an easy task.

14.11 A Gentle Warning

We've tried to make the design of the protocol look as easy as possible. Please don't be fooled by this. Protocol design is fiendishly difficult, and requires a lot of experience. Even with lots of experience, it is very easy to get wrong. Though we've tried very hard to get everything right in this book, there is always a possibility that the key negotiation protocol we designed here is wrong. It is important to have professional paranoia and treat all protocols with skepticism.

14.12 Key Negotiation from a Password

So far, we've assumed there is an authentication system to base the key negotiation on. In many situations, all you have is a password. You could just use a MAC keyed with the password to run this protocol, but there is a problem: given a transcript from this protocol (acquired by eavesdropping on the communications), you can test for any particular password. Just compute the authentication value and see whether it is correct.

The problem with passwords is that people don't choose them from a very large set. There are programs that search through all likely passwords. Ideally we'd like a key negotiation protocol where an eavesdropper cannot perform an offline dictionary attack.

Such protocols exist; probably the best-known example is SRP [129]. They provide a significant security improvement. We do not describe password-based key negotiation protocols here. If you are interested in using a password-based key negotiation protocol, you should also be aware of the fact that there are multiple patents in this area.

14.13 Exercises

Exercise 14.1 In Section 14.5, we stated that a property of the protocol could result in providing erroneous information to investigating administrators. Give a concrete scenario where this could be a problem.

Exercise 14.2 Suppose Alice and Bob implement the final protocol in Section 14.7. Could an attacker exploit a property of this protocol to mount a denial-of-service attack against Alice? Against Bob?

Exercise 14.3 Find a new product or system that uses (or should use) a key negotiation protocol. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on the security and privacy issues surrounding the key negotiation protocol.

CHAPTER 15

Implementation Issues (II)

The key negotiation protocol we designed leads to some new implementation issues.

15.1 Large Integer Arithmetic

The public-key computations all depend on large integer arithmetic. As we already mentioned, it is not easy to implement large integer arithmetic properly.

Large integer routines are almost always platform-specific in one way or another. The efficiencies that can be gained by using platform-specific features are just too great to pass up. For example, most CPUs have an add-with-carry operation to implement addition of multiword values. But in C or almost any other higher-level language, you cannot access this instruction. Doing large integer arithmetic in a higher-level language is typically several times slower than an optimized implementation for the platform. And these computations also form the bottleneck in public-key performance, so the gain is too important to ignore.

We won't go into the details of how to implement large integer arithmetic. There are other books for that. Knuth [75] is a good start, as is Chapter 14 of the *Handbook of Applied Cryptography* [90]. To us, the real question is how to *test* large integer arithmetic.

In cryptography, we have different goals from those of most implementers. We consider a failure rate of 2^{-64} (about one in 18 million trillion) unacceptable, whereas most engineers would be very happy to achieve this. Many programmers seem to think that a failure rate of 2^{-20} (about one in a million) is acceptable, or even good. We have to do much better, because we're working in an adversarial setting.

Most block ciphers and hash functions are comparatively easy to test.[1] Very few implementation bugs lead to errors that are hard to find. If you make a mistake in the S-box table of AES, it will be detected by testing a few AES encryptions. Simple, random testing exercises all the data paths in a block cipher or hash function and quickly finds all systematic problems. The code path taken does not depend on the data provided, or only in a very limited way. Any decent test set for a symmetric primitive will exercise all the possible flows of control in the implementation.

Large integer arithmetic is different. The major difference is that in most implementations, the code path depends on the data. Code that propagates the last carry is used only rarely. Division routines often contain a piece of code that is used only once every 2^{32} divisions or even once every 2^{64} divisions. A bug in this part of the code will not be found by random testing. This problem gets worse as we use larger CPUs. On a 32-bit CPU, you could still run 2^{40} random test cases and expect that each 32-bit word value had occurred in each part of the data path. But this type of testing simply does not work for 64-bit CPUs.

The consequence is that you have to do extremely careful testing of your large integer arithmetic routines. You have to verify that every code path is in fact taken during the tests. To achieve this, you have to carefully craft test vectors: something that takes some care and precision. Not only do you have to use every code path, but you also need to run through all the boundary conditions. If there is a test with $a < b$, then you should test this for $a = b - 1$, $a = b$, and $a = b + 1$, but of course only as far as these conditions are possible to achieve.

Optimization makes this already bad situation even worse. As these routines are part of a performance bottleneck, the code tends to be highly optimized. This in turn leads to more special cases, more code path, etc., all of which make the testing even harder.

A simple arithmetic error can have catastrophic security effects. Here is an example. While Alice is computing an RSA signature, there is a small error in the exponentiation modulo p but not modulo q. (She is using the CRT to speed up her signature.) Instead of the proper signature σ, she sends out $\sigma + kq$ for some value of k. (The result Alice gets is correct modulo q but wrong modulo p, so it must be of the form $\sigma + kq$.) The attacker knows $\sigma^3 \bmod n$, which is the number Alice is computing a root of, and which only depends on the message. But $(\sigma + kq)^3 - \sigma^3$ is a multiple of q, and taking the greatest common divisor of this number and n will reveal q and thus the factorization of n. Disaster!

[1] Two notable exceptions are IDEA and MARS, which often use separate code for special cases.

So what are we to do? First of all, don't implement your own large integer routines. Get an existing library. If you want to spend any time on it, spend your time understanding and testing the existing library. Second, run really good tests on your library. Make sure you test every possible code path. Third, insert additional tests in the application. There are several techniques you can use.

By the way, we've discussed the testing problem in terms of different code paths. Of course, to avoid side-channel attacks (see Sections 8.5 and 15.3), the library should be written in such a way that the code path doesn't change depending on the data. Most of the code path differences that occur in large integer arithmetic can be replaced by masking operations (where you compute a mask from the ''if'' condition and use that to select the right result). This addresses the side-channel problem, but it has the same effect on testing. To test a masked computation, you have to test both conditions, so you have to generate test cases that achieve both conditions. This is exactly the testing problem we mentioned. We merely explained it in terms of code paths, as that seems to be easier to understand.

15.1.1 Wooping

The technique we describe in this section has the rather unusual name of *wooping*. During an intense discussion between David Chaum and Jurjen Bos, there was a sudden need to give a special verification value a name. In the heat of the moment, one of them suggested the name ''woop,'' and afterward the name stuck to the entire technique. Bos later described the details of this technique in his PhD thesis [18, ch. 6], but dropped the name as being insufficiently academic.

The basic idea behind wooping is to verify a computation modulo a randomly chosen small prime. Think of it as a cryptographic problem. We have a large integer library that tries to cheat and give us the wrong results. Our task is to check whether we get the right results. Just checking the results with the same library is not a good idea, as the library might make consistent errors. Using the wooping technique, we can verify the library computations, as long as we assume that the library is not actually malicious in the sense that it tries very hard to corrupt our verification computations.

First, we generate a relatively small random prime t, on the order of 64–128 bits long. The value of t should not be fixed or predictable, but that is what we have a PRNG for. The value of t is kept secret from all other parties. Then, for every large integer x that occurs in the computations, we also keep $\tilde{x} := (x \bmod t)$. The $\tilde{x}$ value is called the *woop* of x. The woop values have a fixed size, and are generally much smaller than the large integers. Computing the woop values is therefore not a great extra cost.

So now we have to keep woop values with every integer. For any input x to our algorithm, we compute $\tilde{x}$ directly as $x \bmod t$. For all our internal computations, we shadow the large integer computations in the woop values to compute the woop of the result without computing it from the large integer result.

A normal addition computes $c := a + b$. We can compute $\tilde{c}$ using $\tilde{c} = \tilde{a} + \tilde{b} \pmod{t}$. Multiplication can be handled in the same way. We could verify the correctness of $\tilde{c}$ after every addition or multiplication by checking that $c \bmod t = \tilde{c}$, but it is more efficient to do all of the checks at the very end.

Modular addition is only slightly more difficult. Instead of just writing $c = (a + b) \bmod n$, we write $c = a + b + k \cdot n$ where k is chosen such that the result c is in the range $0, \ldots, n - 1$. This is just another way to write the modulo reduction. In this case, k is either 0 or -1, assuming both a and b are in the range $0, \ldots, n - 1$. The woop version is $\tilde{c} = (\tilde{a} + \tilde{b} + \tilde{k} \cdot \tilde{n}) \bmod t$. Somewhere inside the modulo addition routine, the value of k is known. All we have to do is convince the library to provide us with k, so we can compute $\tilde{k}$.

Modular multiplication is somewhat more difficult to do. Again we have to write $c = a \cdot b + k \cdot n$; and to compute $\tilde{c} = \tilde{a} \cdot \tilde{b} + \tilde{k} \cdot \tilde{n} \pmod{t}$, we need $\tilde{a}$, $\tilde{b}$, $\tilde{n}$, and $\tilde{k}$. The first three are readily available, but $\tilde{k}$ will have to be teased out of the modular multiplication routine in some way. That can be done when you create the library, but it is very hard to retrofit to an existing library. A generic method is to first compute $a \cdot b$, and then divide that by n using a long division. The quotient of the division is the k we need for the woop computation. The remainder is the result c. The disadvantage of this generic method is that it is significantly slower.

Once you can keep the woop value with modular multiplications, it is easy to do so with the modular exponentiation as well. Modular exponentiation routines simply construct the modular exponentiation from modular multiplications. (Some use a separate modular squaring routine, but that can be extended with a woop value just like the modular multiplication routine.) Just keep a woop value with every large integer, and have every multiplication compute the woop of the result from the woops of the inputs.

The woop-extended algorithms compute the woop value of the results based on the woop values of the inputs: if one or more of the woop inputs is wrong, the woop output is almost certainly wrong, too. So once a woop value is wrong, the error propagates to the final result.

We check the woop values at the end of our computation. If the result is x, all you have to do is check that $(x \bmod t) = \tilde{x}$. If the library made any mistakes, the woop values will not match. We assume the library doesn't carefully craft its mistakes in a way that depends on the value t that we chose. After all, the library code was fixed long before we chose t, and the library code is not under control of the attacker. It is easy to show that any error the library might make will be caught by the overwhelming majority of t values. So adding a woop verification to an existing library gives us an extremely good verification of the computations.

What we really want is a large integer library that has a built-in woop verification system. But we don't know of one.

How large should your woop values be? That depends on many factors. For random errors, the probability of the woop value not detecting the error is about $1/t$. But nothing is ever random in our world. Suppose there is a software error in our library. We've got to assume that the attacker knows this. She can choose the inputs to our computation, and not only trigger the error but also choose the difference that the error induces. This is why t must be a random, secret number; without knowing t, the attacker cannot target the error in the final result to a difference that won't be caught by our wooping.

So what would you do if you were an attacker? You would try to trigger the error, of course, but you would also try to force the difference to be zero modulo as many t's as you can. The simplest countermeasure is to require that t be a prime. If the attacker wants to cheat modulo 16 different 64-bit primes, then she will need to carefully select at least $16 \cdot 64 = 1024$ bits of the input. As most computations have a limited number of input bits that can be chosen by an attacker, this limits the probability of success of the attack.

Larger values for t are better. There are so many more primes of larger sizes that the probability of success rapidly disappears for the attacker. If we were to keep to our original goal of 128-bit security, we would need a 128-bit t, or something in that region.

Woop values are not the primary security of the system; they are only a backup. If a woop verification ever fails, we know we have a bug in our software that needs to be fixed. The program should abort whatever it is doing and report a fatal error. This also makes it much harder for an attacker to perform repeated attacks on the system. Therefore, we suggest using a 64-bit random prime for t. This will reduce the overhead significantly, compared to using a 128-bit prime, and in practice, it is good enough. If you cannot afford the 64-bit woop, a 32-bit woop is better than nothing. Especially on most 32-bit CPUs, a 32-bit woop can be computed very efficiently, as there are direct multiplication and division instructions available.

If you ever have a computation where the attacker could provide a large amount of data, you should check the intermediate woop values as well. Each check is simple: $(x \bmod t) \overset{?}{=} \tilde{x}$. By checking intermediate values that depend on only a limited number of bits from the attacker, you make it harder for her to cheat the woop system.

Using a large integer library with woop verifications is our strong preference. It is a relatively simple method of avoiding a large number of potential security problems. And we believe it is less work to add woop verification to the library once than to add application-specific verifications to each of the applications that uses the library.

15.1.2 Checking DH Computations

If you don't have a woop-enabled library, you will have to work without one. The DH protocol we described already contains a number of checks; namely, that the result should not be 1 and that the order of the result should be q. Unfortunately, the checks are not performed by the party doing the computation, but by the party receiving the result of the computation. In general, you don't want to send out any erroneous results, because they could leak information, but in this particular case it doesn't seem to do much harm. If the result is erroneous, the protocol will fail in one way or another, so the error will be noticed. The protocol safety only breaks down when your arithmetic library returns x when asked to compute g^x, but that is a type of error that normal testing is very likely to find.

Where needed, we would probably run DH on a library without woop-verification. The type of very rare arithmetical errors that we worry about here are unlikely to reveal x from a g^x computation. Any other mistake seems harmless, especially since DH computations have no long-term secrets. Still, we prefer to use a wooping library wherever possible, just to feel safe.

15.1.3 Checking RSA Encryption

RSA encryption is more vulnerable and needs extra checks. If something goes wrong, you might leak the secret that you are encrypting, or even your secret key.

If woop-verification is not available, there are two other methods to check the RSA encryption. Suppose the actual RSA encryption consists of computing $c = m^5 \bmod n$, where m is the message and c the ciphertext. To verify this, we could compute $c^{1/5} \bmod n$ and compare it to m. The disadvantages are that this is a very slow verification of a relatively fast computation, and that it requires knowledge of the private key, which is typically not available when we do RSA encryption.

Probably a better method is to choose a random value z and check that $c \cdot z^5 = (m \cdot z)^5 \bmod n$. Here we have three computations of fifth powers: the $c = m^5$; the computation of z^5; and then finally the check that $(mz)^5$ matches $c \cdot z^5$. Random arithmetical errors are highly likely to be caught by this verification. By choosing a random value z, we make it impossible for any attacker to target the error-producing values. In our designs, we only use RSA encryption to encrypt random values, so the attacker cannot do any targeting at all.

15.1.4 Checking RSA Signatures

RSA signatures are really easy to check. The signer only has to run the signature verification algorithm. This is a relatively fast verification, and arithmetical errors are highly likely to be caught. Every RSA signature computation should verify the results by checking the signature just produced. There is no excuse not to do this.

15.1.5 Conclusion

Let us make something quite clear. The checks we have been talking about are *in addition* to the normal testing of the large integer libraries. They do not replace the normal testing that any piece of software, especially security software, should undergo.

If any of these checks ever fail, you know that your software just failed. There is not much you can do in that situation. Continuing with the work you are doing is unsafe; you have no idea what type of software error you have. The only thing you can really do is log the error and abort the program.

15.2 Faster Multiplication

There are a lot of ways in which you can do a modulo multiplication faster than a full multiply followed by a long division. If you have to do a lot of multiplications, then Montgomery's method [93] is the most widely used one; see [39] for a readable description.

The basic idea behind Montgomery's method is a technique to compute $(x \bmod n)$ for some x much larger than n. The traditional ''long division'' method is to subtract suitable multiples of n from x. Montgomery's idea is simpler: divide x repeatedly by 2. If x is even, we divide x by two by shifting the binary representation one bit to the right. If x is odd, we first add n (which does not change the value modulo n, of course) and then divide the even result by 2. (This technique only works if n is odd, which is always the case in our systems. There is a simple generalization for even values of n.) If n is k bits long, and x is not more than $(n-1)^2$, we perform a total of k divisions by 2. The result will always be in the interval $0, \ldots, 2n-1$, which is an almost fully reduced result modulo n.

But wait! We've been dividing by 2, so this gives us the wrong answer. Montgomery's reduction does not actually give you $(x \bmod n)$, but rather $x/2^k \bmod n$ for some suitable k. The reduction is faster, but you get an extra factor of 2^{-k}. There are various tricks to deal with this extra factor.

One bad idea is to simply redefine your protocol to include an extra factor 2^{-k} in the computations. This is bad because it mixes different levels. It modifies the cryptographic protocol specification to favor a particular implementation technique. Perhaps you'll want to implement the protocol on another platform later, where you'll find that you don't want to use Montgomery multiplication at all. (Maybe that platform is slow but has a large integer coprocessor that performs modular multiplication directly.) In that case, the 2^{-k} factors in the protocol become a real hindrance.

The standard technique is to change your number representation. A number x is represented internally by $x \cdot 2^k$. If you want to multiply x and y, you do a Montgomery multiplication on their respective representations. You get $x \cdot 2^k \cdot y \cdot 2^k$, but you also get the extra 2^{-k} factor from the Montgomery reduction, so the final result is $x \cdot y \cdot 2^k \bmod n$, which is exactly the representation of xy. The overhead cost of using Montgomery reduction therefore consists of the cost of converting the input numbers into the internal representation (multiplication by 2^k) and the cost of converting the output back to the real result (division by 2^k). The first conversion can be done by performing a Montgomery multiplication of x and ($2^{2k} \bmod n$). The second conversion can be done by simply running the Montgomery reduction for another k bits, as that divides by 2^k. The final result of a Montgomery reduction is not guaranteed to be less than n, but in most cases it can be shown to be less than $2n - 1$. In those situations, a simple test and an optional subtraction of n will give the final correct result.

In real implementations, the Montgomery reduction is never done on a bit-by-bit basis, but per word. Suppose the CPU uses w-bit words. Given a value x, find a small integer z such that the least significant word of $x + zn$ is all zero. You can show that z will be one word, and can be computed by multiplying the least significant word of x with a single word constant factor that only depends on n. Once the least significant word of $x + zn$ is zero, you divide by 2^w by shifting the value a whole word to the right. This is much faster than a bit-by-bit implementation.

15.3 Side-Channel Attacks

We discussed timing attacks and other side-channel attacks briefly back in Section 8.5. The main reason we were brief there is not because these attacks are benign. It is, rather, that timing attacks are also useful against public-key computations and we now consider both.

Some ciphers invite implementations that use different code paths to handle special situations. IDEA [84, 83] and MARS [22] are two examples. Other ciphers use CPU operations whose timing varies depending on the data they process. On some CPUs, multiplication (used by RC6 [108] and MARS) or data-dependent rotation (used by RC6 and RC5 [107]) has an execution time that depends on the input data. This can enable timing attacks. The primitives we have been using in this book do not use these types of operations. AES is, however, vulnerable to cache timing attacks [12], or attacks that exploit the difference in the amount of time it takes to retrieve data from cache rather than main memory.

Public-key cryptography is also vulnerable to timing attacks. Public-key operations often have a code path that depends on the data. This almost always leads to different processing times for different data. Timing information, in turn, can lead to attacks. Imagine a secure Web server for e-commerce. As part of the SSL negotiations, the server has to decrypt an RSA message chosen by the client. The attacker can therefore connect to the server, ask it to decrypt a chosen RSA value, and wait for the response. The exact time it takes the server to respond can give the attacker important information. Often it turns out that if some bit of the key is one, inputs from set *A* are slightly faster than inputs from set *B*, and if the key bit is zero, there is no difference. The attacker can use this difference to attack the system. She generates millions of queries from both sets *A* and *B*, and tries to find a statistical difference in the response times to the two groups. There might be many other factors that influence the exact response time, but she can average those out by using enough queries. Eventually she will gather enough data to measure whether the response times for *A* and *B* are different. This gives the attacker one bit of information about the key, after which the attack can proceed with the next bit.

This all sounds far-fetched, but it has been done in the laboratory, and could very well be done in practice [21, 78].

15.3.1 Countermeasures

There are several ways to protect yourself against timing attacks. The most obvious one is to ensure that every computation takes a fixed amount of time. But this requires the entire library be designed with this goal in mind. Furthermore, there are sources of timing differences that are almost impossible to control. Some CPUs have a multiplication instruction that is faster for some values than for others. Many CPUs have complicated cache systems, so as soon as your memory access pattern depends on nonpublic data, the cache delays might introduce a timing difference. It is almost impossible to rid operations of all timing differences. We therefore need other solutions.

An obvious idea is to add a random delay at the end of each computation. But this does not eliminate the timing difference. It just hides it in the noise of the delay. An attacker who can take more samples (i.e., get your machine to do more computations) can average the results and hope to average out the random delay that was added. The exact number of tries the attacker needs depends on the magnitude of the timing difference the attacker is looking for, and the magnitude of the random delay that is added. In real timing attacks, there is almost always a lot of noise, so any attacker who tries a timing attack is already doing the averaging. The only question is the ratio of the signal to the noise.

A third method is to make an operation constant-time by forcing it to last a standardized amount of time. During development, you choose a duration d that is longer than the computation will ever take. You then mark the time t at which the computation started, and after the computation you wait until time $t + d$. This is slightly wasteful, but it is not too bad. We like this solution, but it only provides protection against pure timing attacks. If the attacker can listen in on the RF radiation that your machine emits or measure the power consumption, the difference between the computation and the delay is probably detectable, which in turn allows timing attacks as well as other attacks. Still, an RF-based attack requires the attacker to be physically close to the machine. That enormously reduces the threat, compared to timing attacks that can be done over the Internet.

You can also use techniques that are derived from blind signatures [78]. For some types of computations they can hide (almost) all of the timing variations.

There is no perfect solution to the problem of timing attacks. It is simply not possible to secure the computers you can buy against a really sophisticated attack such as an RF-based one. But although you can't create a perfect solution, you can get a reasonably good one. Just be really careful with the timing of your public-key operations. An even better solution than just making your public-key operations fixed-time is to make the entire transaction fixed-time, using the technique mentioned above. That is, you not only make the public-key operation fixed-time, but you also fix the time between when the request comes in and the response goes out. If the request comes in at a time t, you send the response at time $t + C$ for some constant C. But to make sure you never leak any timing information, you had better be sure that the response will be ready at time $t + C$. To guarantee this, you will probably have to limit the frequency at which you accept incoming requests to some fixed upper bound.

15.4 Protocols

Implementing cryptographic protocols is not that different from implementing communication protocols. The simplest method is to maintain the state in the program counter, and simply perform each of the steps of the protocol in turn. Unless you use multithreading, this stops everything else in the program while you wait for an answer. As the answer might not be forthcoming, this is often a bad idea.

A better solution is to keep an explicit protocol state, and update the state each time a message arrives. This message-driven approach is slightly more work to implement, but it provides much more flexibility.

15.4.1 Protocols Over a Secure Channel

Most cryptographic protocols are executed over insecure channels, but sometimes you run a cryptographic protocol over a secure channel. This makes sense in some situations. For example, each user has a secure channel to a key distribution center; the key distribution center uses a simple protocol to distribute keys to the users to allow them to communicate to each other. (The Kerberos protocol does something like this.) If you are running a cryptographic protocol with a party you have already exchanged a key with, you should use the full secure channel functionality. In particular, you should implement replay protection. This is very easy to do, and it prevents a large number of attacks on the cryptographic protocol.

Sometimes the secure channel allows the protocol to use shortcuts. For example, if the secure channel provides replay protection, the protocol itself does not have to. Still, the old modularization rule states that the protocol should minimize its dependency on the secure channel.

For the rest of our protocol implementation discussion, we are going to assume that the protocol runs over an insecure channel. Some of the discussion does not quite apply to the secure channel case, but the solutions can never hurt.

15.4.2 Receiving a Message

When a protocol state receives a message, there are several checks that have to be made. The first is to see if the message belongs to the protocol at all. Each message should start with the following fields:

Protocol identifier. Identifies exactly which protocol and protocol version this is. Version identifiers are important.

Protocol instance identifier. Identifies which instance of the protocol this message belongs to. Perhaps Alice and Bob are running two key negotiation protocols simultaneously, and we don't want to confuse the two runs.

Message identifier. Identifies the message within the protocol. The easiest method is to simply number them.

Depending on the situation, some of these identifiers can be implicit. For example, for protocols that run over their own TCP connection, the port number and its associated socket uniquely identify the protocol instance on the local machine. The protocol identifier and version information only need to be exchanged once. Note that it is important to exchange them at least once to make sure they get included in any authentication or signature used in the protocol.

After checking the protocol identifier and instance identifier, we know which protocol state to send the message to. Let us assume that the protocol state has just received message $n - 1$ and is expecting to receive message n.

If the received message is indeed message n, things are easy. Just process it as the protocol rules specify. But what if it has a different number?

If the number is larger than n or less than $n - 1$, something very weird is going on. Such a message should not have been generated, and therefore must be a forgery of some kind. You must ignore the contents of the forged message.

If the received message is message $n - 1$, the reply message you sent might not have arrived. At least, this could happen if you are running the protocol over an unreliable transport system. As we want to minimize dependencies on other parts of the system, this is exactly what we will assume.

First of all, check that the newly received message $n - 1$ is absolutely identical to the previous message with number $n - 1$ that you received. If they are different, you must ignore the new message. Sending a second answer will break the security of many protocols. If the messages are identical, just resend your reply. Of course, the version that you resend must be identical to the previous reply that you sent.

If you ignored the received message due to any of these rules, you have a second decision to make. Should you abort the protocol? The answer depends to some extent on the application and situation. If you have been running a protocol over a secure channel, something is very wrong. Either the secure channel is compromised, or the party you are talking to is misbehaving. In either case, you should abort the protocol and the channel. Simply delete the protocol state and the channel state, including the channel key.

If you're running the protocol over an insecure channel, then any of the ignored messages could be from an attacker trying to interfere with the protocol. Ideally, you would ignore the attacker's messages and just complete the protocol. This is, of course, not always possible. For example, if the attacker's forged message $n - 1$ reaches you first, you will send a reply. If you later receive the ''real'' message $n - 1$, you are forced to ignore it. There is no recovery from this situation, as you cannot safely send a second reply. But you have no idea which of the two messages $n - 1$ you received was the real one, so in order to have the best chance of completing the protocol successfully, you should just log the second message $n - 1$ as an error and continue as usual. If the message you replied to came from the attacker, the protocol will fail eventually because cryptographic protocols are specifically designed to prevent attackers from successfully completing the protocol with one of the participants.

15.4.3 Timeouts

Any protocol run includes timeouts. If you don't get a response to a message within a reasonable time, you can resend your last message. After a few resends, you have to give up. There is no point continuing with a protocol when you cannot communicate with the other party.

The easiest way to implement timeouts is to send timing messages to the protocol state. You can use timers explicitly set by the protocol, or use timing messages that are sent every few seconds or so.

One well-known attack is to send lots of ''start-of-protocol'' messages to a particular machine. Each time you receive a start-of-protocol message, you initialize a new protocol execution state. After receiving a few million of these, the machine runs out of memory, and everything stops. A good example is the SYN flood attack. There is no easy method to protect yourself against these flooding attacks in general, especially in the age of botnets and distributed attacks, but they do show that it is important to delete old protocol states. If a protocol is stalled for too long, you should delete it.

The proper timing for resends is debatable. In our experience, a packet on the Internet either arrives within a second or so, or is lost forever. Resending a message if you haven't received a reply within five seconds seems reasonable. Three retries should be enough; if the message loss rate is so high that you lose four consecutive messages spread out over 15 seconds, you're not going to get a whole lot done over that connection. We prefer to inform the user of a problem after 20 seconds, rather than require the user to sit there and wait for a minute or two.

15.5 Exercises

Exercise 15.1 Consider all the operations a computer might perform with a cryptographic key. Which ones might have timing characteristics that could leak information about the key?

Exercise 15.2 Find a new product or system that manipulates secret data. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on issues surrounding side-channel attacks.

Part IV

Key Management

In This Part

CHAPTER

16

The Clock

Before we begin the detailed discussion of key management in the next chapter, we need to discuss one more primitive function: the clock. At first glance, this is a decidedly *un*-cryptographic primitive, but because the current time is often used in cryptographic systems, we need a reliable clock.

16.1 Uses for a Clock

There are several cryptographic uses for a clock. Key management functions are often linked to deadlines. The current time can provide both a unique value and a complete ordering of events. We will discuss each of these uses in more detail.

16.1.1 Expiration

In many situations, we want to limit the validity period of a document. In the real world, we often see limited validity periods too. Checks, open airline tickets, vouchers, coupons, and even copyrights all have limited validity periods. The standard way to limit the validity period of a digital document is to include the expiration time in the document itself. But to check whether a document has expired, we need to know the current time. Hence, the need for a clock.

16.1.2 Unique Value

Another useful function of a clock—if its resolution is high enough—is to provide a unique value for a single machine. We've been using nonces in several places. The important property of a nonce is that any single value is never used twice, at least within some defined scope. Sometimes the scope is limited, such as the nonce we use in the secure channel, and the nonce can be generated using a counter. In other situations, the nonce has to be unique across reboots of the computer. There are two generic ways of generating nonce values. The first is to use the current time of the clock with some mechanism to ensure you never use the same time code twice. The second is to use a PRNG, which we discussed in some detail in Chapter 9. The disadvantage of using a random nonce is that it needs to be rather large. To achieve a security level of 128 bits, we would need to use a 256-bit random nonce. Not all primitives support such a large nonce. Furthermore, a PRNG can be very hard to implement on certain platforms. A reliable clock is an attractive alternative way to generate nonces.

16.1.3 Monotonicity

One of the useful properties of time is that it always keeps going forward. It never stops or reverses. There are cryptographic protocols that use this property. Including the time in a cryptographic protocol prevents an attacker from trying to pass off old messages as ones that belong to the current protocol. After all, the time encoded in those messages is not within the time-span of the current protocol.

Another really important application of the clock is auditing and logging. In any kind of transaction system, it is very important to keep a log of what happened. If there is ever a dispute, the audit logs provide the necessary data to trace the exact sequence of events. Including the time in each logging event is important; without a time stamp, it is very hard to know which events belong to the same transaction, and in which order the events occurred. As well-synchronized clocks do not deviate significantly from each other, the time stamps allow events from different logs on different machines to be correlated.

16.1.4 Real-Time Transactions

Our next example comes from Niels's work on electronic payment systems. To support real-time payments, the bank needs to run a real-time financial transaction system. To allow an audit to be performed, there should be a clear sequence of transactions. Given two transactions *A* and *B*, it is important to know which of the two was performed first, because the result of one of them could depend on whether the other one has been performed yet or not. The simplest way to record this sequence is to give a time stamp to each transaction. This only works if you have a reliable clock.

An unreliable clock might give the wrong time. There is little harm done if the clock accidentally moves backward: it is easy to check that the current time is greater than the time stamp of the last transaction performed. There is a problem, however, if the clock moves forward. Suppose half an hour's worth of transactions were done with the clock set in 2020. You can't just change the time stamps of those transactions; it is not acceptable to modify financial records by hand. You can't perform any new transactions with a time stamp before 2020 because that would upset the order of the transactions, which is determined by the time stamp. There are solutions to this problem, but a reliable clock is certainly preferable.

16.2 Using the Real-Time Clock Chip

Most desktop computers contain a real-time clock chip and a small battery. This is really a small digital watch built into your machine. This is how your computer knows what time it is when you start it up in the morning. Why not simply use this clock time?

The real-time clock chip is adequate for normal use, but in a security system we have to impose higher standards. As part of the security system, the clock should give the correct time even if an enemy tries to manipulate the clock. A second reason is the consequences of a failed clock. For normal uses, a clock that shows the wrong time is irritating but not dangerous. If the clock is part of the security system, clock failures can result in much greater damage.

The real-time clocks in typical hardware are not as reliable and secure as we need. We have personally experienced several real-time clock chip failures in the last decade. Moreover, those failures were spontaneous, without a malicious attacker trying to corrupt the clock. Most failures are simple. On an old machine, the battery runs low and the clock stops or resets to 1980. Or one day you start the machine and the clock has been set to some date in 2028. Sometimes a clock just gradually drifts faster or slower than the real time.

Apart from accidental errors in real-time clocks, we have to consider active attacks. Someone might try to manipulate the clock in some way. Depending on the details of the computer, changing the clock time can be easy or hard. On some systems, you need special administrator access to change the clock; others have clocks that can be changed by anyone.

16.3 Security Dangers

There are several types of attack that can be mounted against a system with a clock.

16.3.1 Setting the Clock Back

Suppose the attacker can set the clock to some arbitrary time in the past. This might allow all kinds of mischief. The machine mistakenly believes it lives in the past. Maybe an attacker once had access to some data because he was a temporary employee, but that access has now expired. With the wrong time on the clock, a computer might now allow this ex-employee access to the sensitive data. This problem has the potential of occurring every time some access is revoked from a user. Setting the clock back might restore his access, depending on how the rest of the system was designed.

Another interesting avenue of attack is automated tasks. Suppose an HR computer makes salary payments automatically at the end of the month, using direct deposit. Automated tasks like this are initiated by a program that checks the time and has a list of tasks to perform. Repeatedly setting the clock back can trigger the tasks repeatedly. If the task is set to start at midnight, the attacker sets the clock to 23:55 (11:55 pm), and waits for the task to be started. After the task finishes, the attacker sets the clock back again. He can repeat this until the bank balance of the company is exhausted.

Another problem occurs in financial systems. It is important to get the time of a transaction right because interest computations give different results depending on when a transaction was performed. If you carry a large balance on your credit card, it would be very advantageous to convince your bank's computer that the online payment you just made actually happened six months ago, and avoid paying six months of interest.

16.3.2 Stopping the Clock

Every designer lives with the instinctive understanding that time does not stand still. It is an unspoken assumption, too obvious to even document. The systems they design rely on time behaving normally. But if the clock is stopped, time appears to stand still. Things might not get done. And many systems behave in unpredictable ways.

The simple problems are things like getting the wrong time on audit logs and reports. The exact time of a transaction can have large financial consequences, and sending out formal paperwork with the wrong date and time on it can lead to serious complications.

Other problems might occur with real-time displays. Maybe the GUI programmer uses a simple system to display the current situation at the real-time broker. Every ten seconds, he refreshes the display with the latest data. But not all reports of financial transactions arrive with the same speed, due to various delays. Just reporting the latest data that was received is going to give an inconsistent view of the financial situation. Maybe one part of a transaction has already been reported, but the other half has not. The money could show up on the bank balance before the shares move from the stock holdings. Accountants do not like to get reports where the numbers do not add up.

So the programmer does something clever. Each report of a financial transaction is time-stamped and stored in a local database. To display a consistent report, he takes a particular point in time and reports the financial situation at that point in time. For example, if the slowest system has a five-second delay in reporting, he displays the financial situation of seven seconds ago. It increases the display delay a bit, but it guarantees a consistent report. That is, until the clock is stopped. Suddenly, the display reports the same situation over and over again: the situation of seven seconds ago relative to the (failed) clock. Oops!

16.3.3 Setting the Clock Forward

Setting the clock forward makes the computer think it lives in the future. This leads to simple denial-of-service attacks. With the clock set four years in the future, all credit card transactions are suddenly refused because all the cards have expired. You cannot book online airline tickets either, because there is no airline schedule out yet for those dates.

Substantial bidding at eBay auctions happens in the last seconds. If you can move eBay's clock forward just a little bit, you cut out many of the other bidders and can obtain the item at a cheaper price.

A friend of ours had a problem of this nature with his billing system. Due to a software error, the clock jumped ahead by about 30 years. The billing system started to bill all his customers for 30 years of unpaid bills. In this case, it didn't result in a direct financial loss, but it could have been different if he had been using automatic debits from bank accounts or credit cards. It certainly wasn't good customer relations.

There are also direct security risks involved with clocks set to a future time. There are many situations in which certain data is to be kept secret until a specific time, and made public after that time. In an automated system, setting the clock forward provides access to the data. If this is a profit warning for a publicly traded company, quite a bit of profit can be made from accessing this data prematurely.

16.4 Creating a Reliable Clock

We don't have a simple solution to the clock problem. We can suggest some ideas and techniques, but the details depend too much on the exact working environment and the risk analysis for us to be able to give universal answers. Our goal here is therefore multifold. We wish to increase understanding, encourage minimal reliance on a clock, identify key issues to consider, and provide an example for how to think about building a reliable clock.

Most computers have, or can implement, a counter of some sort that starts when the computer is booted. This might be a count of the number of CPU clock cycles, a refresh counter, or something similar. This counter can be used to keep track of the time since the last reboot. It is not a clock, as it provides no information about what the actual time is, but it can be used to measure elapsed time between events as long as both events happened since the last reboot.

The main use for this type of counter, at least in relation to our clock problem, is to check for accidental errors in the real-time clock. If the real-time clock doesn't run properly, it will show discrepancies with the clock counter. This is simple to test for, and provides some warning for certain error modes of the clock chip. Note that the correspondence between clock time and counter value has to be modified if the clock time is changed by an authorized user.

A second simple check is to keep track of the time of the last shutdown, or the last time data was written to disk. The clock should not jump backwards. If your machine suddenly boots in the year 1980, it is obvious that something is wrong. It is also possible to stop the clock jumping forward too much. Most computers are booted at least once a week. Perhaps you should get the user to confirm the correct date if the machine hasn't been booted for a week.[1] That would catch the case of the clock jumping more than a week forward. Of course, we're assuming here that the user is not the adversary.

There are other methods of checking the time. You could ask a time server on the Internet or an intranet. There are widely used time synchronization protocols such as NTP [92] or SNTP [91]. Some of these protocols even provide for authentication of the time data so an attacker cannot spoof the machine. Of course, the authentication requires some kind of keying infrastructure. The shared key with the time server could be a manually configured symmetric key, but manually configuring keys is a hassle. It can also be done using a PKI, but as we will see in Chapter 18, most PKI systems need a clock, which results in a chicken-and-egg problem. Be careful if you rely on the cryptographic protection offered by a clock synchronization protocol. The security of your entire system could hinge on the security of the protocol.

[1] As most users will hit the OK button without bothering to look at the message, it is probably better to ask the user to enter the current date, without showing him what the clock-date is.

16.5 The Same-State Problem

This brings us to a serious problem that you find on some hardware platforms. We're talking here about small embedded computers—something like a door lock or a remote smart card reader. These typically consist of a small CPU, a small amount of RAM, nonvolatile memory (e.g., flash) to store the program, some communication channels, and further task-specific hardware.

You will notice that a real-time clock is often not included. Adding a real-time clock requires an extra chip, an oscillator crystal, and most importantly, a battery. Apart from the extra cost, adding a battery complicates the device. You now have to worry about the battery running out. Batteries can be sensitive to temperature fluctuations, and the toxic chemicals in some batteries can even lead to problems with shipping the hardware. For all of these reasons, many small computers do not have a real-time clock.

Every time such a small computer is booted, it starts in exactly the same state. It reads the same program from the same nonvolatile memory, initializes the hardware, and starts operations. As this is a book about cryptography, we will assume that some kind of cryptographic protocol is used in the communication with other pieces of the system. But here is the problem: without a clock or hardware random number generator, the embedded system will always repeat the exact same behavior. Suppose the attacker waits until the gate computer needs to open the gate because a truck needs to pass through. She reboots the gate computer just before the gate needs to open (e.g., by interrupting the power supply momentarily). After some initialization procedures, the central system will command the gate computer to open the gate via the communication channel. The next day, the attacker reboots the gate computer again, and sends exactly the same messages as were sent the first time. As the gate computer starts in the same state and sees the same inputs, it behaves the same and opens the gate. This is bad. Note that it doesn't matter if the gate computer uses a time synchronization protocol. The protocol messages can be replayed from yesterday, and the gate computer has no way of detecting this. The same-state problem is not solved by any protocol.

A real-time clock chip solves this problem. The small embedded computer can encrypt the current time with a fixed secret key to generate highly random data. This data can in turn be used as a nonce in a cryptographic protocol. As the real-time clock never repeats its state, the embedded computer can avoid falling into the same-state trap.

A hardware random number generator has the same effect. It allows the embedded computer to behave differently each time it is rebooted.

But if you don't have a real-time clock or a random number generator, you have a big problem. Sometimes you can fudge a bit and try to extract randomness from the clock skew between the local clock oscillator and the network timing or another oscillator, but it is very hard to extract enough entropy from this within a short time. Taking 10 minutes to reboot an embedded computer is simply unacceptable.

We've seen the same-state problem come up again and again. The upshot is that the hardware has to change before you can do useful cryptography on such small computers. This is hard to sell to managers, especially since the hardware is often already in the field and they don't want to hear that something cannot be done. But there is no magic security sauce that you can pour over an existing insecure system to make it secure. If you don't design the security into the system from the very start, you almost never get good security.

There is one more possible solution, though it rarely works in practice. Sometimes you can keep a reboot counter in the nonvolatile memory. Each time the CPU reboots, it increments a counter in nonvolatile memory. This solution is fraught with problems. Some nonvolatile memories can only be updated a few thousand times, which makes the machine wear out if you keep updating the counter. Some nonvolatile technologies require an additional power voltage to be programmable, which is often not available in the field. In some designs, you can only set bits in nonvolatile memory, or wipe all of the nonvolatile memory. The latter option is not viable, as you'd lose the main program of the machine. Even if all these problems are overcome, it is very difficult to modify nonvolatile memory in such a way that the counter always reliably increases even if the power supply to the machine can be interrupted at arbitrary points in time. This nonvolatile counter option is only viable in a minority of the cases we've seen. When it is feasible, such a counter could be used as part of a PRNG. For example, the counter could be used with CTR mode and an AES key to generate a stream of pseudorandom bits.

16.6 Time

While we're discussing clocks, we have a few short comments on which time base to choose. Stay away from local time. Local time is the time we use on our watches and other clocks. The problem is, local time changes with daylight saving time and time zone. These changes pose problems: some time values are repeated each year when clocks are set back an hour in the fall, which means that the time is no longer unique or monotonic. Some time values are impossible when clocks are set forward an hour in the spring. Furthermore, the exact date on which daylight saving time starts and stops is different in different countries. In some countries, the rules change every few years, and you don't want to have to update your software for that. And people who travel with laptops might change the time on their laptops to the local time, which just makes these problems worse.

The obvious choice is to use UTC time. This is an international time standard based on atomic clocks, and is widely used throughout the world. Any single computer can keep track of the offset of local time with regard to UTC and use this knowledge in interactions with the user.

There is one problem with UTC: the leap seconds. To keep UTC synchronized with the Earth's rotation, there is a leap second once every few years or so. So far, all leap seconds have been extra seconds; there is a particular minute that gets 61 seconds. It is also theoretically possible to have a missing second. It all depends on the rotation of the Earth. The problem for computers is that the leap seconds are unpredictable. Ignoring leap seconds leads to inaccuracies in measuring time intervals across a leap second. This is not really a cryptographic problem, but if you want to make a good clock, you might as well do it right. All computer software always assumes that each minute has 60 seconds. If you synchronize directly to a real UTC clock, the insertion of a leap second can lead to problems. Most likely this results in your internal clock repeating itself for one second. It is a minor problem, but again, it destroys the uniqueness and monotonicity of time values.

For most applications, the exact synchronization of the clock is less important than the monotonicity and uniqueness of the time stamps. As long as you make sure the clock never jumps backwards at a leap second, it doesn't matter how you solve this problem.

16.7 Closing Recommendations

Unfortunately, we have no ideal solution for you. Creating a reliable clock is very tricky, especially in a cryptographic setting where you assume there are malicious attackers. The best solution depends on your local situation. Our recommendations, therefore, are to be aware there are potential security issues associated with the use of a clock, minimize reliance on the clock whenever possible, and be cautious. And again, the most important thing is generally the monotonicity and uniqueness of the time stamps.

16.8 Exercises

Exercise 16.1 Some computers use NTP at boot, or at regular intervals. Turn off NTP for one week on your computer. Write a program that at regular intervals (at least once every two hours) records both the true time and the time reported by your computer. Let t_0 be the initial true time at the start of your experiment. For each time measurement pair, plot the true time minus t_0 on the horizontal axis of a graph and plot your computer's time minus true time on the vertical axis. How different is your computer's clock from true time after one week? Does your graph tell you anything else?

Exercise 16.2 Repeat exercise 16.1, but this time for a collection of five different computers.

Exercise 16.3 Find a new product or system that uses (or should use) a clock. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on the security and privacy issues surrounding the clock.

CHAPTER

17

Key Servers

At last we turn to key management. This is, without a doubt, the most difficult issue in cryptographic systems, which is why we left it to near the end. We've discussed how to encrypt and authenticate data, and how to negotiate a shared secret key between two participants. Now we need to find a way for Alice and Bob to recognize each other over the Internet. As you will see, this gets very complex very quickly. Key management is especially difficult because it involves people instead of mathematics, and people are much harder to understand and predict. Key management is in many ways a capstone to all we have discussed so far. Much of the benefit of cryptography is defeated if key management is done poorly.

Before we start, let us make one thing clear. We talk only about the cryptographic aspects of key management, not the organizational aspects. The organizational aspects include things like a policy covering whom to issue keys to, which keys get access to which resources, how to verify the identity of the people who get keys, policies on the security of the stored keys, mechanisms for verifying that these policies are being adhered to, etc. Every organization will implement these differently, depending on their requirements and their existing organizational infrastructure. We focus only on parts that directly affect the cryptographic system.

One way to handle key management is to have a trusted entity to hand out all the keys. We'll call this entity the *key server*.

17.1 Basics

The basic idea is simple. We assume that everybody sets up a shared secret key with the key server. For example, Alice sets up a key K_A that is known only to her and to the key server. Bob sets up a key K_B that is known only to him and to the key server. Other parties set up keys in the same fashion.

Now suppose Alice wants to communicate with Bob. She has no key she can use to communicate with Bob, but she can communicate securely with the key server. The key server, in turn, can communicate securely with Bob. We could simply send all the traffic to the key server and let the key server act as a giant post office. But that is a bit hard on the key server, as it would have to handle enormous amounts of traffic. A better solution is to let the key server set up a key K_{AB} that is shared by Alice and Bob.

17.2 Kerberos

This is the basic idea behind Kerberos, a widely used key management system [79]. Kerberos is based on the Needham-Schroeder protocol [102].

At a very basic level, here is how it works. When Alice wants to talk to Bob, she first contacts the key server. The key server sends Alice a new secret key K_{AB} plus the key K_{AB} encrypted with Bob's key K_B. Both these messages are encrypted with K_A, so only Alice can read them. Alice sends the message that is encrypted with Bob's key, called the ticket, to Bob. Bob decrypts it and gets K_{AB}, which is now a session key known only to Alice and Bob—and to the key server, of course.

One of the features of Kerberos is that the key server, called the KDC in Kerberos terminology, does not have to update its state very often. Of course, the key server has to remember the key that it shares with each user. But when Alice asks the KDC to set up a key between her and Bob, the KDC performs the function and then forgets all about it. It does not keep track of which keys between users have been set up. This is a nice property because it allows a heavily loaded key server to be distributed over several machines in a simple manner. As there is no state to be updated, Alice can talk to one copy of the key server one moment and to another copy the next moment.

It turns out that the cryptographic protocols needed for a Kerberos-style system are very complicated. Initially, designing such protocols looks quite easy to do, but even experienced cryptographers have published proposals, only to have them broken later on. The flaws that creep in are very subtle. We're not going to explain these protocols here; they are too dangerous to experiment with and modify by hand. Even we shy away from designing this type of protocol anew. If you want to use a protocol of this sort, use the latest version of Kerberos. Kerberos has been around for quite a while, and many competent people have looked at it.

17.3 Simpler Solutions

Sometimes it is not possible to use Kerberos. The protocol is far from simple, and it imposes some restrictions. Servers have to memorize all tickets that they have accepted, and every participant needs a reliable clock. There are several situations in which these requirements cannot be met. Further, we find it more informative to study a simpler design.

We can create a simpler and more robust solution if we don’t put so much emphasis on efficiency. It turns out to be especially useful to allow the key server to maintain state. Modern computers are far more powerful than they were in the days when Kerberos was first designed, and they should not have any trouble maintaining state for tens of thousands of participants. Even a very large system with 100,000 participants is not a problem: if each participant requires a 1 kilobyte state in the key server, storing all states requires only 100 megabytes of memory. The key server still needs to be fast enough to set up all the requested keys, but that too is much less of a problem with modern, fast computers.

We will only discuss the situation in which there is a single key server. There are techniques that you can use to distribute the key server state over several computers, but we won’t go into the details, because you really don’t want to have a key server for tens of thousands of participants; it’s too risky. The danger of large key servers is that *all* the keys are in a single place. That makes the key server a very attractive target for attack. The key server must also be online at all times, which means an attacker can always communicate with the key server at will. The current state of the art does not protect computers from network attacks very well, and putting all your keys in a single place is an invitation to disaster. For smaller systems, the total ‘‘value’’ of the keys kept by the key server is smaller, so this threat is reduced.[1] In the next few chapters we will explore a solution to the key management system that is better suited to very large systems. We will restrict our discussion of key servers to fairly small systems—up to a few thousand participants or so.

[1] We don’t like to leave any unaddressed threat in the system, but in key management, you always end up with a compromise solution.

17.3.1 Secure Connection

Here is a brief description of a simpler solution. First, we assume that Alice and the key server share a key K_A. Instead of using this key directly, they use it to run a key negotiation protocol, like the ones we discussed in Chapter 14. (If K_A is a password, you'd really prefer to use one of the protocols suitable for low-entropy passwords that we discussed in Section 14.12, assuming the patent issues are not a problem for you.) The key negotiation protocol sets up a fresh key K'_A between the key server and Alice. All other participants also perform the same protocol with the key server, and they all set up fresh keys.

Alice and the key server use K'_A to create a secure communication channel (see Chapter 7 for details). Using the secure channel, Alice and the key server can communicate securely. Confidentiality, authentication, and replay protection are all provided by the secure channel. All further communications happen over this secure channel. All other participants create a similar secure channel with the key server.

17.3.2 Setting Up a Key

It is now much easier to design a protocol that sets up a key between Alice and Bob. We only need to consider the case where messages get lost, delayed, or deleted by the attacker, because the secure channel protects us from all other types of manipulation. The protocol can now be something fairly simple. Alice asks the key server to set up a key between her and Bob. The key server responds by sending a new key K_{AB} to both Alice and Bob. The key server can even send the message to Bob through Alice, so that it does not need to communicate with Bob directly. If this happens, Alice simply becomes the equivalent of a network router transiting a secure channel between the key server and Bob.

This does pose one limitation on the system: Bob must run the key negotiation protocol with the key server before Alice asks the key server to set up a shared key with Bob. Whether this turns out to be a problem depends on the exact circumstances, as do the possible solutions to this limitation.

17.3.3 Rekeying

Like all keys, the K'_A key must have a limited lifetime. This is easy to arrange, as Alice can always rerun the key negotiation protocol (using the original key K_A for authentication) to set up a fresh K'_A key. A key lifetime of a few hours seems reasonable for most situations.

Because we can always rekey, the key server does not have to store the secure channel state in a reliable manner. Suppose the key server crashes and loses all state information. As long as it remembers K_A (and the corresponding keys for the other participants), there is no problem. All we have to do to recover is run the key negotiation protocol between the key server and every participant again. So although the key server is not stateless, it does not have to modify its long-term state—the part that is stored on nonvolatile media—when running the protocols.

17.3.4 Other Properties

Perhaps our solution is not simpler than Kerberos from an implementation point of view, but it is simpler from a conceptual point of view. The secure channel makes it much easier to oversee the possible lines of attack against the protocol. Using the key negotiation protocol and the secure channel we already designed is a good example of how modularization can help in the design of cryptographic protocols.

Using the key negotiation protocol to set up the secure channel has another advantage: we get forward secrecy. If Alice's key K_A is compromised today, her old secure channel keys K'_A are not revealed, and therefore all her old communications are still secure.

In the earlier parts of the book, we gave a detailed example design of the cryptographic function we discussed. We won't do that here, nor will we for the rest of the book. The cryptography is fairly straightforward, and we could certainly have described a key server system, but it would not be very useful. Designing key management systems is more a problem of collecting a suitable set of requirements for the particular application and getting the user interface right than a problem of cryptography. To be able to explain the design choices for a concrete example here, we would have to invent and document the entire surrounding social and organizational structure, the threat environment, and the application that needs the key management.

17.4 What to Choose

If you want to implement a central key server, you should use Kerberos if possible. It is widely available and widely used.

In those situations where Kerberos is not suitable, you will have to design and build something like the solution we described, but that will be a major operation. For the most common type of cryptographic applications we have seen, you should count on spending as much time on the key server system as you did on the entire application. Our discussion here should help guide your thinking.

17.5 Exercises

Exercise 17.1 For the protocol in Section 17.3, what is a reasonable lifetime to use for the keys K'_A? Why? What bad things could happen if the lifetime is longer? What bad things could happen if the lifetime is shorter?

Exercise 17.2 For the protocol in Section 17.3, how might an attacker be able to learn K'_A *before* it times out? What bad things would the attacker be able to do with that knowledge? What bad things would the attacker not be able to do with that knowledge?

Exercise 17.3 For the protocol in Section 17.3, how might an attacker be able to learn K'_A *after* it times out? What bad things would the attacker be able to do with that knowledge? What bad things would the attacker not be able to do with that knowledge?

Exercise 17.4 For the protocol in Section 17.3, consider an attacker who intercepts all communications. Can the attacker retroactively read data between Alice and Bob if K_A and K_B are both later exposed?

Exercise 17.5 For the protocol in Section 17.3, could an attacker gain any advantage in breaking the protocol by forcibly rebooting the key server?

Exercise 17.6 For the protocol in Section 17.3, could an attacker mount a denial-of-service attack against two parties wishing to communicate, and if so, how?

Exercise 17.7 For the protocol in Section 17.3, are there policy or legal risks with having the key server generate K_{AB}? Are there things Alice and Bob would not say in a situation where the key server generates K_{AB} that they would say if the key were known only to them?

CHAPTER

18

The Dream of PKI

In this chapter we will give the standard presentation of what a PKI is, and how it solves the key management problem. It is important to understand this first. In the next chapter we'll talk about the challenges with PKIs in practice, but for this chapter we'll visit the perfect world where a PKI solves all your problems.

18.1 A Very Short PKI Overview

A PKI is a *Public-Key Infrastructure*. It is an infrastructure that allows you to recognize which public key belongs to whom. The classical description is as follows.

There is a central authority that is called the *Certificate Authority*, or CA for short. The CA has a public/private key pair (e.g., an RSA key pair) and publishes the public key. We will assume that everybody knows the CA's public key. As this key remains the same over long periods of time, this is easy to accomplish.

To join the PKI, Alice generates her own public/private key pair. She keeps the private key secret, and takes the public key PK_A to the CA and says: ''Hi, I'm Alice and PK_A is my public key.'' The CA verifies that Alice is who she says she is and then signs a digital statement that states something like ''Key PK_A belongs to Alice.'' This signed statement is called the *certificate*. It certifies that the key belongs to Alice.

If Alice now wants to communicate with Bob, she can send him her public key and the certificate. Bob has the CA's public key, so he can verify the signature on the certificate. As long as Bob trusts the CA, he also trusts that PK_A actually belongs to Alice.

Using the same procedures, Bob gets his public key certified by the CA, and sends his public key and certificate to Alice. They now know each other's public key. These keys in turn can be used to run the key negotiation protocol to establish a session key for secure communications.

What is required is a central CA that everybody trusts. Each participant needs to get his or her public key certified, and each participant needs to know the CA's public key. After that, everybody can securely communicate with everybody else.

That sounds simple enough.

18.2 PKI Examples

To make the rest of this chapter easier to understand, we'll first give some examples of how PKIs can be implemented and used.

18.2.1 The Universal PKI

The ultimate dream is a universal PKI. A large organization, like the post office, certifies everybody's public key. The beauty of this is that every person only needs to get a single key certified, as the same key can be used for every application. Because everybody trusts the post office, or whatever other organization becomes the universal CA, everybody can communicate securely with everybody else, and they all live happily ever after.

If our description sounds a bit like a fairy tale, that is because it is. There is no universal PKI, and there never will be.

18.2.2 VPN Access

A more realistic example would be a company that has a VPN (Virtual Private Network) to allow its employees to access the corporate network from home or from their hotel room when they are traveling. The VPN access points must be able to recognize the people who have access and exactly what level of access they have. The IT department of the company acts as the CA and gives every employee a certificate that allows the VPN access points to recognize the employee.

18.2.3 Electronic Banking

A bank wants to allow its customers to perform financial transactions on the bank's website. Properly identifying the customer is vital in this application, as is the ability to produce proof acceptable in court. The bank itself can act as the CA and certify the public keys of its customers.

18.2.4 Refinery Sensors

A refinery complex is very large. Spread out between miles of pipes and access roads are hundreds of sensors that measure things like temperature, flow rate, and pressure. Spoofing sensor data is a very serious attack on the refinery. It might not be too difficult to send false sensor data to the control room, tricking the operators into taking actions that lead to a large explosion. Therefore, it is imperative that the control room get the proper sensor readings. We can use standard authentication techniques to ensure that the sensor data has not been tampered with, but to be sure that the data actually comes from the sensor, some kind of key infrastructure is needed. The company can act as a CA and build a PKI for all the sensors so each sensor can be recognized by the control room.

18.2.5 Credit Card Organization

A credit card organization is a cooperative venture between a few thousand banks spread out all over the world. All of these banks must be able to exchange payments. After all, a user who has a credit card from bank *A* must be able to pay the merchant that banks with bank *B*. Bank *A* will need to settle with bank *B* in some way, and that requires secure communications. A PKI allows all banks to identify each other and perform secure transactions. In this situation, the credit card organization can act as the CA that certifies the keys of each bank.

18.3 Additional Details

In real life, things become somewhat more complicated, so various extensions to the simple PKI scheme are often used.

18.3.1 Multilevel Certificates

In many situations, the CA is split into multiple pieces. For example, the central credit card organization is not going to certify each bank directly. Instead, they will have regional offices to deal with the individual banks. You then get a two-level certificate structure. The central CA signs a certificate on the regional CA's public key that says something like: ''Key PK_X belongs to regional office *X* and is allowed to certify other keys.'' Each regional office can then certify individual bank keys. The certificate on the bank's key consists of two signed messages: the central CA's delegation message that authorizes the regional office's key, and the regional office's certification of the bank's key. This is called the *certificate chain*, and such a chain can be extended to any number of levels.

Such multilevel certificate structures can be very useful. They basically allow the CA functionality to be split into a hierarchy, which is easy to handle for most organizations. Almost all PKI systems have a multilevel structure. One disadvantage of this structure is that the certificates grow larger and require more computations to verify, but this is a relatively small cost in most situations. Another disadvantage is that each extra CA that you add to the system provides another point of attack, and thereby reduces overall system security.

One way to reduce the disadvantage of the large multilevel certificates that we have not seen in practice would be to collapse the certificate hierarchy. To continue with this example, once the bank has its two-level certificate, it could send it to the central CA. The central CA verifies the two-level certificate and replies with a single certificate on the bank's key, using the master CA key. Once the key hierarchy is collapsed like this, the performance cost of adding extra levels to the hierarchy becomes very small. But then again, adding extra layers might not be such a good idea; many-layered hierarchical structures are rarely effective.

You have to be careful when chaining certificates together like this. They add more complexity, and complexity is in general risky. Here is an example. Secure sites on the Internet use a PKI system to allow browsers to identify the correct website. In practice, this system isn't very secure, if only because most users don't verify the name of the website they are using. But a while back, a fatal bug showed up in a library that validates certificates on all Microsoft operating systems. Each element of the certificate chain contains a flag that specifies whether the key it certifies is a CA key or not. CA keys are allowed to certify other keys. Non-CA keys are not allowed to certify other keys. This is an important difference. Unfortunately, the library in question didn't check this flag. So an attacker could buy a certificate for the domain `nastyattacker.com` and use it to sign a certificate for `amazon.com`. Microsoft Internet Explorer used the faulty library. It would accept `nastyattacker.com`'s certification of a fake Amazon key and show the fake website as the real Amazon website. Thus, a worldwide security system that cost a fortune to build was completely outflanked by a simple little bug in a single library. Once the bug was published, a patch was released (it took several tries to fix all the problems), but this remains a good example of a minor bug destroying the security of an entire system.

18.3.2 Expiration

No cryptographic key should be used indefinitely; there is always a risk that the key will be compromised. Regular key changes let you recover from compromise, albeit slowly. A certificate should not be valid forever, either, because both the CA's key and the public key that is being certified expire.

Apart from these cryptographic reasons, expiration is important in keeping information up-to-date. When a certificate expires, a new one will have to be reissued, and this creates an opportunity to update the information in the certificate. A typical expiration interval is somewhere between a few months and a few years.

Almost all certificate systems include an expiration date and time. Nobody should accept the certificate after this date and time. This is why participants in a PKI need a clock.

Many designs include other data in the certificate. Often certificates have a not-valid-before time, in addition to the expiration time. There can be different classes of certificates, certificate serial numbers, date and time of issue, etc. Some of this data is useful, some useless.

The most commonly used format for certificates is X.509 v3, which is overly complicated. See Peter Gutmann's style guide [58] for a discussion of X.509. If you work on a system that doesn't have to be interoperable with other systems, you might strongly consider forgetting about X.509. Of course, X.509 is standardized, and it's hard to fault you for using a standard.

18.3.3 Separate Registration Authority

Sometimes you will see a system with a separate registration authority. The problem is a political one. It is the HR department of a company that decides who is an employee. But the IT department has to run the CA; that is a technical job that they are not going to allow the HR department to do.

There are two good solutions to this. The first one is to use a multilevel certificate structure and let the HR department be its own sub-CA. This automatically provides the necessary flexibility to support multiple sites. The second solution is much like the first one, except that once a user has a two-level certificate, he exchanges it for a one-level certificate at the central CA. This eliminates the overhead of checking a two-level certificate each time it is used, at the cost of adding a simple two-message protocol to the system.

The really bad solution is to add a third party to the cryptographic protocol. The project specifications will talk about the CA and another party that might be called something like the RA (Registration Authority). The CA and RA are treated as completely separate entities, which can add more than 100 pages of documentation to the system. That is bad in itself. Then there is the need to specify the RA–CA interaction. We've even seen three-party protocols in which the RA authorizes the CA to issue a certificate. This is a good example of the problem of imposing user requirements on a technical solution. User requirements only specify the outside behavior of a system. The company needs to have separate functionality for the HR and IT departments. But that does not mean the software has to have different code for the HR and IT departments. In many situations, and certainly in this one, the two departments can use much of the same functionality, and thus much of the same code. Using a single set of certificate functions leads to a design that is simpler, cheaper, more powerful, and more flexible than one based directly on the original requirements that included both a CA and an RA entity. A two-level CA scheme allows HR and IT to share most of the code and protocols. The differences, in this case, are mostly in the user interface and should be easy to implement. That translates to maybe a few hundred lines of extra code, not a few hundred extra pages of specifications that turn into tens of thousands of lines of code.

18.4 Summary

What we have described is a dream, but a very important dream. PKI is the first and last word on key management for most of our industry. People have been brought up on this dream and see it as something so obvious that it doesn't need stating. To be able to understand them, you must understand the PKI dream, because a lot of what they say is within the context of the dream. And it feels *so* good to think that you have a solution to the key management problem . . . .

18.5 Exercises

Exercise 18.1 Suppose a CA is malicious. What bad things could the CA accomplish?

Exercise 18.2 Assume a universal PKI. Can any security problems arise because of the use of this single PKI across multiple applications?

Exercise 18.3 What policy or organizational challenges might impede or prevent the deployment of a worldwide universal PKI?

Exercise 18.4 In addition to the examples in Sections 18.2.2–18.2.5, give three example scenarios for which a PKI might be viable.

CHAPTER 19

PKI Reality

While very useful, there are some fundamental problems with the basic idea of a PKI. Not in theory, but then, theory is something very different from practice. PKIs simply don't work in the real world the way they do in the ideal scenario discussed in Chapter 18. This is why much of the PKI hype has never matched the reality.

When talking about PKIs, our view is much broader than just e-mail and the Web. We also consider the role of PKIs in authorization and other systems.

19.1 Names

We'll start with a relatively simple problem: the concept of a name. The PKI ties Alice's public key to her name. What is a name?

Let's begin in a simple setting. In a small village, everybody knows everybody else by sight. Everybody has a name, and the name is either unique or will be made unique. If there are two Johns, they will quickly come to be called something like Big John and Little John. For each name there is one person, but one person might have several names; Big John might also be called Sheriff or Mr. Smith.

The name we are talking about here is not the name that appears on legal documents. It is the name that people use to refer to you. A name is really any kind of signifier that is used to refer to a person, or more generally, to an entity. Your ''official'' name is just one of many names, and for many people it is one that is rarely used.

As the village grows into a town, the number of people increases until you no longer know them all. Names start losing their immediate association with a person. There might only be a single J. Smith in town, but you might not know him. Names now start to lead a life of their own, divorced from the actual person. You start talking about people you have never actually met. Maybe you end up talking in the bar about the rich Mr. Smith who just moved here and who is going to sponsor the high school football team next year. Two weeks later, you find out that this is the same person who joined your baseball team two months ago, and whom you know by now as John. People still have multiple names, after all. It just isn't obvious which names belong together, and which person they refer to.

As the town grows into a city, this changes even more. Soon you will only know a very small subset of the people. What is more, names are no longer unique. It doesn't really help to know that you are looking for a John Smith if there are a hundred of them in the city. The meaning of a name starts to depend on the context. Alice might know three Johns, but at work when she talks about ''John,'' it is clear from the context that she means John who works upstairs in sales. Later at home, it might mean John the neighbor's kid. The relationship between a name and a person becomes even fuzzier.

Now consider the Internet. Over a billion people are online. What does the name ''John Smith'' mean there? Almost nothing: there are too many of them. So instead of more traditional names we use e-mail addresses. You now communicate with `jsmith533@yahoo.com`. That is certainly a unique name, but in practice it does not link to a person in the sense of someone you will ever meet. Even if you could find out information such as his address and phone number, he is just as likely to live on the other side of the world. You are never going to meet him in person unless you really set out to do so. Not surprisingly, it is not uncommon for people to take on different online personalities. And as always, each person has multiple names. Most users acquire multiple e-mail addresses after a while. (We have more than a dozen among us.) But it is extremely difficult to find out whether two e-mail addresses refer to the same person. And to make things more complicated, there are people who share an e-mail address, so that ''name'' refers to them both.

There are large organizations that try to assign names to everybody. The best-known ones are governments. Most countries require each person to have a single official name, which is then used on passports and other official documents. The name itself is not unique—there are many people with the same name—so in practice it is often extended with things like address, driver's license number, and date of birth. This still does not guarantee a unique identifier for a person, however.[1] Also, several of these identifiers can change over the course of a person's life. People change their addresses,

[1] Driver's license numbers are unique, but not everybody has one.

driver's license numbers, names, and even gender. Just about the only thing that doesn't change is the date of birth, but this is compensated for by the fact that plenty of people lie about their date of birth, in effect changing it.

Just in case you thought that each person has a single government-sanctioned official name, this isn't true, either. Some people are stateless and have no papers at all. Others have dual nationalities, with two governments each trying to establish an official name—and for various reasons, they may not agree on what the official name should be. The two governments might use different alphabets, in which case the names cannot be the same. Some countries require a name that fits the national language and will modify foreign names to a similar ''proper'' name in their own language.

To avoid confusion, many countries assign unique numbers to individuals, like the Social Security number (SSN) in the United States or the SoFi number in the Netherlands. The whole point of this number is to provide a unique and fixed name for an individual, so his actions can be tracked and linked together. To a large degree these numbering schemes are successful, but they also have their weaknesses. The link between the actual human and the assigned number is not very tight, and false numbers are used on a large scale in certain sectors of the economy. And as these numbering schemes work on a per-country basis, they do not provide global coverage, nor do the numbers themselves provide global uniqueness.

One additional aspect of names deserves mention. In Europe, there are privacy laws that restrict what kind of information an organization can store about people. For example, a supermarket is not allowed to ask for, store, or otherwise process an SSN or SoFi number for its loyalty program. This restricts the reuse of government-imposed naming schemes.

So what name should you use in a PKI? Because many people have many different names, this becomes a problem. Maybe Alice wants to have two keys, one for her business and one for her private correspondence. But she might use her maiden name for her business and her married name for her private correspondence. Things like this quickly lead to serious problems if you try to build a universal PKI. This is one of the reasons why smaller application-specific PKIs work much better than a single large one.

19.2 Authority

Who is this CA that claims authority to assign keys to names? What makes that CA authoritative with respect to these names? Who decides whether Alice is an employee who gets VPN access or a customer of the bank with restricted access?

For most of our examples, this is a question that is simple to answer. The employer knows who is an employee and who isn't; the bank knows who is a customer. This gives us our first indication of which organization should be the CA. Unfortunately, there doesn't seem to be an authoritative source for the universal PKI. This is one of the reasons why a universal PKI cannot work.

Whenever you are planning a PKI, you have to think about who is authorized to issue the certificates. For example, it is easy for a company to be authoritative with regard to its employees. The company doesn't decide what the employee's name is, but it does know what name the employee is known by *within the company*. If ''Fred Smith'' is officially called Alfred, this does not matter. The name ''Fred Smith'' is a perfectly good name within the context of the employees of the company.

19.3 Trust

Key management is the most difficult problem in cryptography, and a PKI system is one of the best tools that we have to solve it with. But everything depends on the security of the PKI, and therefore on the trustworthiness of the CA. Think about the damage that can be done if the CA starts to forge certificates. The CA can impersonate anyone in the system, and security completely breaks down.

A universal PKI is very tempting, but trust is really the area where it fails. If you are a bank and you need to communicate with your customers, would you trust some dot-com on the other side of the world? Or even your local government bureaucracy? What is the total amount of money you could lose if the CA does something horribly wrong? How much liability is the CA willing to take on? Will your local banking regulations allow you to use a foreign CA? These are all enormous problems. Just imagine the damage that can occur if the CA's private key is published on a website.

Think of it in traditional terms. The CA is the organization that hands out the keys to the buildings. Most large office buildings have guards, and most guards are hired from an outside security service. The guards verify that the rules are being obeyed: a rather straightforward job. But deciding who gets which keys is not something that you typically outsource to another company, because it is a fundamental part of the security policy. For the same reason, the CA functionality should not be outsourced.

No organization in the world is trusted by everybody. There isn't even one that is trusted by *most* people. Therefore, there will never be a universal PKI. The logical conclusion is that we will have to use lots of small PKIs. And this is exactly the solution we suggest for our examples. The bank can be its own CA; after all, the bank trusts itself, and all the customers already trust the bank with their money. A company can be its own CA for the VPN, and the credit card organization can also run its own CA.

An interesting observation here is that the trust relationships used by the CA are ones that already exist and are based on contractual relationships. This is always the case when you design cryptographic systems: the basic trust relationships you build on are all based on contractual relationships.

19.4 Indirect Authorization

Now we come to a big problem with the classic PKI dream. Consider authorization systems. The PKI ties keys to names, but most systems are not interested in the name of the person. The banking system wants to know which transactions to authorize. The VPN wants to know which directories to allow access to. None of these systems cares *who* the key belongs to, only *what* the keyholder is authorized to do.

To this end, most systems use some kind of *access control list*, or ACL. This is just a database of who is authorized to do what. Sometimes it is sorted by user (e.g., Bob is allowed the following things: access files in the directory `/homes/bob`, use of the office printer, access to the file server), but most systems keep the database indexed by action (e.g., charges to this account must be authorized by Bob or Betty). Often there are ways to create groups of people to make the ACLs simpler, but the basic functionality remains the same.

So now we have three different objects: a key, a name, and permission to do something. What the system wants to know is which key authorizes which action, or in other words, whether a particular key has a particular permission. The classic PKI solves this by tying keys to names and using an ACL to tie names to permissions. This is a roundabout method that introduces additional points of attack [45].

The first point of attack is the name–key certificate provided by the PKI. The second point of attack is the ACL database that ties names to permissions. The third point of attack is name confusion: with names being such fuzzy things, how do you compare whether the name in the ACL is the same as the name in the PKI certificate? And how do you avoid giving two people the same name?

If you analyze this situation, you will clearly see that the technical design has followed the naive formulation of the requirements. People think of the problem in terms of identifying the key holder and who should have access—that is how a security guard would approach the problem. Automated systems can use a much more direct approach. A door lock doesn't care who is holding the key, but allows access to anyone with the key.

19.5 Direct Authorization

A much better solution is generally to directly tie the permissions to the key, using the PKI. The certificate no longer links the key to a name; it links the key to a set of permissions [45].

All systems that use the PKI certificates can now decide directly whether to allow access or not. They just look at the certificate provided and see if the key has the appropriate permissions. It is direct and simple.

Direct authorization removes the ACL and the names from the authorization process, thereby eliminating these points of attack. Some of the problems will, of course, reappear at the point where certificates are issued. Someone must decide who is allowed to do what, and ensure that this decision is encoded in the certificates properly. The database of all these decisions becomes the equivalent of the ACL database, but this database is less easy to attack. It is easy to distribute to the people making the decisions, removing the central ACL database and its associated vulnerabilities. Decision makers can just issue the appropriate certificate to the user without further security-critical infrastructure. This also removes much of the reliance on names, because the decision makers are much further down in the hierarchy and have a much smaller set of people to deal with. They often know the users personally, or at least by sight, which helps a great deal in avoiding name confusion problems.

So can we just get rid of the names in the certificates, then?

Well, no. Though the names will not be used during normal operations, we do need to provide logging data for audits and such. Suppose the bank just processed a salary payment authorized by one of the four keys that has payment authority for that account. Three days later, the CFO calls the bank and asks why the payment was made. The bank knows the payment was authorized, but it has to provide more information to the CFO than just a few thousand random-looking bits of public-key data. This is why we still include a name in every certificate. The bank can now tell the CFO that the key used to authorize the payment belonged to ''J. Smith,'' which is enough for the CFO to figure out what happened. But the important thing here is that the names only need to be meaningful to humans. The computer never tries to figure out whether two names are the same, or which person the name belongs to. Humans are much better at dealing with the fuzzy names, whereas computers like simple and well-specified things such as sets of permissions.

19.6 Credential Systems

If you push this principle further, you get a full-fledged credential system. This is the cryptographer's super-PKI. Basically, it requires that you need a credential in the form of a signed certificate for every action you perform.

If Alice has a credential that lets her read and write a particular file, she can delegate some or all of her authority to Bob. For example, she could sign a certificate on Bob's public key that reads something like ''Key PK_{Bob} is authorized to read file *X* by delegated authority of key PK_{Alice}.'' If Bob wants to read file *X*, he has to present this certificate and a certificate proof that Alice has read access to file *X*.

A credential system can add additional features. Alice could limit the time validity of the delegation by including the validity period in the certificate. Alice might also limit Bob's ability to delegate the authority to read file *X*.[2]

In theory, a credential system is extremely powerful and flexible. In practice, they are rarely used. There are several reasons for this.

First of all, credential systems are quite complex and can impose a noticeable overhead. Your authority to access a resource might depend on a chain of half-a-dozen certificates, each of which has to be transmitted and checked.

The second problem is that credential systems invite a micromanagement of access. It is so easy to split authorities into smaller and smaller pieces that users end up spending entirely too much time deciding exactly how much authority to delegate to a colleague. This time is often wasted, but a bigger problem is the loss of the colleague's time when it turns out he doesn't have enough access to do his job. Maybe this micromanagement problem can be solved with better user education and better user interfaces, but that seems to be an open problem. Some users avoid the micromanagement problem by delegating (almost) all their authority to anyone who needs any kind of access, effectively undermining the entire security system.

The third problem is that you need to develop a credential and delegation language. The delegation messages need to be written in some sort of logical language that computers can understand. This language needs to be powerful enough to express all the desired functionality, yet simple enough to allow fast chaining of conclusions. It also has to be future-proof. Once a credential system is deployed, every program will need to include code to interpret the delegation language. Upgrading to a new version of the delegation language can be very difficult, especially since security functionality spreads into every piece of a system. Yet it is effectively impossible to design a delegation language that is general enough to satisfy all future requirements, since we never know what the future will bring. This remains an area of research.

The fourth problem with credential systems is probably insurmountable. Detailed delegation of authority is simply too complex a concept for the average user. There doesn't seem to be a way of presenting access rules to

[2] This is an often-requested feature, but we believe it may not always be a good one. Limiting Bob's ability to delegate his authority just invites him to run a proxy program so that other people can use his credential to access a resource. Such proxy programs undermine the security infrastructure and should be banned, but this is only tenable if there are no operational reasons to run a proxy. And there are always operational reasons why someone needs to delegate authority.

users in a manner they can understand. Asking users to make decisions about which authorities to delegate is bound to fail. We see that in the real world already. In some student houses it is customary for one person to go to the ATM and get cash for several people. The other students lend him their ATM card and PIN code. This is an eminently risky thing to do, yet it is done by some of the supposedly more well-educated people in our society. As consultants, we've visited many companies and sometimes had work-related reasons to have access to the local network. It is amazing how much access we got. We've had system administrators give us unrestricted access to the research data, when all we needed to do was look at a file or two. If system administrators have a hard time getting this right, ordinary users certainly will, too.

As cryptographers, we'd love the idea of a credential system if only the users were able to manage the complexity. There is undoubtedly a lot of interesting research to do on human interactions with security systems.

There is, however, one area where credentials are very useful and should be mandatory. If you use a hierarchical CA structure, the central CA signs certificates on the keys of the sub-CAs. If these certificates do not include any kind of restriction, then each sub-CA has unlimited power. This is problematic; we've just multiplied the number of places where system-critical keys are stored.

In a hierarchical CA structure, the power of a sub-CA should be limited by including restrictions in the certificate on its key. This requires a credential-like delegation language for CA operations. Exactly what type of restrictions you'd want to impose depends on the application. Just think about what type of sub-CAs you want to create and how their power should be limited.

19.7 The Modified Dream

Let's summarize all the criticism of PKIs we've presented so far and present a modified dream. This is a more realistic representation of what a PKI should be.

First of all, each application has its own PKI with its own CA. The world consists of a large number of small PKIs. Each user is a member of many different PKIs at the same time.

The user must use different keys for each PKI, as he cannot use the same key in different systems without careful coordination in the design of the two systems. The user's key store will therefore contain dozens of keys, requiring tens of kilobytes of storage space.

The PKI's main purpose is to tie a credential to the key. The bank's PKI ties Alice's key to the credential that allows access to Alice's account. Or the company's PKI ties Alice's key to a credential that allows access to the VPN. Significant changes to a user's credentials require a new certificate to be issued. Certificates still contain the user's name, but this is mainly for management and auditing purposes.

This modified dream is far more realistic. It is also more powerful, more flexible, and more secure than the original dream. It is very tempting to believe that this modified dream will solve your key management problems. But in the next section, we will encounter the hardest problem of all—one that will never be solved fully and will always require compromises.

19.8 Revocation

The hardest problem to solve in a PKI is *revocation*. Sometimes a certificate has to be withdrawn. Maybe Bob's computer was hacked and his private key was compromised. Maybe Alice was transferred to a different department or even fired from the company. You can think of all kinds of situations where you want to revoke a certificate.

The problem is that a certificate is just a bunch of bits. These bits have been used in many places and are stored in many places. You can't make the world forget the certificate, however hard you try. Bruce lost a PGP key more than a decade ago; he still gets e-mail encrypted with the corresponding certificate.[3] Even trying to make the world forget the certificate is unrealistic. If a thief breaks into Bob's computer and steals his private key, you can be certain he also made a copy of the certificate on the corresponding public key.

Each system has its own requirements, but in general, revocation requirements differ in four variables:

- *Speed of revocation*. What is the maximum amount of time allowed between the revocation command and the last use of the certificate?
- *Reliability of revocation*. Is it acceptable that under some circumstances revocation isn't fully effective? What residual risk is acceptable?
- *Number of revocations*. How many revocations should the revocation system handle at a time?
- *Connectivity*. Is the party checking the certificates online at the time of certificate verification?

There are three workable solutions to the revocation problem: revocation lists, fast expiration, and online certificate verification.

19.8.1 Revocation List

A *certificate revocation list*, or CRL, is a database that contains a list of revoked certificates. Everybody who wants to verify a certificate must check the CRL database to see if the certificate has been revoked.

[3] PGP has its own strange PKI-like structure called the *web of trust*. Those interested in PGP's web of trust should read [130].

A central CRL database has attractive properties. Revocation is almost instantaneous. Once a certificate has been added to the CRL, no further transactions will be authorized. Revocation is also very reliable, and there is no direct upper limit on how many certificates can be revoked.

The central CRL database also has significant disadvantages. Everybody must be online all the time to be able to check the CRL database. The CRL database also introduces a single point of failure: if it is not available, no actions can be performed. If you try to solve this by authorizing parties to proceed whenever the CRL is unavailable, attackers will use denial-of-service attacks to disable the CRL database and destroy the revocation capability of the system.

An alternative is to have a distributed CRL database. You could make a redundant mirrored database using a dozen servers spread out over the world and hope it is reliable enough. But such redundant databases are expensive to build and maintain and are normally not an option. Don't forget, people rarely want to spend money on security.

Some systems simply send copies of the entire CRL database to every device in the system. The U.S. military STU-III encrypted telephone works in this manner. This is similar to the little booklets of stolen credit card numbers that used to be sent to each merchant. It is relatively easy to do. You can just let every device download the updated CRL from a Web server every half hour or so, at the cost of increasing the revocation time. However, this solution restricts the size of the CRL database. Most of the time you can't afford to copy hundreds of thousands of CRL entries to every device in the system. We've seen systems where the requirements state that every device must be capable of storing a list of 50 CRL entries, which can be problematic.

In our experience, CRL systems are expensive to implement and maintain. They require their own infrastructure, management, communication paths, and so on. A considerable amount of extra functionality is required just to handle the comparatively rarely used functionality of revocation.

19.8.2 Fast Expiration

Instead of revocation lists, you can use *fast expiration*. This makes use of the already existing expiration mechanism. The CA simply issues certificates with a very short expiration time, ranging anywhere from 10 minutes to 24 hours. Each time Alice wants to use her certificate, she gets a new one from the CA. She can then use it for as long as it remains valid. The exact expiration speed can be tuned to the requirements of the application, but a certificate validity period of less than 10 minutes does not seem to be very practical.

The major advantage of this scheme is that it uses the already available certificate issuing mechanism. No separate CRL is required, which significantly reduces the overall system complexity. All you need to do to revoke a permission is inform the CA of the new access rules. Of course, everybody still needs to be online all the time to get the certificates reissued.

Simplicity is one of our main design criteria, so we prefer fast expiration to a CRL database. Whether fast expiration is possible depends mostly on whether the application demands instantaneous revocation, or whether a delay is acceptable.

19.8.3 Online Certificate Verification

Another alternative is *online certificate verification*. This approach, which is embodied in the Online Certificate Status Protocol (OCSP), has seen a lot of headway in some domains, such as Web browsers.

To verify a certificate, Alice queries a trusted party—such as the CA or a delegated party—with the serial number of the certificate in question. The trusted party looks up the status of the certificate in its own database, and then sends a signed response to Alice. Alice knows the trusted party's public key and can verify the signature on the response. If the trusted party says the certificate is valid, Alice knows that the certificate has not been revoked.

Online certificate verification has a number of attractive properties. As with CRLs, revocation is almost instantaneous. Revocation is also very reliable. Online certificate verification also shares some disadvantages with CRLs. Alice must be online to verify a certificate, and the trusted party becomes a point of failure.

In general, we prefer online certificate verification to CRLs. Online certificate verification avoids the problem of massively distributing the CRLs and avoids the need to parse and verify the CRLs on the client. The design of online certificate verification protocols can therefore be made cleaner, simpler, and more scalable than CRLs.

In most situations online certificate verification is inferior to fast expiration, however. With online certificate verification, you can't trust the key without a trusted party's signature. If you view that signature as a new certificate on the key, you have a fast-expiration system with very short expiration times. The disadvantage of online certificate verification is that every verifier has to query the trusted party, whereas for fast expiration the prover can use the same CA signature for many verifiers.

19.8.4 Revocation Is Required

Because revocation can be hard to implement, it becomes very tempting not to implement it at all. Some PKI proposals make no mention of revocation. Others list the CRL as a future extension possibility. In reality, a PKI without some form of revocation is pretty useless. Real-life circumstances mean that keys *do* get compromised, and access has to be revoked. Operating a PKI without a working revocation system is somewhat like operating a ship without a bilge pump. In theory, the ship should be watertight and it shouldn't need a bilge pump. In practice, there is always water collecting in the bottom of the ship, and if you don't get rid of it, the ship eventually sinks.

19.9 So What Is a PKI Good For?

At the very beginning of our PKI discussion, we stated that the purpose of having a PKI is to allow Alice and Bob to generate a shared secret key, which they use to create a secure channel, which they in turn use to communicate securely with each other. Alice wants to authenticate Bob (and vice versa) without talking to a third party. The PKI is supposed to make this possible.

But it doesn't.

There is *no* revocation system that works entirely offline. It is easy to see why. If neither Alice nor Bob contacts any outside party, neither of them can ever be informed that one of their keys has been revoked. So the revocation checks force them to go online. Our revocation solutions require online connections.

But if we are online, we don't need a big complex PKI. We can achieve our desired level of security by simply setting up a central key server, such as those described in Chapter 17.

Let's compare the advantages of a PKI over a key server system:

- A key server requires everybody to be online in real time. If you can't reach the key server, you can't do anything at all. There is no way Alice and Bob can recognize each other. A PKI gives you some advantages. If you use expiration for revocation, you only need to contact the central server once in a while; for applications that use certificates with validity periods of hours, the requirement for real-time online access and processing is significantly relaxed. This is useful for non-interactive applications like e-mail. This is also useful for certain authorization systems, or cases where communications are expensive. Even if you use a CRL database, you might have rules on how to proceed if the CRL database cannot be reached. Credit card systems have rules like this. If you can't get automatic authorization, any transaction up to a certain amount is okay. These rules would have to be based on a risk analysis, including the risk of a denial-of-service attack on the CRL system, but at least you get the option of proceeding; the key server solution provides no alternatives.

- The key server is a single point of failure. Distributing the key server is difficult, since it contains all the keys in the system. You really don't want to start spreading your secret keys throughout the world. The

CRL database, in contrast, is much less security-critical and is easier to distribute. The fast-expiration solution makes the CA a point of failure. But large systems almost always have a hierarchical CA, which means that the CA is already distributed, and failures affect only a small part of the system.

- In theory, a PKI should provide you with nonrepudiation. Once Alice has signed a message with her key, she should not be able to later deny that she signed the message. A key server system can never provide this; the central server has access to the same key that Alice uses and can therefore forge an arbitrary message to make it look as if Alice sent it. In real life, nonrepudiation doesn't work because people cannot store their secret keys sufficiently well. If Alice wants to deny that she signed a message, she is simply going to claim that a virus infected her machine and stole her private key.
- The most important key of a PKI is the CA root key. This key does not have to be stored in a computer that is online. Rather, it can be stored securely and only loaded into an offline computer when needed. The root key is only used to sign the certificates of the sub-CAs, and this is done only rarely. In contrast, the key server system has the master key material in an online computer. Computers that are offline are much harder to attack than those that are online, so this makes a PKI potentially more secure.

So there are a few advantages to PKIs. They are nice to have, but none of them gives you a really critical advantage in some environments. These advantages only come at a stiff price. A PKI is much more complex than a key server system, and the public-key computations require a lot more computational power.

19.10 What to Choose

So how should you set up your key management system? Should you use a key server-type scheme or a PKI-type scheme? As always, this depends on your exact requirements, the size of your system, your target application, and so on.

For small systems, the extra complexity of a PKI is in general not warranted. We think it is easier to use the key server approach. This is mainly because the advantages of a PKI over the key server approach are more relevant for large installations than for small ones.

For large systems, the additional flexibility of a PKI is still attractive. A PKI can be a more distributed system. Credential-style extensions allow the central

CA to limit the authority of the sub-CAs. This in turn makes it easy to set up small sub-CAs that cover a particular area of operations. As the sub-CA is limited in the certificates it can issue by the certificate on its own key, the sub-CA cannot pose a risk to the system as a whole. For large systems, such flexibility and risk limitation are important.

If you are building a large system, we would advise you to look very seriously at a PKI solution, but do compare it to a key server solution. You'll have to see if the PKI advantages outweigh its extra cost and complexity. One problem might be that you really want to use credential-style limitations for your sub-CAs. To do this, you must be able to express the limitations in a logical framework. There is no generic framework in which this can be done, so this ends up being a customer-specific part of the design. It probably also means that you cannot use an off-the-shelf product for your PKI, as they are unlikely to have appropriate certificate restriction language.

19.11 Exercises

Exercise 19.1 What bad things could happen if Alice uses the same keys with multiple PKIs?

Exercise 19.2 Suppose a system employs devices that are each capable of storing a list of 50 CRL entries. How can this design decision lead to security problems?

Exercise 19.3 Suppose a system uses a PKI with a CRL. A device in that system is asked to verify a certificate but cannot access the CRL database because of a denial-of-service attack. What are the possible courses of action for the device, and what are the advantages and disadvantages of each course of action?

Exercise 19.4 Compare and contrast the advantages and disadvantages of PKIs and key servers. Describe one example application for which you would use a PKI. Describe one example application for which you would use a key server. Justify each of your decisions.

Exercise 19.5 Compare and contrast the advantages and disadvantages of CRLs, fast revocation, and online certificate verification. Describe one example application for which you would use a CRL. Describe one example application for which you would use fast revocation. Describe one example application for which you would use online certificate verification. Justify each of your decisions.

CHAPTER

20

PKI Practicalities

In practice, if you need a PKI, you will have to decide whether to buy it or build it. We'll now discuss some of the practical considerations that occur when designing a PKI system.

20.1 Certificate Format

A certificate is just a data type with multiple required and optional fields. It is important that the encoding of a particular data structure be unique, because in cryptography we often hash a data structure to sign it or compare it. A format like XML, which allows several representations of the same data structure, requires extra care to ensure that signatures and hashes always work as they should. Although we dislike their complexity, X.509 certificates are another alternative.

20.1.1 Permission Language

For all but the simplest of PKI systems, you really want to be able to restrict the certificates that a sub-CA can issue. To do that, you need to encode a restriction into the sub-CA's certificate, which in turn requires a language in which to express the key's permissions. This is probably the hardest point of the PKI design. The restrictions you are going to need depend on your application. If you can't find sensible restrictions, you should rethink your decision to use a PKI. Without restrictions in the certificates, every sub-CA effectively has a master key—and that is a bad security design. You could restrict yourself to a single CA, but then you would lose many of the advantages of a PKI over a key server system.

20.1.2 The Root Key

To do anything, the CA must have a public/private key pair. Generating this pair is straightforward. The public key needs to be distributed to every participant, together with some extra data, such as the validity period of this key. To simplify the system, this is normally done using a *self-certifying certificate*, which is a rather odd construction. The CA signs a certificate on its own public key. Although it is called self-certifying, it is nothing of the sort. The name self-certification is a historical misnomer that we are stuck with. The certificate doesn't certify the key at all, and it proves nothing about the security properties of the key, because anyone can create a public key and self-certify it. What the self-certification does is tie additional data to the public key. The permission list, validity period, human contact data, etc., are all included in the self-certificate. The self-certificate uses the same data format as all other certificates in the system, and all participants can reuse the existing code to check this additional data. The self-certificate is called the *root certificate* of the PKI.

The next step is to distribute the root certificate to all the system's participants in a secure manner. Everybody must know the root certificate, and everybody must have the right root certificate.

The first time a computer joins the PKI, it will have to be given the root certificate in a secure manner. This can be as simple as pointing the computer at a local file or a file on a trusted Web server, and telling the machine that this is the root certificate for the PKI in question. Cryptography cannot help with this initial distribution of the root certificate, because there are no keys that can be used to provide the authentication. The same situation occurs if the private key of the CA is ever compromised. Once the root key is no longer secure, an entirely new PKI structure will have to be initialized, and this involves giving every participant the root certificate in a secure manner. This should provide a good motivation for keeping the root key secure.

The root key expires after a while, and the central CA will have to issue a new key. Distributing the new root certificate is easier. The new root certificate can be signed with the old root key. Participants can download the new root certificate from an insecure source. As it is signed with the old root key, it cannot be modified. The only possible problem is if a participant does not get the new root certificate. Most systems overlap the validity of the root keys by a few months to allow sufficient time for switching to the new root key.

There is a small implementation issue here. The new CA root certificate should probably have two signatures—one with the old root key so users can recognize the new root certificate, and one (self-certifying) signature with the new root key to be used by new devices that are introduced after the old key expires. You can do this either by including support for multiple signatures in your certificate format, or by simply issuing two separate certificates for the same new root key.

20.2 The Life of a Key

Let's consider the lifetime of a single key. This can be the CA's root key or any other public key. A key goes through several phases in its life. Not all keys require all phases, depending on the application. As an example, we'll use Alice's public key.

Creation The first step in the life of a key is creation. Alice creates a public/private key pair and stores the private part in a secure manner.

Certification The next step is certification. Alice takes her public key to the CA or the sub-CA and has it certify her key. This is the point where the CA decides which permissions to give to Alice's public key.

Distribution Depending on the application, Alice might have to distribute her certified public key before she can use it. If, for example, Alice uses her key for signatures, each party that could potentially receive Alice's signature should have her public key first. The best way to do this is to distribute the key for a while before Alice uses it the first time. This is especially important for a new root certificate. When the CA switches to a new root key, for example, everybody should be given the chance to learn the new root certificate before being presented with a certificate signed with the new key.

Whether you need a separate distribution phase depends on your application. If you can avoid it, do so. A separate distribution phase has to be explained to the users and becomes visible in the user interface. That, in turn, creates lots of extra work, because many users won't understand what it means and will not use the system properly.

Active use The next phase is when Alice uses her key actively for transactions. This is the normal situation for a key.

Passive use After the active use phase, there must be a period of time where Alice no longer uses her key for new transactions, but everybody still accepts the key. Transactions are not instantaneous; sometimes they get delayed. A signed e-mail could very well take a day or two to reach its destination. Alice should stop using her key actively and allow a reasonable period for all pending transactions to be completed before the key expires.

Expired Finally, the key expires and it is not considered to be valid anymore.

How are the key phases defined? The most common solution is to include explicit times for each phase transition in the certificate. The certificate contains the start of the distribution phase, the start of the active use phase, the start of the inactive use phase, and the expiration time. Unfortunately, all of these times have to be presented to the user, because they affect the way the certificate works, and this is probably too complicated for ordinary users to handle.

A more flexible scheme is to have a central database that contains the phase of each key, but this introduces a whole new raft of security issues, which we'd rather not do. And if you have a CRL, it can override the chosen phase periods and expire a key immediately.

Things become even more complicated if Alice wants to use the same key in several different PKIs. In general, we think this is a bad idea, but sometimes it cannot be avoided. But extra precautions need to be taken if it cannot be avoided. Suppose Alice uses a small tamper-resistant module that she carries with her. This module contains her private keys and performs the necessary computations for a digital signature. Such modules have a limited storage capacity. Alice's certificates on her public key can be stored on the corporate intranet without size limitations, but the small module cannot store an unlimited number of private keys. In situations like this, Alice ends up using the same key for multiple PKIs. It also implies that the key lifetime schedule should be similar for all the PKIs Alice uses. This might be difficult to coordinate.

If you ever work on a system like this, make sure that a signature used in one PKI cannot be used in another PKI. You should always use a single digital signature scheme, such as the one explained in Section 12.7. The signed string of bytes should not be the same in two different PKI systems or in two different applications. The simplest solution is to include data in the string to be signed that uniquely identifies the application and the PKI.

20.3 Why Keys Wear Out

We've mentioned several times that keys have to be replaced regularly, but why is this?

In a perfect world, a key could be used for a very long time. An attacker who has no system weaknesses to work with is reduced to doing exhaustive searches. In theory, that reduces our problem to one of choosing large enough keys.

The real world isn't perfect. There are always threats to the secrecy of a key. The key must be stored somewhere, and an attacker might try to get at it. The key must also be used, and any use poses another threat. The key has to be transported from the storage location to the point where the relevant computations are done. This will often be within a single piece of equipment, but it opens up a new avenue of attack. If the attacker can eavesdrop on the communication channel used for this transport, then she gets a copy of the key. Then there is the cryptographic operation that is done with the key. There are no useful cryptographic functions that have a full proof of security. At their core, they are all based on arguments along the lines of: ''Well, none of us has found a way to attack this function, so it looks pretty safe.''[1] And as we have already discussed, side-channels can leak information about keys.

The longer you keep a key, and the more you use it, the higher the chance an attacker might manage to get your key. If you want to limit the chance of the attacker knowing your key, you have to limit the lifetime of the key. In effect, a key wears out.

There is another reason to limit the lifetime of a key. Suppose something untoward happens and the attacker gets the key. This breaks the security of the system and causes damage of some form. (Revocation is only effective if you find out the attacker has the key; a clever attacker would try to avoid detection.) This damage lasts until the key is replaced with a new key, and even then, data previously encrypted under the old key will remain compromised. By limiting the lifetime of a single key, we limit the window of exposure to an attacker who has been successful.

There are thus two advantages to short key lives. They reduce the chance that an attacker gets a key, and they limit the damage that is done if he nevertheless succeeds.

So what is a reasonable lifetime? That depends on the situation. There is a cost to changing keys, so you don't want to change them too often. On the other hand, if you only change them once a decade, you cannot be sure that the change-to-a-new-key function will work at the end of the decade. As a general rule, a function or procedure that is rarely used or tested is more likely to fail.[2] Probably the biggest danger in having long-term keys is that the change-key function is never used, and therefore will not work well when it is needed. A key lifetime of one year is probably a reasonable maximum.

Key changes in which the user has to be involved are relatively expensive, so they should be done infrequently. Reasonable key lifetimes are from one month and upwards. Keys with shorter lifetimes will have to be managed automatically.

[1] What is often called a ''proof of security'' for cryptographic functions is actually not a complete proof. These proofs are generally reductions: if you can break function *A*, you can also break function *B*. They are valuable in allowing you to reduce the number of primitive operations you have to assume are secure, but they do not provide a complete proof of security.

[2] This is a generally applicable truism and is the main reason you should always test emergency procedures, such as fire drills.

20.4 Going Further

Key management is not just a cryptographic problem. It is a problem of interfacing with the real world. The specific choice of which PKI to use, along with how the PKI is configured, will depend on the specifics of the application and the environment in which it is supposed to be deployed. We have outlined the key issues to consider.

20.5 Exercises

Exercise 20.1 What fields do you think should appear in a certificate, and why?

Exercise 20.2 What are the root SSL keys hard-coded within your Web browser of choice? When were these keys created? When do they expire?

Exercise 20.3 Suppose you have deployed a PKI, and that the PKI uses certificates in a certain fixed format. You need to update your system. Your updated system needs to be backward compatible with the original version of the PKI and its certificates. But the updated system also needs certificates with extra fields. What problems could arise with this transition? What steps could you have taken when originally designing your system to best prepare for an eventual transition to a new certificate format?

Exercise 20.4 Create a self-signed certificate using the cryptography packages or libraries on your machine.

Exercise 20.5 Find a new product or system that uses a PKI. This might be the same product or system that you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on the security and privacy issues surrounding the use of the PKI.

CHAPTER

21

Storing Secrets

We discussed the problem of storing transient secrets, such as session keys, back in Section 8.3. But how do we store long-term secrets, such as passwords and private keys? We have two opposing requirements. First of all, the secret should be kept secret. Second, the risk of losing the secret altogether (i.e., not being able to find the secret again) should be minimal.

21.1 Disk

One of the obvious ideas is to store the secret on the hard drive in the computer or on some other permanent storage medium. This works, but only if the computer is kept secure. If Alice stores her keys (without encryption) on her PC, then anyone who uses her PC can use her keys. Most PCs are used by other people, at least occasionally. Alice won't mind letting someone else use her PC, but she certainly doesn't want to grant access to her bank account at the same time! Another problem is that Alice probably uses several computers. If her keys are stored on her PC at home, she cannot use them while at work or while traveling. And should she store her keys on her desktop machine at home or on her laptop? We really don't want her to copy the keys to multiple places; that only weakens the system further.

A better solution would be for Alice to store her keys on her PDA or smart phone. Such a device is less likely to be lent out, and it is something that she takes with her everywhere she goes. But small devices such as these can also easily be lost or stolen, and we don't want someone later in possession of the device to have access to the secret keys.

You'd think that security would improve if we encrypt the secrets. Sure, but with what? We need a master key to encrypt the secrets with, and that master key needs to be stored somewhere. Storing it next to the encrypted secrets doesn't give you any advantage. This *is* a good technique to reduce the number and size of secrets though, and it is widely used in combination with other techniques. For example, a private RSA key is several thousand bits long, but by encrypting and authenticating it with a symmetric key, we can reduce the size of the required secure storage by a significant factor.

21.2 Human Memory

The next idea is to store the key in Alice's brain. We get her to memorize a password and encrypt all the other key material with this password. The encrypted key material can be stored anywhere—maybe on a disk, but it can also be stored on a Web server where Alice can download it to whatever computer she is using at the moment.

Humans are notoriously bad at memorizing passwords. If you choose very simple passwords, you don't get any security. There are simply not enough simple passwords for them to be really secret: the attacker can just try them all. Using your mother's maiden name doesn't work very well; her name is quite often public knowledge—and even if it isn't, there are probably only a few hundred thousand surnames that the attacker has to try to find the right one.

A good password must be unpredictable. In other words, it must contain a lot of entropy. Normal words, such as passwords, do not contain much entropy. There are about half a million English words—and that is counting all the very long and obscure words in an unabridged dictionary—so a single word as password provides at most 19 bits of entropy. Estimates of the amount of entropy per character in English text vary a bit, but are in the neighborhood of 1.5–2 bits per letter.

We've been using 256-bit secret keys throughout our systems to achieve 128 bits of security. In most places, using a 256-bit key has very little additional cost. However, in this situation the user has to memorize the password (or key), and the additional cost of larger keys is high. Trying to use passwords with 256 bits of entropy is too cumbersome; therefore, we will restrict ourselves to passwords with only 128 bits of entropy.[1]

Using the optimistic estimate of 2 bits per character, we'd need a password of 64 characters to get 128 bits of entropy. That is unacceptable. Users will simply refuse to use such long passwords.

[1] For the mathematicians: passwords chosen from a probability distribution with 128 bits of entropy.

What if we compromise and accept 64 bits of security? That is already very marginal. At 2 bits of entropy per character, we need the password to be at least 32 characters long. Even that is too long for users to deal with. Don't forget, most real-world passwords are only 6–8 letters long.

You could try to use assigned passwords, but have you ever tried to use a system where you are told that your password is ''719327582742994690518 6''? Or how about ''aoekjk3ncmakwe''? Humans simply can't remember such passwords, so this solution doesn't work. (In practice, users will write the password down, but we'll discuss that in the next section.)

A much better solution is to use a *passphrase*. This is similar to a password. In fact, they are so similar that we consider them equivalent. The difference is merely one of emphasis: a passphrase is much longer than a password.

Perhaps Alice could use the passphrase, ''Pink curtains meander across the ocean.'' That is nonsensical, but fairly easy to remember. It is also 38 characters long, so it probably contains about 57–76 bits of entropy. If Alice expands it to ''Pink dotty curtains meander over seas of Xmas wishes,'' she gets 52 characters for a very reasonable key of 78–104 bits of entropy. Given a keyboard, Alice can type this passphrase in a few seconds, which is certainly much faster than she can type a string of random digits. We rely on the fact that a passphrase is much easier to memorize than random data. Many mnemonic techniques are based on the idea of converting random data to things much closer to our passphrases.

Some users don't like to do a lot of typing, so they choose their passphrases slightly differently. How about ''Wtnitmtstsaaoof,ottaaasot,aboet''? This looks like total nonsense; that is, until you think of it as the first letters of the words of a sentence. In this case we used a sentence from Shakespeare: ''Whether 'tis nobler in the mind to suffer the slings and arrows of outrageous fortune, or to take arms against a sea of troubles, and by opposing end them.'' Of course, Alice should not use a sentence from literature; literary texts are too accessible for an attacker, and how many suitable sentences would there be in the books on Alice's bookshelf? Instead, she should invent her own sentence, one that nobody else could possibly think of.

Compared to using a full passphrase, the initial-letters-from-each-word technique requires a longer sentence, but it requires less typing for good security because the keystrokes are more random than consecutive letters in a sentence. We don't know of any estimate for the number of bits of entropy per character for this technique.

Passphrases are certainly the best way of storing a secret in a human brain. Unfortunately, many users still find it difficult to use them correctly. And even with passphrases, it is extremely difficult to get 128 bits of entropy in the human brain.

21.2.1 Salting and Stretching

To squeeze the most security out of a limited-entropy password or passphrase, we can use two techniques that sound as if they come from a medieval torture chamber. These are so simple and obvious that they should be used in every password system. There is really no excuse not to use them.

The first is to add a *salt*. This is simply a random number that is stored alongside the data that was encrypted with the password. If you can, use a 256-bit salt.

The next step is to *stretch* the password. Stretching is essentially a very long computation. Let p be the password and s be the salt. Using any cryptographically strong hash function h, we compute

$$\begin{aligned} x_0 &:= 0 \\ x_i &:= h(x_{i-1} \parallel p \parallel s) \qquad \text{for } i = 1, \ldots, r \\ K &:= x_r \end{aligned}$$

and use K as the key to actually encrypt the data. The parameter r is the number of iterations in the computation and should be as large as practical. (It goes without saying that x_i and K should be 256 bits long.)

Let's look at this from an attacker's point of view. Given the salt s and some data that is encrypted with K, you try to find K by trying different passwords. Choose a particular password p, compute the corresponding K, decrypt the data and check whether it makes sense and passes the associated integrity checks. If it doesn't, then p must have been false. To check a single value for p, you have to do r different hash computations. The larger r is, the more work the attacker has to do.

It is sometimes useful to be able to check whether the derived key is correct before decrypting the data. When this is helpful, a key check value can be computed. For example, the key check value could be $h(0 \parallel x_{r-1} \parallel p \parallel s)$, which because of the properties of hash functions is independent from K. This key check value would be stored alongside the salt and could be used to check the password before decrypting the data with K.

In normal use, the stretching computation has to be done every time a password is used. But remember, this is at a point in time where the user has just entered a password. It has probably taken several seconds to enter the password, so using 200 ms for password processing is quite acceptable. Here is our rule to choose r: choose r such that computing K from (s, p) takes 200−1000 ms on the user's equipment. Computers get faster over time, so r should be increasing over time as well. Ideally, you determine r experimentally when the user first sets the password and store r alongside s. (Do make sure that r is a reasonable value, not too small or too large.)

How much have we gained? If $r = 2^{20}$ (just over a million), the attacker has to do 2^{20} hash computations for each password she tries. Trying 2^{60} passwords would take 2^{80} hash computations, so effectively using $r = 2^{20}$ makes the effective key size of the password 20 bits longer. The larger r you choose, the larger the gain.

Look at it another way. What r does is stop the attacker from benefiting from faster and faster computers, because the faster computers get, the larger r gets, too. It is a kind of Moore's law compensator, but only in the long run. Ten years from now, the attacker can use the next decade's technology to attack the password you are using today. So you still need a decent security margin and as much entropy in the password as you can get.

This is another reason to use a key negotiation protocol with forward secrecy. Whatever the application, it is quite likely that Alice's private keys end up being protected by a password. Ten years from now, the attacker will be able to search for Alice's password and find it. But if the key that is encrypted with the password was only used to run a key negotiation protocol with forward secrecy, the attacker will find nothing of value. Alice's key is no longer valid (it has expired), and knowing her old private key does not reveal the session keys used ten years ago.

The salt stops the attacker from taking advantage of an economy of scale when she is attacking a large number of passwords simultaneously. Suppose there are a million users in the system, and each user stores an encrypted file that contains her keys. Each file is encrypted with the user's stretched password. If we did not use a salt, the attacker could attack as follows: guess a password p, compute the stretched key K, and try to decrypt each of the key files using K. The stretch function only needs to be computed once for every password, and the resulting stretched key can be used in an attempt to decrypt each of the files.

This is no longer possible when we add the salt to the stretching function. All the salts are random values, so each user will use a different salt value. The attacker now has to compute the stretching function once for each password/file combination, rather than once for each password. This is a lot more work for the attacker, and it comes at a very small price for the users of the system. Since bits are cheap, for simplicity we suggest using a 256-bit salt.

By the way, do take care when you do this. We once saw a system that implemented all this perfectly, but then some programmer wanted to improve the user interface by giving the user a faster response as to whether the password he had typed was correct or not. So he stored a checksum on the password, which defeated the entire salting and stretching procedure. If the response time is too slow, you can reduce r, but make sure there is no way to recognize whether a password is correct or not without doing at least *r* hash computations.

21.3 Portable Storage

The next idea is to store key material outside the computer. The simplest form of storage is a piece of paper with passwords written on it. Most people have that in one form or another, even for noncryptographic systems like websites. Most users have at least half a dozen passwords to remember, and that is simply too much, especially for systems where you use your password only rarely. So to remember passwords, users write them down. The limitation to this solution is that the password still has to be processed by the user's eyes, brain, and fingers every time it is used. To keep user irritation and mistakes within reasonable bounds, this technique can only be used with relatively low-entropy passwords and passphrases.

As a designer, you don't have to design or implement anything to use this storage method. Users will use it for their passwords, no matter what rules you make and however you create your password system.

A more advanced form of storage would be portable memory of some form. This could be a memory-chip card, a magnetic stripe card, a USB stick, or any other kind of digital storage. Digital storage systems are always large enough to store at least a 256-bit secret key, so we can eliminate the low-entropy password. The portable memory becomes very much like a key. Whoever holds the key has access, so this memory needs to be held securely.

21.4 Secure Token

A better—and more expensive—solution is to use something we call a *secure token*. This is a small computer that Alice can carry around. The external shape of tokens can differ widely, ranging from a smart card (which looks just like a credit card), to an iButton, USB dongle, or PC Card. The main properties are nonvolatile memory (i.e., a memory that retains its data when power is removed) and a CPU.

The secure token works primarily as a portable storage device, but with a few security enhancements. First of all, access to the stored key material can be limited by a password or something similar. Before the secure token will let you use the key, you have to send it the proper password. The token can protect itself against attackers who try a brute-force search for the password by disabling access after three or five failed attempts. Of course, some users mistype their password too often, and then their token has to be resuscitated, but you can use longer, higher-entropy passphrases or keys that are far more secure for the resuscitation.

This provides a multilevel defense. Alice protects the physical token; for example, by keeping it in her wallet or on her key chain. An attacker has to steal the token to get anywhere, or at least get access to it in some way. Then the attacker needs to either physically break open the token and extract the data, or find the password to unlock the token. Tokens are often tamper-resistant to make a physical attack more difficult.[2]

Secure tokens are currently one of the best and most practical methods of storing secret keys. They can be relatively inexpensive and small enough to be carried around conveniently.

One problem in practical use is the behavior of the users. They'll leave their secure token plugged into their computer when going to lunch or to a meeting. As users don't want to be prompted for their password every time, the system will be set to allow hours of access from the last time the password was entered. So all an attacker has to do is walk in and start using the secret keys stored in the token.

You can try to solve this through training. There's the ''corporate security in the office'' video presentations, the embarrassingly bad ''take your token to lunch'' poster that isn't funny at all, and the ''if I ever again find your token plugged in unattended, you are going to get another speech like this'' speeches. But you can also use other means. Make sure the token is not only the key to access digital data, but also the lock to the office doors, so users have to take their token to get back into their office. Fix the coffee machine to only give coffee after being presented with a token. These sorts of tactics motivate employees to bring their token to the coffee machine and not leave it plugged into their computer while they are away. Sometimes security consists of silly measures like these, but they work far better than trying to enforce take-your-token-with-you rules by other means.

21.5 Secure UI

The secure token still has a significant weakness. The password that Alice uses has to be entered on the PC or some other device. As long as we trust the PC, this is not a problem, but we all know PCs are not terribly secure, to say the least. In fact, the whole reason for not storing Alice's keys on the PC is because we don't trust it enough. We can achieve a much better security if the token itself has a secure built-in UI. Think of a secure token with a built-in keyboard and display. Now the password, or more likely a PIN, can be entered directly into the token without the need to trust an outside device.

[2] They are tamper-resistant, not tamper-proof; tamper-resistance merely makes tampering more expensive. Tamper-responding devices may detect tampering and self-destruct.

Having a keyboard on the token protects the PIN from compromise. Of course, once the PIN has been typed, the PC still gets the key, and then it can do anything at all with that key. So we are still limited by the security of the whole PC.

To stop this, we have to put the cryptographic processes that involve the key into the token. This requires application-specific code in the token. The token is quickly growing into a full-fledged computer, but now a trusted computer that the user carries around. The trusted computer can implement the security-critical part of each application on the token itself. The display now becomes crucial, since it is used to show the user what action he is authorizing by typing his PIN. In a typical design, the user uses the PC's keyboard and mouse to operate the application. When, for example, a bank payment has to be authorized, the PC sends the data to the token. The token displays the amount and a few other transaction details, and the user authorizes the transaction by typing her PIN. The token then signs the transaction details, and the PC completes the rest of the transaction.

At present, tokens with a secure UI are too expensive for most applications. Maybe the closest thing we have is a PDA or smart phone. However, people download programs onto their PDAs and phones, and these devices are not designed from the start as secure units, so perhaps these devices are not significantly more secure than a PC. We hope that tokens with secure UIs become more prevalent in the future.

21.6 Biometrics

If we want to get really fancy, we can add biometrics to the mix. You could build something like a fingerprint or iris scanner into the secure token. At the moment, biometric devices are not very useful. Fingerprint scanners can be made for a reasonable price, but the security they provide is generally not very good. In 2002, cryptographer Tsutomu Matsumoto, together with three of his students, showed how he was able to consistently fool all the commercially available fingerprint scanners he could buy, using only household and hobby materials [87]. Even making a fake finger from a latent fingerprint (i.e., the type you leave on every shiny surface) is nothing more than a hobby project for a clever high-school student.

The real shock to us wasn't that the fingerprint readers could be fooled. It was that fooling them was so incredibly simple and cheap. What's worse, the biometrics industry has been telling us how secure biometric identification is. They never told us that forging fingerprints was this easy. Then suddenly a mathematician (not even a biometrics expert) comes along and blows the whole process out of the water. A recent 2009 paper shows that these issues are still a problem [3].

Still, even though they are easy to fool, fingerprint scanners can be very useful. Suppose you have a secure token with a small display, a small keyboard, and a fingerprint scanner. To get at the key, you need to get physical control of the token, get the PIN, and forge the fingerprint. That is more work for the attacker than any of our previous solutions. It is probably the best practical key storage scheme that we can currently make. On the other hand, this secure token is going to be rather expensive, so it won't be used by many people.

Fingerprint scanners could also be used on the low-security side rather than the high-security side. Touching a finger to a scanner can be done very quickly, and it is quite feasible to ask the user to do that relatively often. A fingerprint scanner could thus be used to increase the confidence that the proper person is in fact authorizing the actions the computer is taking. This makes it more difficult for employees to lend their passwords to a colleague. Rather than trying to stop sophisticated attackers, the fingerprint scanner could be used to stop casual breaches of the security rules. This might be a more important contribution to security than trying to use the scanner as a high-security device.

21.7 Single Sign-On

Because the average user has so many passwords, it becomes very appealing to create a single sign-on system. The idea is to give Alice a single master password, which in turn is used to encrypt all the different passwords from her different applications.

To do this well, all the applications must talk to the single sign-on system. Any time an application requires a password, it should not ask the user, but rather the single sign-on program, for it. There are numerous challenges for making this a reality on a wide scale. Just think of all the different applications that would have to be changed to automatically get their passwords from the single sign-on system.

A simpler idea is to have a small program that stores the passwords in a text file. Alice types her master password and then uses the copy and paste functionality to copy the passwords from the single sign-on program to the application. Bruce designed a free program called Password Safe to do exactly this. But it's just an encrypted digital version of the piece of paper that Alice writes her passwords on. It is useful, and an improvement on the piece of paper if you always use the same computer, but not the ultimate solution that the single sign-on idea would really like to be.

21.8 Risk of Loss

But what if the secure token breaks? Or the piece of paper with the passwords is left in a pocket and run through the washing machine? Losing secret keys is always a bad thing. The cost can vary from having to reregister for each application to get a new key, to permanently losing access to important data. If you encrypt the PhD thesis you have been working on for five years with a secret key and then lose the key, you no longer have a PhD thesis. You just have a file of random-looking bits. Ouch!

It is hard to make a key storage system both easy to use and highly reliable. A good rule of thumb, therefore, is to split these functions. Keep two copies of the key—one that is easy to use, and another one that is very reliable. If the easy-to-use system ever forgets the key, you can recover it from the reliable storage system. The reliable system could be very simple. How about a piece of paper in a bank vault?

Of course, you want to be careful with your reliable storage system. By design, it will quickly be used to store all of your keys, and that would make it a very tempting target for an attacker. You'll have to do a risk analysis to determine whether it is better to have a number of smaller reliable key storage places or a single large one.

21.9 Secret Sharing

There are some keys that you need to store super-securely—for example, the private root key of your CA. As we have seen, storing a secret in a secure manner can be difficult. Storing it securely and reliably is even more difficult.

There is one cryptographic solution that can help in storing secret keys. It is called *secret sharing* [117], which is a bit of a misnomer because it implies that you share the secret with several people. You don't. The idea is to take the secret and split it into several different shares. It is possible to do this in such a way that, for example, three out of the five shares are needed to recover the secret. You then give one share to each of the senior people in the IT department. Any three of them can recover the secret. The real trick is to do it in a manner such that any two people together know absolutely nothing about the key.

Secret sharing systems are very tempting from an academic point of view. Each of the shares is stored using one of the techniques we talked about before. A k-out-of-n rule combines high security (at least k people are necessary to retrieve the key) with high reliability ($n - k$ of the shares may be lost without detrimental effect). There are even fancier secret sharing schemes that allow more complex access rules, along the lines of (Alice and Bob) or (Alice and Carol and David).

In real life, secret sharing schemes are rarely used because they are too complex. They are complex to implement, but more importantly, they are complex to administrate and operate. Most companies do not have a group of highly responsible people who distrust each other. Try telling the board members that they will each be given a secure token with a key share, and that they will have to show up at 3 a.m. on a Sunday in an emergency. Oh yes, and they are not to trust each other, but to keep their own shares secure even from other board members. They will also need to come down to the secure key-management room to get a new key share every time someone joins or leaves the board. In practice, this means that using the board members is out. The CEO isn't very useful for holding a share either, because the CEO tends to travel quite a bit. Before you know it, you are down to the two or three senior IT management people. They could use a secret sharing scheme, but the expense and complexity make this unattractive. Why not use something much simpler, such as a safe? Physical solutions such as safes or bank vaults have several advantages. Everybody understands how they work, so you don't need extensive training. They have already been tested extensively, whereas the secret-reconstruction process is hard to test because it requires such a large number of user interactions—and you really don't want to have a bug in the secret-reconstruction process that results in you losing the root key of your CA.

21.10 Wiping Secrets

Any long-term secret that we store eventually has to be wiped. As soon as a secret is no longer needed, its storage location should be wiped to avoid any future compromise. We discussed the problems of wiping memory in Section 8.3. Wiping long-term secrets from permanent storage is much harder.

The schemes for storing long-term secrets that we discussed in this chapter use a variety of data storage technologies, ranging from paper to hard disks to USB sticks. None of these storage technologies comes with a documented wiping functionality that guarantees the data it stored is no longer recoverable.

21.10.1 Paper

Destroying a password written down on paper is typically done by destroying the paper itself. One possible method is to burn the paper, and then grind the ashes into a fine powder, or mix the ashes into a pulp with just a little bit of water. Shredding is also an option, although many shredders leave the paper in large enough pieces that reconstructing a page is relatively easy.

21.10.2 Magnetic Storage

Magnetic media are very hard to wipe. There is surprisingly little literature about how to do this; the best paper we know of is by Peter Gutmann [57], although the technical details of that paper are probably outdated now.

Magnetic media store data in tiny magnetic domains; the direction of magnetization of a domain determines the data it encodes. When the data is overwritten, the magnetization directions are changed to reflect the new data. But there are several mechanisms that prevent the old data from being completely lost. The read/write head that tries to overwrite old data is never exactly aligned and will tend to leave some parts of the old data untouched. Overwriting does not completely destroy old data. You can think of it as repainting a wall with a single coat of paint. You can still vaguely see the old coat of paint under it. The magnetic domains can also migrate away from the read/write head either to the side of the track or deeper down into the magnetic material, where they can linger for a long time. Overwritten data is typically not recoverable with the normal read/write head, but an attacker who takes apart a disk drive and uses specialized equipment might be able to retrieve some or all of the old data.

In practice, repeatedly overwriting a secret with random data is probably the best option. There are a few points to keep in mind:

- Each overwrite should use fresh random data. Some researchers have developed particular data patterns that are supposed to be better at wiping old data, but the choice of patterns depends on the exact details of the disk drive. Random data might require more overwriting passes for the same effect, but it works in all situations and is therefore safer.

- Overwrite the actual location that stored the secret. If you just change a file by writing new data to it, the file system might decide to store the new data in a different location, which would leave the original data intact.

- Make sure that each overwrite pass is actually written to disk and not just to one of the disk caches. Disk drives that have their own write-cache are a particular danger, as they might cache the new data and optimize the multiple overwrite operations into a single write.

- It is probably a good idea to wipe an area that begins well before the secret data and that ends well after it. Because the rotational speed of a disk drive is never perfectly constant, the new data will not align perfectly with the old data.

As far as we know, there is no reliable information on how many overwrite passes are required, but there is no reason to choose a small number. You only have to wipe a single key. (If you have a large amount of secret data, store that data encrypted under a key, and only wipe the key.) We consider 50 or 100 overwrites with random data perfectly reasonable.

It is theoretically possible to erase a tape or disk using a degaussing machine. However, modern high-density magnetic storage media resist degaussing to such an extent that this is not a reliable wiping method. In practice, users do not have access to degaussing machines, so this is a nonissue.

Even with extensive overwriting, you should expect that a highly specialized and well-funded attacker could still recover the secret from the magnetic medium. To completely destroy the data, you will probably have to destroy the medium itself. If the magnetic layer is bonded to plastic (floppy disk, tape), you can consider shredding and then burning the media. For a hard disk, you can use a belt sander to remove the magnetic layer from the platters, or use a blowtorch to melt the disk platters down to liquid metal. In practice, you are unlikely to convince users to take such extreme measures, so repeated overwriting is the best practical solution.

21.10.3 Solid-State Storage

Wiping nonvolatile memory, such as EPROM, EEPROM, and flash, poses similar problems. Overwriting old data does not remove all traces, and the data retention mechanisms we discussed in Section 8.3.4 are also at work. Again, repeatedly overwriting the secret with random data is the only practical solution, but it is by no means perfect. As soon as the solid-state device is no longer needed, it should be destroyed.

21.11 Exercises

Exercise 21.1 Investigate how login passwords are stored on your machine. Write a program that, given a stored (encrypted or hashed) password, exhaustively searches for the real password. How long would it take your program to exhaustively search through the top one million passwords?

Exercise 21.2 Investigate how private keys are stored with GNU Privacy Guard (GPG). Write a program that, given a stored encrypted GPG private key, exhaustively searches for the password and recovers the private key. How long would it take your program to exhaustively search through the top one million passwords?

Exercise 21.3 Consider a 24-bit salt. Given a group of 64 users, would you expect two users to have the same salt? 1024 users? 4096 users? 16,777,216 users?

Exercise 21.4 Find a new product or system that maintains long-term secrets. This might be the same product or system you analyzed for Exercise 1.8. Conduct a security review of that product or system as described in Section 1.12, this time focusing on issues surrounding how the system might store these secrets.

Part V

Miscellaneous

In This Part

CHAPTER

22

Standards and Patents

Aside from cryptography itself, there are two things you really should know about: standards and patents. We offer our perspectives on both here.

22.1 Standards

Standards are a double-edged sword. On one hand, no one will fault you for using a standard. We said this in the context of AES. On the other hand, many security standards don't work. This is a conundrum. We focus on the engineering aspects of cryptography in this book. But if you do any cryptographic engineering, you are going to encounter standards. So you need to know a bit about them, and we consider them now.

22.1.1 The Standards Process

For those who have not been involved in the standards development process, we'll first describe how many standards are made. It starts out with some standardization body, such as the Internet Engineering Task Force (IETF), the Institute of Electrical and Electronics Engineers (IEEE), the International Organization for Standardization (ISO), or the European Committee for Standardization (CEN). This standardization body sets up a committee in response to some perceived need for a new or improved standard. The committee goes by different names: working group, task group, or whatever. Sometimes there are hierarchical structures of committees, but the basic idea remains the same. Committee membership is typically voluntary. People apply to join, and pretty much anyone is accepted. Often there are some procedural hoops to jump through, but there is no significant selection of members. These committees vary in size up to several hundred members, but big committees split themselves into smaller subcommittees (called task groups, study groups, or whatever). Most work is done in a committee of up to a few dozen people.

Standardization committees have regular meetings, once every few months. All members travel to a city and meet in a hotel for a few days. In the months between meetings, members of the committee will do some work, create proposals and presentations, etc. At the meetings, the committee decides which way to proceed. Usually there is a single editor who gets the job of putting all the proposals together into a single standards document. Creating a standard is a slow process and often takes many years.

So who turns up to join these committees? Well, being a member is expensive. Apart from the time it takes, the travel and hotels are not cheap. So everybody there is sent by their company. Companies have several motivations to be represented. Sometimes they want to sell products that must interoperate with products from other companies. This requires standards, and the best way to keep track of the standardization process is to be there. Companies also want to keep an eye on their competitors. You don't want to let your competitor write the standards, because they will do something to put you at a competitive disadvantage—perhaps skew it toward their own technology or requirements or include techniques for which they themselves hold a patent. Sometimes companies don't want a standard, so they show up at the committee meetings to try to slow the process down to allow their proprietary solution time to capture the market. In real life, all these motivations, plus several more, are all mixed together in varying proportions to create a very complex political environment.

Not surprisingly, quite a number of standardization committees fail. They never produce anything, or produce something atrociously bad, or end up being deadlocked and overtaken by the market, and then define whatever system captured the market. Successful committees manage to produce a standards document after a few years.

Once the standard has been written, everybody goes and implements it. This, of course, leads to systems that do not interoperate, so there is a secondary process where interoperability is tested and the different manufacturers adapt their implementations to work together.

There are many problems with this process. The political structure of the committee puts very little emphasis on creating a good technical standard. The most important thing is to reach consensus. The standard is finished when everybody is equally *un*happy with the result. To pacify the different factions, standards have many options, extended functionalities, useless alternatives, etc. And as each faction has its own ideas, opinions, and focus points, the best compromise is often contradictory. Many standards are internally inconsistent, or even contradict themselves.

This whole process is made even more complex by the fact that companies are creating implementations while the standardization process is still going on, based on drafts of the standard. This makes it even harder to change something, because somebody has already implemented it and doesn't want to do it again. Of course, different companies will implement things in different ways and then fight in the committee to get the standard adjusted to fit their implementation. Sometimes the only compromise is to choose something that no company has implemented, just to ensure that they are all equally unhappy. Technical merit does not really feature in this type of discussion.

22.1.1.1 The Standard

One of the results is that most standards are extremely difficult to read. The standards document is a design by committee, and there is little pressure within the process to make the document clear, concise, accurate, or readable. In fact, a highly unreadable document is easier to work with, because only a few of the committee members will understand it, and they can work on it without being bothered by the other members. Digging through hundreds of pages of badly written documentation is no fun, so most committee members end up not reading the full draft and only checking the limited portions of the standard they are interested in.

22.1.1.2 Functionality

As we already mentioned, interoperability testing is always required. And of course, different companies implement different things. Quite often, what ends up being implemented is subtly different from what the standard defines, and as each company is already marketing its products, it is sometimes too late to change things. We've seen products of brand *A* that recognize the implementations of brand *B* by their deviations from the standard, and then adjust their own behavior to make things work.

Standards often include a very large number of options, but the actual implementations will only use a particular set of options, with a few restrictions and extensions, of course, because the standards document itself describes something that doesn't work. And the difference between the actual implementations and the standard are, of course, not documented.

Overall, the entire process works—kind of—but only for central functionality. A wireless network will allow you to connect, but management functionality is unlikely to work across products from different vendors. Simple HTML pages will display correctly on all browsers, but more advanced layout features give different results on different browsers. We've all become so used to this that we hardly notice it.

This situation is unfortunate. As an industry, we seem unable to create standards that are even readable or correct, let alone provide interoperability of different products for all but the most basic functionality.

22.1.1.3 Security

These failings mean that the typical method of producing standards simply doesn't work for security purposes. In security, we have an active attacker who will seek out the most remote corner of the standard. Security also depends on the weakest link: any single mistake can be fatal.

We've already hammered on the importance of simplicity. Standards are anything but simple. The committee process precludes simplicity and invariably produces something that is more complex than anyone in the committee can fully understand. For that reason alone, the result can never be secure.

When we've spoken to some standardization people about this problem, we often get responses along the lines of: ''Well, the techies always want to make a perfect standard.'' ... ''Political realities are that we have to make a compromise.'' ... ''That is just how the system works.'' ... ''Look at what we have achieved.'' ... ''Things are working pretty well.'' In security, that is not good enough. The very fact that interoperability testing is required after the standard has been set demonstrates that committee standards don't work in security. If the functional part of the standard (i.e., the easy part) isn't good enough to result in interoperable systems without testing, then the security part cannot possibly achieve security without testing. And as we know, it isn't possible to test for security. Sure, it might be possible to create an implementation that includes a subset of the functionality of the standard that is also secure, but that is not sufficient for a security standard. A security standard claims that if you adhere to it, you will achieve a certain level of security. Security is simply too difficult to leave to a committee. So whenever someone suggests using a committee-written cryptography standard, we are always extremely reluctant to do so.

There are a few useful standards in this field, none of which was written by a committee. Sometimes you get just a small group of people who create a single coherent design. And sometimes the result is adopted as a standard without a lot of political compromises. These standards can be quite good. We'll discuss the most important two below.

22.1.2 SSL

SSL is the security protocol used by Web browsers to connect securely to Web servers. The first widely used version was SSL 2, which contained several security flaws. The improved version is known as SSL 3 [53]. It was designed by three people without any further committee process. SSL 3 has been widely used and is generally acknowledged as a good protocol.

A warning: SSL is a good protocol, but that does not mean that any system that uses SSL is secure. SSL relies on a PKI to authenticate the server, and the PKI client embedded in most browsers is so permissive that the overall security level is rather low. One of our browsers has approximately 150 different root certificates from 70 different CAs. So even before we start looking at active attacks, there are 35 different organizations spread throughout the world that we have to trust with all of our Web information.

SSL was never really standardized. It was simply implemented by Netscape, and became a de facto standard. Standardization and further development of SSL is being done under the name TLS by an IETF working group. The changes between SSL and TLS seem fairly minor, and we have no reason to believe that TLS is not as good as SSL 3. But given the IETF's recent record with designing security protocols such as IPsec [51], there is a danger of the committee effect once again asserting itself and unnecessarily complicating a good standard.

22.1.3 AES: Standardization by Competition

To us, AES is the shining example of how to standardize security systems. AES is not a design by committee, but rather a design by competition. The new SHA-3 standardization process is proving to be very similar. The process is rather simple. First you specify what the system is supposed to achieve. Developing the specifications can be done in a reasonably small group with inputs from many different sources.

The next step is a call for proposals. You ask experts to develop complete solutions that satisfy the given requirements. Once the proposals are in, all that remains is to choose among the proposals. This is a straightforward competition in which you judge them by a variety of criteria. As long as you make security the primary criterion, the submitters have a vested interest in finding security weaknesses in their competitors' proposals. With a bit of luck, this will lead to useful feedback. In other situations, you might have to pay to get security evaluations done by outside experts.

In the end, if things go right, you will be able to select a single proposal, either unchanged or with minor changes. This is not the time to make an amalgamation of the different proposals; that will just lead to another committee design. If none of the proposals satisfies the requirements, and it seems possible to create something better, you should probably ask for new proposals.

This is exactly how NIST ran the AES competition, and it worked incredibly well. The 15 original proposals were evaluated in a first round and reduced to five finalists. A second round of evaluations on the finalists led to the selection of the winner. Amazingly enough, any one of the five finalists would have been a good standard, and certainly a better standard than any committee design. Our main concern is that the AES standardization process may have been just a little too quick, and there might not have been enough time to thoroughly analyze all the finalists. But still the process was very good.

The competition model of standardization only works if you have enough experts to create at least a few competing designs. But in our opinion, if you don't have enough experts to generate several proposals, you should not be standardizing any security systems. For reasons of simplicity and consistency, which are crucial to the overall security, a security system must be designed by a small group of experts. Then you need other experts to analyze the proposal and attack it, looking for flaws. To have any hope of getting a good result—whatever process you use—you need enough experts to form at least three proposal groups. If you have that many experts, you should use the competition model, as it is a model that has demonstrated it can produce good security standards.

22.2 Patents

We had a long discussion about the role of patents in cryptography in the first edition of this book. The ecosystem surrounding patents has changed somewhat since then, and we've also learned more. In this book, we're going to refrain from offering specific advice about patents. But we do want to make you aware of the fact that patents play a role in cryptography. Patents may affect which cryptographic protocols you choose to use or not use. We suggest you contact your lawyer for specific advice regarding patents.

CHAPTER 23

Involving Experts

There is something strange about cryptography: everybody thinks they know enough about it to design and build their own system. We never ask a second-year physics student to design a nuclear power plant. We wouldn't let a trainee nurse who claims to have found a revolutionary method for heart surgery operate on us. Yet people who have read a book or two think they can design their own cryptographic system. Worse still, they are sometimes able to convince management, venture capitalists, and even some customers that their design is the neatest thing since sliced bread.

Among cryptographers, Bruce's first book, *Applied Cryptography* [111, 112], is both famous and notorious. It is famous for bringing cryptography to the attention of tens of thousands of people. It is notorious for the systems that these people then designed and implemented on their own.

A recent example is 802.11, the wireless network standard. The initial design included a secure channel called *wired equivalent privacy* (WEP) to encrypt and authenticate wireless communications. The standard was designed by a committee without any cryptographers. The results were horrible from a security perspective. The decision to use the RC4 encryption algorithm was not the best one, but not a fatal flaw in and of itself. However, RC4 is a stream cipher and needs a unique nonce. WEP didn't allocate enough bits for the nonce, with the result that the same nonce value had to be reused, which in turn resulted in many packets being encrypted with the same key stream. That defeated the encryption properties of the RC4 stream cipher and allowed a smart attacker to break the encryption. A more subtle flaw was not hashing the secret key and nonce together before using it as the

RC4 key, which eventually led to key-recovery attacks [52]. A CRC checksum was used for authentication, but since CRC computations are linear, it was trivial (using some linear algebra) to modify any packet without any chance of detection. A single shared key was used for all users in a network, making key updates much more difficult to do. The network password was used directly as the encryption key for all communications, without using any kind of key negotiation protocol. And finally, encryption was turned off by default, which meant that most implementations never bothered turning encryption on in the first place. WEP wasn't just broken; it was robustly broken.

Designing a replacement for WEP was not easy, because it had to be retrofitted to existing hardware. But there was no choice; the security of the original standard was abysmal. The replacement became WPA.

The WEP story is not exceptional. It got more press than most bad cryptographic designs because 802.11 is such a successful product, but we have seen many similar situations in other systems. As a colleague once told Bruce: ''The world is full of bad security systems designed by people who have read *Applied Cryptography*.''

Cryptography Engineering could have the same effect.

That makes this a very dangerous book. Some people will read this book, and then turn around and design a cryptographic algorithm or protocol. When they're finished, they'll have something that looks good to them, and maybe even works, but will it be secure? Maybe they'll get 70% right. If they're very lucky, they may get 90% right. But there is no prize for being almost right in cryptography. A security system is only as strong as its weakest link; to be secure, *everything* must be right. And that is something you simply can't learn from reading books.

So why did we write this book if there's a chance it will lead to bad systems? We wrote it because people who want to learn how to design cryptographic systems must learn it somewhere, and we didn't know of any other suitable books. Consider this book as an introduction to the field, even though it is not a manual. We also wrote it for the other engineers involved in a project. Every part of a security system is of critical importance, and everybody who works on a project has to have a basic understanding of the security issues and techniques involved. This includes the programmers, testers, technical writers, management, and even salespeople. Each person needs to understand enough about security issues to do his or her work properly. We hope this book provides an adequate background to the practical side of cryptography.

We also hope we've instilled in you a sense of professional paranoia. If you've learned that, then you've learned a lot. You can apply professional paranoia to all aspects of your work. You will be extremely skeptical when you design your own protocol or look at someone else's, and that can only help improve security.

If we can leave you with one piece of advice, it is to use cryptographic experts if at all possible. If your project involves cryptography, you will benefit greatly from the insights of an experienced cryptographic designer. Involve one in your project at the beginning. The earlier you consult a cryptographic expert, the cheaper and easier it will be in the long run. Many a time we've been called to projects well underway, only to poke holes in parts that had long since been designed or implemented. The end result is always expensive, either in terms of effort, project schedule, and cost, or in terms of the security for the user of the end product.

Cryptography is fiendishly difficult to do right. Even the systems designed by experts fail regularly. It doesn't matter how clever you are, or how much experience you have in other fields. Designing and implementing cryptographic systems requires specialized knowledge and experience, and the only way to get experience is to do it over and over again. And that, of course, also involves making mistakes. So why get an expert if he makes mistakes as well? For the same reason you get a qualified surgeon to operate on you. It is not that they don't make mistakes; it is that they make a lot fewer and less serious mistakes. And they work in a conservative manner so that the small mistakes do not lead to catastrophic results; they know enough to fail well.

Implementing cryptographic systems is almost as much a specialty as designing them is. Cryptographic designers are available for hire. Cryptographic implementers are much harder to come by, in part because you need more of them. A single designer can create work for ten to twenty implementers. Most people don't think of cryptographic implementation as a specialty. Programmers will move from database programming to GUI work to cryptographic implementations. It's true that database programming and GUI work are also specialties, but an experienced programmer can, with a bit of study, get reasonable results. This does not hold for implementing cryptography, where everything must be right, and there's an attacker trying to make it wrong.

The best way we know to implement cryptographic systems is to take competent programmers and train them for this task. This book might be part of their training, but mostly it requires experience and the right professional paranoia mindset. And just like any other specialized IT skill, it takes years before someone is truly good at it. Given the long time it takes to gather this experience, you must be able to retain these people once they achieve it. That's another problem, and one we will gladly leave to others to solve.

Maybe even more important than this book, or any other, is the project culture. ''Security first'' should not just be a slogan; it has to be woven into the very fabric of the project and the project team. Everybody has to live, breathe, talk, and think security all the time. This is incredibly hard to achieve, but it can be done. DigiCash had a team like that in the 1990s. The aviation industry has a similarly pervasive safety culture. This is something that cannot be achieved in the short term, but it is certainly something that you can strive toward. This book is merely a primer on the most important security issues intended for the more technical people on the team.

As Bruce wrote in *Secrets and Lies*: ''Security is a process, not a product.'' In addition to the security culture, you also need a security process. The aviation industry has an extensive safety process. Most of the IT industry doesn't even have a process for producing software, let alone a process for high-quality software, much less a process for security software. Writing good security software is largely beyond the current state of the art in our industry. That does not mean we should give up, though, and there has been some progress lately. As information technology becomes more and more critical to our infrastructure, our freedom, and our safety, we *must* continue to improve the security of our systems. We have to do the best we can.

We hope this book can contribute somewhat to the improvement of our security systems by teaching those who are working on security systems the basics of practical cryptography.

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